Vibration Suppression Control of a Flexible Arm using Non-linear Observer with Simultaneous Perturbation Stochastic Approximation

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Abstract: The main objective in this study concerns to vibration control of a one-link flexible arm system. A variable structure system (VSS)-non-linear observer has been proposed in order to reduce the oscillation in controlling the angle of the flexible arm. The non-linear observer parameters are optimized using a novel version of simultaneous perturbation stochastic approximation (SPSA) algorithm. The SPSA algorithm is especially useful when the number of parameters to be adjusted is large, and makes it possible to estimate them very efficiently. As for the vibration and position control, a model reference sliding-mode control (MR-SMC) has been proposed. The simulations show that the vibration control of a one-link flexible arm system can be achieved more efficiently using our method.

Keywords: Non-linear Observer, Simultaneous Perturbation Method, Flexible Arm System, Fisher Information Matrix, Sliding Mode Control.

I. INTRODUCTION

The robot manipulators are widely used in various industrial applications. In this paper, the main goal is to control the vibration of the flexible arm. Since the feedback of only the motor angle will not be sufficient to suppress the oscillation, a variable structure system (VSS)-non-linear observer is incorporated. Also, a model reference-sliding mode control (MR-SMC) is established as a very efficient control method. However, there are many design parameters for the observer and controller to be determined, so it is difficult to design them in advance. Hence in order to overcome the problem, the simultaneous perturbation stochastic approximation (SPSA) algorithm is used to obtain the parameters of the VSS non-linear observer and controller.

II. DYNAMIC MODELING

The physical configuration of flexible arm is given by Fig.1. The deflection $y(x,t)$ is described by an infinite series of separable modes [1].

$$y(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) \tag{1}$$

where $\phi_i(x)$ is a characteristic function and $q_i(t)$ is a mode function. The kinetic and potential energies can be determined as follows:

$$\begin{aligned} T_e &= \frac{1}{2}\dot{\theta}^2 J + \frac{m}{2L}\sum_{i=1}^{n} A_i \dot{q}_i^2 + \frac{m}{2L}\dot{\theta}\sum_{i=1}^{n} A_i \dot{q}_i^2 \\ &+ \frac{m}{L}\dot{\theta}\sum_{i=1}^{n} B_i \dot{q}_i + \frac{M}{2}(L^2\dot{\theta}^2 + \sum_{i=1}^{n} C_i^2 \dot{q}_i^2 \\ &+ \dot{\theta}^2 \sum_{i=1}^{n} C_i^2 q_i^2 + 2L\dot{\theta}\sum_{i=1}^{n} C_i \dot{q}_i) \end{aligned} \tag{2}$$

$$V = \frac{EI}{2}\sum_{i=1}^{n} D_i q_i^2 \tag{3}$$

where

$A_i = \int_0^L \phi_i^2(x)dx,\ B_i = \int_0^L x\phi_i(x)dx,\ C_i = \phi_i(L)$ and

$D_i = \int_0^L \left[d^2\phi_i(x)/dx^2\right]^2 dx.$

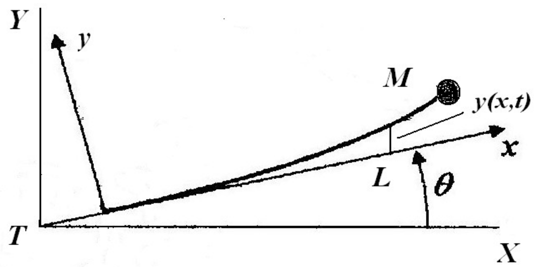

Fig.1. One-link flexible arm.

In the Fig. 1, L is the length of the flexible arm that has a mass m, T is the torque that rotates the elastic arm, M is the payload at the end of the arm and θ is the angle of the joint. The equation of motion is written as follows:

$$EIL\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \tag{4}$$

and the boundary conditions are

$$y(0,t) = 0 \tag{5}$$

$$\frac{dy}{dx}(0,t) = 0 \tag{6}$$

$$\frac{d^2 y}{dx^2}(L,t) = 0 \tag{7}$$

$$EI\frac{d^3 y}{dx^3}(L,t) = m\frac{d^2 y}{dt^2}(L,t). \tag{8}$$

From (4) and (5)-(8), we have

$$y_i(x,t) = \phi_i(x)\cos\omega_i t\,. \tag{9}$$

Moreover

$$\begin{aligned}\phi_i(x) = c_{1i}\cos\beta_i x + c_{2i}\cosh\beta_i x +\\ c_{3i}\sin\beta_{ix} + c_{4i}\sinh\beta_i x\end{aligned} \tag{10}$$

$$\omega_i^2 = \frac{EI}{\rho a}\beta_i^4\,. \tag{11}$$

Substituting $\phi_i(x)$ from (10) into (9) and using (5)-(8), β_i and c_{1i}-c_{4i} are determined.

