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Hierarchies Based on the Number of Cooperating Systems of Three-Dimensional Finite Automata

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Abstract

The question of whether processing three-dimensional digital patterns is much more difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. Recently, due to the advances in many application areas such as computer vision, robotics, and so forth, it has become increasingly apparent that the study of three-dimensional pattern processing has been of crucial importance. Thus, the study of three-dimensional automata as a computational model of three-dimensional pattern processing has been meaningfull. This paper introduces a cooperating system of three-dimensional finite automata as one model of three-dimensional automata. A cooperating system of three-dimensional finite automata consists of a finite number of three-dimensional finite automata and a three-dimensional input tape where these finite automata work independently (in parallel). Those finite automata whose input heads scan the same cell of the input tape can communicate with each other, that is, every finite automaton is allowed to know the internal states of other finite automata on the same cell it is scanning at the moment. In this paper, we continue the study of cooperating systems of three-dimensional finite automata, and mainly investigate hierarchies based on the number of their cooperating systems.

Key Words **:** computational complexity, cooperating system, finite automaton, hierarchy, three-dimension

1 Introduction

It has become increasingly apparent that the study of three-dimensional pattern processing has been of crucial importance. Thus, the study of three-dimensional automata as a computational model of three-dimensional pattern processing has also been meaningful. For example, in 1982, three-dimensional finite automata were introduced.

A cooperating system of three-dimensional finite automata (*CS*-2-*FA*) [2,3,4] consists of a finite number of three-dimensional finite automata and a three-dimensional input tape where these finite automata work independently (in parallel). Those finite automata whose input heads scan the same cell of the input tape can communicate with each other, that is, every finite automaton is allowed to know the internal states of other finite automata on the same cell it is scannig at the moment.

In this paper, we propose a cooperating system of five-way three-dimensional finite automata (*CS*-*FV*3-*FA*) which is a resticted version of *CS*-3-*FA*'s, and mainly investigate the hierarchies can be obtained by varying the number of finite automata in the system for classes of sets accepted by *CS*-*FV*3-*FA*'s and *CS*-3-*FA*'s.The five-way three-dimensional finite automaton [7] is a three-dimensional finite automaton [1] whose input head can move east, west, south, north, or down, but not up.

2 Preliminaries

Definition 2.1. Let Σ be a finite set of symbols. A *three-dimesional tape* over Σ is a three-dimensional rectangular array of elements of Σ. The set of all three-dimesional tapes over Σ is denoted by $\Sigma^{(3)}$. Given a tape $x \in \Sigma^{(3)}$, for each integer $j (1 \leq j \leq 3)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x)=n_1$, $l_2(x)=n_2$, and $l_3(x)=n_3$ is denoted by $\Sigma^{(n_1,n_2,n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each $j (1 \leq j \leq 3)$, let $x(i_1, i_2, i_3)$ denote the symbol in x with coordinates (i_1, i_2, i_3). Furthermore, we define

$$x[(i_1, i_2, i_3), (i'_1, i'_2, i'_3)],$$

when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j (1 \leq j \leq 3)$, as the three-dimensional input tape y satisfying the following conditions :

(i)for each $j(1\leq j \leq 3)$, $l_j(y)=i'_j\text{-}i_j+1$;

(ii)for each $r_1, r_2, r_3 (i \leq r_1 \leq l_1(y),\ i \leq r_2 \leq l_2(y),\ i \leq r_3 \leq l_3(y))$, $y(r_1, r_2, r_3)=x(r_1+i_1\text{-}1,\ r_2+i_2\text{-}1,\ r_3+i_3\text{-}1)$. (We call $x[(i_1, i_2, i_3),\ (i'_1, i'_2, i'_3)]$ the $[(i_1, i_2, i_3),\ (i'_1, i'_2, i'_3)]$-*segment* of x.)

We recall a *five-way three-dimensional simple k-head finite automaton* (*FV*3-*SPk*-*HFA*) [5,6]. An *FV*3-*SPk*-*HFA* M is a finite automaton with k read-only input heads operating on a three-dimensional input tape surrounded by boundary symbols #'s. The only one head (called the *'reading'* head) of M is capable of distinguishing the symbols in the input alphabet, and the other heads (called the *'counting'* heads) of M can only detect whether they are on the boundary symbols or a symbol in the input alphabet. When an input tape x is presented to M, M determines the next state of the finite control, the next move direction (east, west, south, nouth, down, or no move) of each input head, depending on the present state of the finite control, the symbol read by the reading head, and on whether or not the symbol read by each counting head is boundary symbol. We say that M *accepts* x if M, when started in its initial state with all its input heads on $x(1,1,1)$, eventually halts in an accepting state with all its heads on the bottom boundary symbols of x. As usual, we define nondeterministic and deterministic *FV*3-*SPk*-*HFA*'s.