Assuming that only the first mode exists, from (2), (3), and using Lagrange's equations as in [2][3], we obtain

$$\frac{d}{dt}\left(\frac{\partial T_e}{\partial\dot\theta}\right) - \frac{\partial T_e}{\partial\theta} + \frac{\partial V}{\partial\theta} = T \tag{12}$$

$$\frac{d}{dt}\left(\frac{\partial T_e}{\partial\dot q_1}\right) - \frac{\partial T_e}{\partial q_1} + \frac{\partial V}{\partial q_1} = 0 \tag{13}$$

thus, we obtain

$$\begin{bmatrix}\alpha_{00} & \alpha_{01}\\ \alpha_{01} & \alpha_{11}\end{bmatrix}\begin{bmatrix}\ddot\theta\\ \ddot q_1\end{bmatrix} = \begin{bmatrix}T - 2\dot\theta\alpha_{11}q_1\dot q_1\\ -H_1 q_1 + \alpha_{11}q_1\dot\theta^2\end{bmatrix} \tag{14}$$

$$y = \theta$$

where $\alpha_{00} = J + ML^2 + \alpha_{11}q_1^2$, T is the motor's shaft torque, J is the moment of inertia, $\alpha_{01} = \omega_1 + ML\phi_{1e}, \alpha_{11} = v_1 + ML + \phi_{1e}^2, v_1 = \rho a\int_0^L \phi_1^2 dx_1$ $H_1 = EI\int_0^L \ddot\phi_1^2 dx_1, \phi_{1e} = \phi_1(L), \omega_1 = \rho a\int_0^L x_1\phi_1 dx_1$, a is the area of the cross section, ρ is the density, E is Young's modulus, I is the area moment of inertia and y is the observation of θ. Defining the state variables such that

$x_1 = \theta$ $x_2 = \dot\theta$, $x_3 = q_1, x_4 = \dot q_1$, we have

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\end{bmatrix} = \begin{bmatrix}x_2\\ f_1(x_2,x_3,x_4)\\ x_4\\ f_2(x_2,x_3,x_4)\end{bmatrix} + \begin{bmatrix}0\\ b_1\\ 0\\ b_2\end{bmatrix}T \tag{15}$$

where

$$f_1(x_2,x_3,x_4) = \frac{1}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}$$

$$\cdot\left[-2\alpha_{11}^2 x_2x_3x_4 - \alpha_{01}(-H_1x_3 + \alpha_{11}x_3x_2^2)\right]$$

$$f_2(x_2,x_3,x_4) = \frac{1}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}$$

$$\cdot\left[2\alpha_{01}\alpha_{11}^2 x_2x_3x_4 - \alpha_{00}(-H_1x_3 + \alpha_{11}x_3x_2^2)\right]$$

$$b_1 = \frac{\alpha_{11}}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}$$

$$b_2 = \frac{\alpha_{01}}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}\,.$$

III. PROPOSED MODIFIED SPSA ALGORITHM

The SPSA is an efficient algorithm for high-dimensional problems in terms of proving a good solution for a relatively small number of measurements of the objective function. The recursions for SPSA algorithms are(basic SPSA algorithm [4]):

$$\hat\theta_{k+1} = \hat\theta_k - \bar a_k \bar{\bar H}_k^{-1}\hat g_k(\hat\theta_k). \tag{16}$$