A *five-way three-dimensional sensing simple k-head finite automaton* (*FV*3-*SNSPk*-*HFA*) is the same device as an *FV*3-*SPk*-*HFA* except that the former can detect coincidence of the input heads.

We denote a deterministic (nondeterministic) *FV*3-*SPk*-*HFA* by *FV*3-*SPk*-*HDFA* (*FV*3-*SPk*-*HNFA*), and denote a deterministic (nondeterministic) *FV*3-*SNSPk*-*HFA* by *FV*3-*SNSPk*-*HDFA* (*FV*3-*SNSPk*-*HNFA*).

We now give a formal definition of a *cooperating system of k three-dimensional deterministic finite automata* (*CS*-3-*DFA*(k)) as an acceptor.

Definition 2.2. A *CS*-3-*DFA*(k) is a k-tuple $M = (FA_1, FA_2, \cdots, FA_k)$, $k \geq 1$, such that for each $1 \leq i \leq k$,

$$FA_i = (\Sigma, Q_i, X_i, \delta_i, q_{0i}, F_i, \phi, \#),$$

where

1. Σ is a finite set of *input symbols*.
2. Q_i is a finite set of *states*.
3. $X_i = (Q_1 \cup \{\phi\}) \times \cdots \times (Q_{i-1} \cup \{\phi\}) \times (Q_{i+1} \cup \{\phi\}) \times \cdots \times (Q_k \cup \{\phi\})$, where '$\phi$' is a special state not in $(Q_1 \cup Q_2 \cup \cdots \cup Q_k)$.
4. $\delta_i = (\Sigma \cup \{\#\}) \times X_i \times Q_i \rightarrow Q_i \times \{$east(=(0,+1,0)), west(=(0,-1,0)),south(=(+1,0,0)),north(=(-1,0,0)),up(=(0,0,-1)),down(=(0,0,-1)),no move (=(0,0,0))$\}$ is the *next move function*, where '#' is the *boundary symbol* not in Σ.
5. $q_{0i} \in Q_i$ is the *initial state* of FA_i.
6. $F_i \subseteq Q_i$ is the set of *accepting state* of FA_i.

Every automaton of M independently (in parallel) works step by step on the same three-dimensional tape x over Σ surrounded by boundary symbols #'s. Each step is assumed to require exactly one time for its completion. For each i ($1 \leq i \leq k$), let q_i be the state of FA_i at time 't'. Then each FA_i enters the next state 'p_i' at time '$t+1$' according to the function

$$\begin{aligned}\delta_i(x(\alpha,\beta,\gamma),&(q'_1,\cdots,q'_{i-1},q'_{i+1},\cdots,q'_k),q_i)\\ &=(p_i,(d_1,d_2,d_3)),\end{aligned}$$

where $x(\alpha,\beta,\gamma)$ is the symbol read by the input head of FA_i at time 't' and for each $j \in \{1,\cdots,i-1,i+1,\cdots,k\}$,

$$q'_j = \begin{cases} q_j \in Q_j & \text{if the input heads of } FA_i \text{ and } FA_j \text{ are on the same input position at the moment '}t\text{';} \\ \phi & \text{otherwise,} \end{cases}$$

and moves 1st input head to $x(\alpha+d_1, \beta+d_2, \gamma+d_3)$ at time '$t+1$'. We assume that the input head of FA_i never falls off the tape beyond boundary symbols.

When an input tape $x \in \Sigma^{(3)}$ is presented to M, we say that M *accepts* the tape x if each automaton of M, when started in its initial state with its input head on $x(1,1,1)$, eventually enters an accepting state with its input head on one of the bottom boundary symbols.

We next introduce a *cooperating system of k five-way three-dimensional deterministic finite automata* (*CS*-*FV*3-*DFA*(k)), with which we are mainly concerned in this paper.

Definition 2.3. A *CS*-*FV*3-*DFA*(k) is a *CS*-3-*DFA*(k) $M = (FA_1, FA_2, \cdots, FA_k)$ such that the input head of each FA_i can only move east, west, south, north, or down, but not up.

To give the formal definition of a *cooperating system of k three-dimensional nondeterministic finite automata* (*CS*-3-*NFA*(k)) and a *cooperating system of k five-way three-dimensional nondeterministic finite automata* (*CS*-*FV*3-*NFA*(k)) is left to the reader.

For each $X \in \{$*FV*3-*SPk*-*HDFA*, *FV*3-*SPk*-*HNFA*, *FV*3-*SNSPk*-*HDFA*, *FV*3-*SNSPk*-*HNFA*, *CS*-3-*DFA*(k), *CS*-3-*NFA*(k), *CS*-*FV*3-*DFA*(k), *CS*-*FV*3-*NFA*(k)$\}$, by X^c we denote an X whose input tapes are restricted to cubic ones; by $\mathcal{L}[X]$ ($\mathcal{L}[X^c]$) we denote the class of sets of input tapes accepted by X's (X^c's). We will focus our attention on the acceptors whose input tapes are restricted to cubic ones.

3 Hierarchies Based on the Number of Automata

3.1. Six-Way Case

We first investigate how the number of automata of *CS*-3-*FA*^{c}'s affects the accepting power.

Theorem 3.1.1.

For each $k \geq 1$ *and each* $X \in \{N, D\}$, $\mathcal{L}[CS\text{-}3\text{-}XFA_{\{0\}}(k)^c] \subsetneq \mathcal{L}[CS\text{-}3\text{-}XFA_{\{0\}}(k+2)^c]$, *where* $\mathcal{L}[CS\text{-}3\text{-}XFA_{\{0\}}(k)^c]$ *denote the class of sets of cubic tapes over a one-letter alphabet accepted by* $CS\text{-}3\text{-}XFA(k)$*'s.*

Proof. It is easy to prove that every *CS*-3-*DFA*(*k*)[*CS*-3-*NFA*(*k*)] can be simulated by a (six-way) three-dimesional sensing deterministic [nondeterministic] *k*-head finite automaton, and every (six-way) three-dimensional sensing deterministic [nondeterministic] *k*-head finite automaton can be simulated by a *CS*-3-*DFA*(*k*+1)[*CS*-3-*NFA*(*k*+1)]. From [8], for sets of cubic tapes over a one-letter alphabet, (six-way) three-dimensional sensing deterministic [nondeterministic] (*k*+1)-head finite automata are more powerful than the cooresponding *k*-head finite automata. From these facts, the theorem follows. □

Unfortunately, it is unknown whether $\mathcal{L}[CS\text{-}3\text{-}XFA_{\{0\}}(k)^c] \subsetneq \mathcal{L}[CS\text{-}3\text{-}XFA_{\{0\}}(k+1)^c]$ for any $k \geq 1$ and for any $X \in \{D, N\}$. It is also unknown wtether $\mathcal{L}[CS\text{-}3\text{-}XFA(k)^c] \subsetneq \mathcal{L}[CS\text{-}3\text{-}XFA(k+1)^c]$ for any $k \geq 2$ and for any $X \in \{D, N\}$. (It is easy to show that $\mathcal{L}[CS\text{-}3\text{-}XFA(1)^c] \subsetneq \mathcal{L}[CS\text{-}3\text{-}XFA(2)^c]$.)

3.2. Five-Way Case

We next investigate how the number of automata of $CS\text{-}FV3\text{-}FA^c$'s affects the accepting power.

For each $n \geq 1$, let $T(n) = \{x \in \{0,1\}^{(3)} | (^\exists m \geq n)$ $[l_1(x) = l_2(x) = l_3(x) = m$ & $x[(1,1,1),(m,m,1)] = x[(1,1,2),(m,m,2)]\} \in R_n(m)$ & $x[(1,1,3),(m,m,m)] \in \{0\}^{(3)}]\}$, where $R_n(m) = \{x \in \{0,1\}^{(3)} | \ l_1(x) = m, l_2(x) = m, l_3(x) = m$ & (x has exactly n 1's)$\}$ for each $m \geq n$. It is obvious that for any fixed positive integer n, $T(n)$ can be accepted by a $CS\text{-}FV3\text{-}DFA(n)$.

We first consider the following problem: given a fixed positive integer n, find a *CS*-*FV*3-*FA* which accepts $T(n)$ and uses the minimum number of automata. Unfortunately, we cannot generally solve the problem in the present paper, but we give the lower and upper bounds. Let $f(n)$ denote the minimum number of automata required for deterministic $CS\text{-}FV3\text{-}FA^c$'s to accept $T(n)$, and $g(n)$ denote the minimum number of automata required for nondeterministic $CS\text{-}FV3\text{-}FA^c$'s to accept $T(n)$. Clearly, $g(n) \leq f(n)$ for any $n \geq 1$.