The second order of simultaneous perturbation stochastic approximation (2SPSA) [5] is :

$$\hat\theta_{k+1} = \hat\theta_k - \bar a_k \bar{\bar H}_k^{-1}\hat g_k(\hat\theta_k),\ \bar{\bar H}_k = f_k(\bar H_k) \tag{17a}$$

$$\bar H_k = \frac{k}{k+1}\bar H_{k-1} + \frac{1}{k+1}\hat H_k,\ k=0,1,... \tag{17b}$$

where $\bar a_k$ is a scalar gain, $\hat g_k$ is the simultaneous perturbation that estimates the loss function, $\hat H_k$ is the estimate of the Hessian matrix, and f_k maps an usual non-positive-definite $\bar H_k$ to a positive definite pxp matrix. In our proposed SPSA algorithm, all the parameters are perturbed simultaneously, also it is possible to modify parameters with only two measurements of an evaluation function regardless of the dimension of the parameter. First, we compute the eigenvalues of $\bar H_k$ and sort them into descending order:

$$\Lambda_k = \mathrm{diag}[\lambda_1,...,\lambda_{q-1},\lambda_q,\lambda_{q+1},...,\lambda_p] \tag{18}$$

where

$$\hat\lambda_q = \varepsilon\lambda_{q-1}, \hat\lambda_{q+1} = \varepsilon\hat\lambda_q,...,\hat\lambda_p = \varepsilon\hat\lambda_{p-1} \tag{19}$$

and

$$\varepsilon = (\lambda_{q-1}/\lambda_1)^{q-2}\,. \tag{20}$$

Then, we use the mapping f_k as follows:

$$f_k(\bar H_k) = P_k\hat\Lambda_k P_k^T \tag{21}$$

where $\hat\Lambda_k$ is the diagonal matrix of Λ_k, the 2SPSA algorithm based on the mapping (21) makes the procedure of eliminating the non-positive definiteness of $\bar H_k$. In this part, we use the Fisher information matrix $F(n)$ instead of the Hessian matrix $\bar H_k$ in order to keep and guarantee the estimation matrix be positive-definite. In these estimates, the gain series at each iterations are determined using the 2SPSA algorithm (17a) by replacing $\hat\Lambda_k$ in the mapping f_k of (21) with

$\hat{\Lambda}_k$ that contains constant diagonal elements

$$\hat{\theta}_{k+1} = \hat{\theta}_k - \overline{a}_k \overline{\lambda}_k^{-1} \hat{g}_k(\hat{\theta}_k) \quad (22)$$

where $\overline{\lambda}_k$ is the geometric mean of all the eigenvalues of $F(n)$

$$\overline{\lambda}_k = (\lambda_1 \ldots \lambda_{q-1} \hat{\lambda}_q \hat{\lambda}_{q+1} \ldots \hat{\lambda}_p)^{1/p}. \quad (23)$$

Recursions (22) and (17b) together with (18)–(20) and (23) form a modified version of the 2SPSA (called 3SPSA). The parameters estimation using our proposed SPSA algorithm is given as follows:

$$\hat{\chi}_{k+n} = \hat{\chi}_{k-1} - \psi_{\frac{k-1}{n+1}} \cdot \left\{ \frac{1}{2} \frac{(W_{k+n} - W_k^T \hat{\chi}_{k-1}^+)^2 - (W_{k+n} - W_{k+n}^T \hat{\chi}_{k-1}^+)^2}{c_{\frac{k-1}{n+1}}} Xs_{k-1} - \begin{bmatrix} \upsilon^2 I_n & 0 \\ 0 & 0 \end{bmatrix} \hat{\chi}_{k-1} \right\} \quad (24)$$

where W_k is measured output, c is the perturbation, υ represents the variance, n, k are sampling time, χ is the parameter to be estimated, and ψ is a gain coefficient and the subscript in this coefficient represents a fraction. By making use of this, the perturbation vector c defined by our proposed SPSA algorithm is added at the same time to all parameters, and finally $\hat{\chi}_{k-1}^+$ is calculated by

$$\hat{\chi}_{k-1}^+ = \hat{\chi}_{k-1} + c_{\frac{k-1}{n+1}} s_{k-1} \quad (25)$$

where s_k is a signed vector.