Theorem 3.2.1. *For each* $k \geq 1$, (1) $f(k^2+k-1) \leq 2k-1$, (2) $f(k^2+2k) \leq 2k$, *and* (3) $f(k(k-1)/2+1) \geq k$.

Proof. The proofs of (1) and (2) are similar. We only give the proof of (2) here. To prove (2) is equivalent to proving that: for each $k \geq 1$, $T(k^2+2k) \in \mathcal{L}[CS\text{-}FV3\text{-}DFA(2k)^c]$.

For each $n \leq 1$, let $T'(n) = \{x[(1,1,1),(l_1(x), l_2(x), 2)] | x \in T(n)\}$. For convenience, we prove by induction on k that $T'(k^2+2k) \in \mathcal{L}[CS\text{-}FV3\text{-}DFA(2k)]$. It will be obvious that (2) follows from this fact.

We now prove (3). Suppose that there is a $CS\text{-}FV3\text{-}DFA(k-1)^c$ $M(k-1) = (FA_1, FA_2, \cdots, FA_{k-1})$ accepting $T(k(k-1)/2+1)$. Let h_i denote the input of FA_i for each $i \in \{1, 2, \cdots, k-1\}$.

For each $m \geq k(k-1)/2+1$, let $V(m) = \{x \in T(k(k-1)/2+1) | l_1(x) = l_2(x) = l_3(x) = m\}$, and for each permutation $\sigma : \{1, 2, \cdots, k-1\} \to \{1, 2, \cdots, k-1\}$, let $W_\sigma(m)$ be the set of all input tapes $x \in V(m)$ such that during the accepting computation of $M(k-1)$ on x, input heads $h_{\sigma(1)}, h_{\sigma(2)}, \cdots, h_{\sigma(k-1)}$ leave the first plane of x in this order. Then there must exist some permutation τ such that

$$|W_\tau(m)| \geq |V(m)|/(k-1)! = \Omega(m^{2(k(k-1)/2+1)}).$$

For each $x \in W_\tau(m)$ and each $1 \leq i \leq k-1$, let $q_{\tau(i)}(x), p_{\tau(i)}(x)$ and $t_{\tau(i)}(x)$ denote the internal state of $FA_{\tau(i)}$, the position of $h_{\tau(i)}$ and the time, respectively, when $h_{\tau(i)}$ leaves the first plane during the accepting computation of $M(k-1)$ on x.

For each $x \in W_\tau(m)$, let

$$t(x) = (t_{\tau(2)}(x) - t_{\tau(1)}(x), t_{\tau(3)}(x) - t_{\tau(2)}(x), \cdots, t_{\tau(k-1)}(x) - t_{\tau(k-2)}(x)),$$

and

$$u(x) = ((q_{\tau(1)}(x), p_{\tau(1)}(x)), \cdots, (q_{\tau(k-1)}(x), p_{\tau(k-1)}(x)), t(x)).$$

Clearly, for each $2 \leq i \leq k-1$, $t_{\tau(i)}(x) - t_{\tau(i-1)}(x) = O(m^{k-i})$, because otherwise $FA_{\tau(i)}, \cdots, FA_{\tau(k-1)}$ would enter a loop on the first plane, and thus $M(k-1)$ would never accept x. So $|\{u(x) | x \in W_\tau(m)\}| = O(m^{k(k-1)})$. Therefore, it follows that for large m

$$|W_\tau(m)| > |\{u(x) | x \in W_\tau(m)\}|,$$

and so there exist two diffirent input tapes $x, y \in W_\tau(m)$ such that $u(x) = u(y)$. Let z be the tape obtained from x by replacing the second plane of x with the second plane of y. It follows that z is also accepted by $M(k-1)$. This is a contradiction, because z in not in $T(k(k-1)/2+1)$. This completes the proof of (3). □

Theorem 3.2.2. $g(2k^2-5k+4) \geq k$, for $k \geq 1$.

Proof. The proof is very similar to that of (3) of Theorem 3.2.1. Suppose, to the contrary, that there is a $CS\text{-}FV3\text{-}NFA(k-1)^c$ $M(k-1) = (FA_1, FA_2, \cdots, FA_{k-1})$ accepting $T(2k^2-5k+4)$. Let h_i denote the input head of FA_i for each $i \in \{1, 2, \cdots, k-1\}$.