IV. DESIGN OF NON-LINEAR OBSERVER

Since only the motor angle x_1 is the measurable state variable, the remaining states x_2, x_3 and x_4 are predicted using intelligent state observer design [6]. For this, (14) can be written as

$$\dot{x} = f(x) + g(x)T \quad (26)$$

$$y = Cx, \quad C = [1 \quad 0 \quad 0 \quad 0]. \quad (27)$$

For this non-linear system, we consider a robust VSS observer, which predicts system states and is defined as follows:

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})T + M(\overline{y}) + K(\hat{y} - y) \quad (28)$$

$$\hat{y} = C\hat{x} \quad (29)$$

$$M(\overline{y}) = -g(x)\frac{\overline{y}}{\|\overline{y}\| + \gamma}\varsigma \quad (30)$$

$$\overline{y} = \hat{y} - y = C(\hat{x} - x) \quad (31)$$

where $\hat{x}$ represents the predicted values of system states, K is the observer gain matrix, $M(\overline{y})$ is the observer non-linearity term, ς represents the gain, and $\gamma > 0$ is an averaging constant for removing chattering. Now defining the estimation error as

$$e = \hat{x} - x \quad (32)$$

then, we have

$$\dot{e} = f(\hat{x}) - f(x) + [g(\hat{x}) - g(x)]T + KC(\hat{x} - x) + M(\overline{y}). \quad (33)$$

The error system is given as follows:

$$\dot{e} = [f'(x_d) + g'(x_d)T + KC]e + M(\overline{y}) = A_0 e + M(\overline{y}) \quad (34)$$

where

$$A_0 = A + GT + KC \quad (35)$$

$$A = \partial f_i / \partial x_j \quad (36)$$

$$G = \partial g_i / \partial x_j \qquad (i,j\text{=2,3,4}). \quad (37)$$

Choosing a Lyapunov function of e as

$$V = \frac{1}{2}e^2 \quad (38)$$

and integrating V with respect to e yields

$$\dot{V} = e\dot{e} = e^2(A_0 - |g(x)|C\frac{1}{C\|e\| + \gamma}\varsigma). \quad (39)$$

The observer parameters determined by our novel SPSA algorithm are computed with A and G, so as to ensure the stability of (40) minimizing the following evaluation function:

$$J_0 = \sum (y - \hat{y})^2. \quad (40)$$

The parameters determined are
K=[-339 -19002 15.20 -10109]^T, ς =0.012 , γ =0.013.

V. SLIDING MODE CONTROLLER

The purpose of sliding mode control is to make the states converge to the sliding mode surface. Therefore, we choose the desired response based on a second order reference model given as [6]

$$\begin{bmatrix} \dot{x}_m \\ \ddot{x}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\omega_n \end{bmatrix} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} U_m \quad (41)$$

where ω_n is the eigenvalue of angular frequency and U_m is the model input. Assuming the sliding mode hyper-plane for the system of (14) with the states variables predicted by the observer as

$$\sigma = s_1(x_1 - x_m) + s_2(x_2 - \dot{x}_m) + s_3 x_3 + s_4 x_4 \quad (42)$$

when the sliding mode is in operation, then

$$\sigma = 0 \quad (43)$$

$$\dot{\sigma} = 0. \quad (44)$$

The equivalent control input can be obtained by substituting equation (14) to (44). This gives

$$T_{eq} = 2\alpha_{11} x_2 x_3 x_4 + \frac{\alpha_{01}}{\alpha_{11}}(-H_1 x_3 + \alpha_{11} x_2^2 x_3) - \frac{\Delta}{s_2}\left[s_1(x_2 - \dot{x}_m) - s_2 \ddot{x}_m + s_3 x_4 + s_4 \dot{x}_4\right] \quad (45)$$

where

$$\Delta = (\alpha_{00} - \alpha_{01}^2 / \alpha_{11}) > 0.$$

Now, we consider the design of sliding mode controller, which is the non-linear input to make the states converge in the hyper-plane. In general, the eventual sliding mode input can be considered which is consisted of two independent inputs, namely, the equivalent control input T_{eq} and non-linear control input T_ℓ [6]. In other words

$$T = T_{eq} + T_\ell = T_{eq} - k(x,t)\mathrm{sat}(\sigma) \tag{46}$$

where

$$\mathrm{sat}(\sigma) = \begin{cases} 1 & if \quad \sigma > \delta \\ \dfrac{\sigma}{\delta} & if \quad |\sigma| \le \delta \\ -1 & if \quad \sigma < -\delta \end{cases} \tag{47}$$