For each $m \geq 2k^2-5k+4$, let $V(m) = \{x \in T(2k^2-5k+4) | l_1(x) = l_2(x) = l_3(x) = m\}$. With each $x \in V(m)$, we associate one fixed accepting computation, $c(x)$, of $M(k-1)$ on x in which $M(k-1)$ operates in $O(m^{4(k-1)})$ time. Furthermore, for each permutation σ:$\{1, 2, \cdots, k-1\} \rightarrow \{1, 2, \cdots, k-1\}$, let $W_\sigma(m)$ be the set of all input tapes $x \in V(m)$ such that during $c(x)$, input heads $h_{\sigma(1)}, h_{\sigma(2)}, \cdots, h_{\sigma(k-1)}$ leave the first plane of x in this order. Then there must exist some permutation τ such that

$$|W_\tau(m)| \geq |V(m)|/(k-1)! = \Omega(m^{4k^2-10k+8}).$$

For each $x \in W_\tau(m)$ and each $1 \leq i \leq k-1$, let $q_{\tau(i)}(x), p_{\tau(i)}(x)$ *and* $t_{\tau(i)}(x)$ denote the internal state of $FA_{\tau(i)}$, the position of $h_{\tau(i)}$ and the time, respectively, when $h_{\tau(i)}$ leaves the first plane of x during $c(x)$.

For each $x \in W_\tau(m)$, let

$$t(x) = (t_{\tau(2)}(x) - t_{\tau(1)}(x), t_{\tau(3)}(x) - t_{\tau(2)}(x), \cdots, t_{\tau(k-1)}(x) - t_{\tau(k-2)}(x)),$$

and

$$u(x) = ((q_{\tau(1)}(x), p_{\tau(1)}(x)), \cdots, (q_{\tau(k-1)}(x), p_{\tau(k-1)}(x)), t(x)).$$

Clearly, for each $2 \leq i \leq k-1, t_{\tau(i)}(x) - t_{\tau(i-1)}(x) = O(m^{4(k-1)})$. So $|\{u(x)|x \in W_\tau(m)\}| = O(m^{4k^2-10k+6})$. Therefore, it follows that for large m

$$|W_\tau(m)| > |\{u(x)|x \in W_\tau(m)\}|,$$

and so there exist two different input tapes $x, y \in W_\tau(m)$ such that $u(x) = u(y)$. Let z be the tape obtained from x by replacing the second plane of x with the second plane of y. Clearly, from $c(m)$ and $c(y)$, we can construct an accepting computation of $M(k-1)$ on z. This is a contradiction, because z in not in $T(2k^2-5k+4)$. This completes the proof of the thereom. □

From Theorems 3.2.1 and 3.2.2, we can get the following theorem.

Theorem 3.2.3. *For each* $k \geq 1$ *and each* $X \in \{D, N\}$, $\mathcal{L}[CS\text{-}FV3\text{-}XFA(k)^c] \subsetneq \mathcal{L}[CS\text{-}FV3\text{-}XFA(k+1)^c]$.

Proof. For each $k \geq 1$, let $D(k)$=max$\{n|f(n) = k\}$ and $N(k)$=max$\{n|g(n) = k\}$. From Theorem 3.2.1 (3) and Thorem 3.2.2, we have

$$D(k) \leq k(k+1)/2 \text{ and } N(k) \leq 2k^2 - k,$$

respectively.

For each $X \in \{D, N\}$, let M be a $CS\text{-}FV3\text{-}XFA(k)^c$ accepting $T(X(k))$. From M, we can easily construct a $CS\text{-}FV3\text{-}XFA(k+1)^c M'$ which accepts $T(X(k)+1)$. Thus $T(X(k)+1) \in \mathcal{L}[CS\text{-}FV3\text{-}XFA(k+1)^c]$. From this and the fact that $T(X(k)+1) \notin \mathcal{L}[CS\text{-}FV3\text{-}XFA(k)^c]$, it follows that $T(X(k)+1) \in \mathcal{L}[CS\text{-}FV3\text{-}XFA(k+1)^c] - \mathcal{L}[CS\text{-}FV3\text{-}XFA(k)^c]$. □

4 Conclusion

We conclude this paper by giving two open problems.

1. For each $k \geq 1$, and each $X \in \{D, N\}$, $\mathcal{L}[CS\text{-}FV3\text{-}XFA_{\{0\}}(k)^c] \subsetneq \mathcal{L}[CS\text{-}FV3\text{-}XFA_{\{0\}}(k+1)^c]$, where $\mathcal{L}[CS\text{-}FV3\text{-}XFA_{\{0\}}(k)^c]$ denote the class of sets of cubic tapes over a one-letter alphabet accepted by *CS-FV3-XFA*(k)'s?

2. For $n \geq 4$, $g(n) < f(n)$? (It is easy to show that for $1 \leq n \leq 3$, $g(n) = f(n)$.)

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