and $k(x,t)$ is the control input function. δ is a constant to eliminate the chattering. We choose a Lyapunov function σ to confirm $\sigma = 0$:

$$V = \frac{1}{2}\sigma^2. \tag{48}$$

With this, $\dot V$ is given by

$$\dot V = \sigma\dot\sigma = \sigma\left\{\begin{matrix}\dfrac{s_2}{\Delta}\left[T - 2\alpha_{11}x_2x_3x_4 - \dfrac{\alpha_{01}}{\alpha_{11}}\begin{pmatrix}-H_1x_3 + \\ \alpha_{11}x_2^2x_3\end{pmatrix}\right] \\ + s_1(x_2 - x_m) - s_2\ddot x_m + s_3x_4 + s_4\dot x_4\end{matrix}\right\}. \tag{49}$$

Substituting (46) into (49), the existence condition for the sliding mode is given as

$$\dot V = \sigma\left\{-\frac{s_2}{\Delta}k(x,t)\,\mathrm{sgn}(\sigma)\right\} = -k(x,t)\frac{s_2}{\Delta}|\sigma| < 0. \tag{50}$$

Since $\frac{s_2}{\Delta} > 0$, if we choose $k(x,t) > 0$, then the state variable x will converge in the sliding mode plane and a stable MR-SMC can be realized. The controller gains are determined using our proposed SPSA algorithm so as to minimize the cost function given by

$$J_h = \sum\left[|L*(x_1 - x_m)| + |x_3|\right]. \tag{51}$$

The parameters values are $s_1 = 3.4$, s_2=2, s_3=11.23, s_4=-0.58 and δ = 0.43, $k(x,t)$=3.45.

VI. SIMULATION RESULTS

The results are compared with previous simulations without our proposed algorithm [6]. Fig. 2 shows a typical responses of the system at the tip position. This result was improved in comparison with previous simulations.

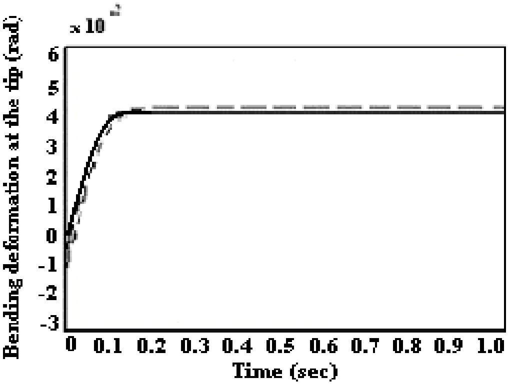

Fig.2. Bending deformation. Without SPSA algorithm (dashed line (- -)). With 3SPSA algorithm and non-linear observer (solid line (-)).

Fig. 3 shows the tip velocity. The algorithm reduces the magnitude of velocity to a small value.

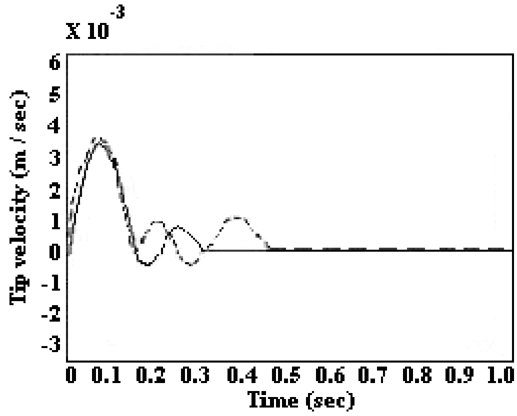

Fig.3. Tip velocity. Without SPSA algorithm (dashed-line(--)). With 3SPSA algorithm and non-linear observer (solid line (-)).

VII. CONCLUSION

We have proposed a MR-SMC method using a non-linear observer for controlling the angular position of a flexible arm by suppressing its oscillation. We also have proposed the use of a novel SPSA algorithm in order to determinate the observer/controller gains. Also, the SPSA algorithm has a very low computational complexity for solving difficult estimation problems in an efficient way. The non-linear observer was successful in predicting the state variables from the motor angular position and the MR-SMC was a very efficient control method. The use of the novel version of SPSA algorithm could determine easily and efficiently, the control parameters and observer gains.

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