The Fourteenth International Symposium on Artificial Life and Robotics 2009 (AROB 14th '09),
B-Con Plaza, Beppu, Oita, Japan, February 5 - 7, 2009

# Current Control of PWM Power Amplifier by Approximate 2-Degree-of-Freedom Digital Controller

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**Abstract**

In this paper, a digital robust controller with bumpless mode switching for controlling the current of PWM power amplifier for satisfying the demands and extending the range of inductive load wider is proposed. It is necessary to measure the value of load to implement this bumpless mode switching automatically according to the load range. Thus, a method of estimating inductive load is shown. The bumpless mode switching is automatically performed by estimating a inductive value without specifying the value of inductive load beforehand. The digital controller equipped with the inductance estimation and the bumpless mode switching is realized by a DSP. Some experiments show that the digital controller with the proposed bumpless mode switching can satisfy larger specifications.

## 1 Introduction

A pulse width modulation (PWM) switching circuit is used for an electric-power conversion circuit, an LCL low-pass filter is inserted between the conversion circuit and the load for noise removal, and a PWM amplifier which constitutes feedback control systems so that the output current supplied to the load might be proportional to a reference input is used as an amplifier itself or as a current supply. If the characteristics and parameters of the load are decided and there is little change in those, satisfactory performances are obtained in general. In many applications, however, loads cannot be specified, i.e., its amplitude is also sharply changed from the zero to the maximum rating. Usually, design conditions are changed for each load and then each controller is re-designed. Then, a so-called robust PWM amplifier which can cover such an extensive load changes and also direct-current power supply voltage changes with one controller is needed. We have recently proposed the different methods for designing a digital controller for PWM voltage amplifiers which can attain such demand. Furthermore, a digital controller which uses mode switching was proposed in order to extend the range of load wider. This mode switching is performed automatically without specifying the value of capacitive load beforehand, i.e., the parameters of the controller are switched automatically. In this paper, a bumpless mode switching method for controlling of the current of PWM power amplifier is proposed. It is necessary to measure the values of the loads to implement this bumpless mode switching automatically according to the load range. Thus, a method of estimating inductive load is shown. The digital controller with the bumpless mode switching function is actually realized by using a DSP. Experimental studies show that the proposed digital controllers can satisfy larger specifications smoothly.

## 2 PWM power amplifier

The PWM amplifier as shown in **Fig.1** is being manufactured. The carrier frequency of triangular waves are 10-100[kHz], and the amplitude $c_m$ is 10[V] and $E$ is 150[V]. The LCL circuit is a filter for removing carrier and switching noises. The values $L_0$, $C_0$ and $L_1$ of the LCL circuit are determined so that control systems can make their sensitivity to load changes low and reduce noise. If the frequency of input $u$ is smaller enough than that of the carrier, the state equation of the PWM amplifier at inductannce load $L_L$

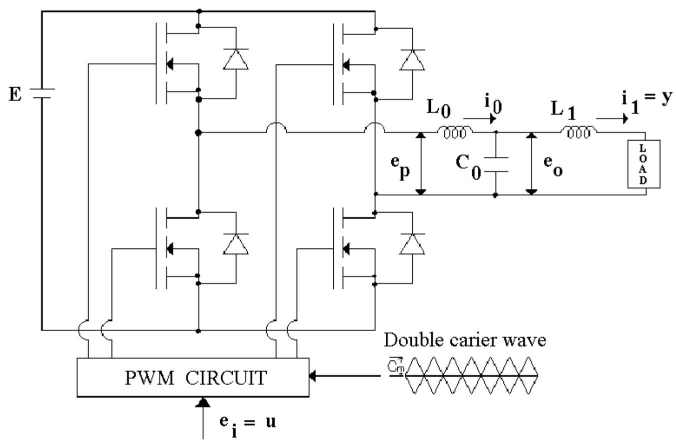


Figure 1: PWM amplifier

in **Fig.1** can be expressed from the state equalizing method as follows:

$$\begin{cases} \dot{x} = A_c x + B_c u \\ y = Cx \end{cases} \quad (1)$$

where

$$\begin{aligned} x &= \left[ \begin{array}{ccc} e_o & i_0 & i_1 \end{array} \right]^T \\ A_c &= \left[ \begin{array}{ccc} 0 & \frac{1}{C_0} & -\frac{1}{C_0} \\ -\frac{1}{L_0} & -\frac{R_0}{L_0} & 0 \\ \frac{1}{L_1+L_L} & 0 & -\frac{R_1+R_L}{L_1+L_L} \end{array} \right] B_c = \left[ \begin{array}{c} 0 \\ \frac{K_p}{L_0} \\ 0 \end{array} \right] \\ C &= \left[ \begin{array}{ccc} 0 & 0 & 1 \end{array} \right] \quad u = e_i \quad y = i_1 \quad K_p = -\frac{E}{c_m} \end{aligned}$$

and $R_0$ is the total resistance of coil, ON resistance of FET, etc. $R_1$ and $R_L$ are resistances of coil($L_1$) and coil($L_L$), respctively. When realizing a digital controller by a DSP, a delay time exists between the start point of sampling operation and the output point of control input due to the input computing time and AD/DA conversion times. This delay time is considered to be equivalent to the input dead time which exists in the controlled object. Then the state equation of the system of **Fig.2** is expressed by

$$\begin{cases} x_d(k+1) = A_d x_d(k) + B_d v(k) \\ y(k) = C_d x_d(k) \end{cases} \quad (2)$$

where

$$\begin{aligned} x_d &= \left[ \begin{array}{c} x \\ \xi \end{array} \right] \quad \xi(k) = u(k) \quad C_d = \left[ \begin{array}{cc} C & 0 \end{array} \right] \\ A_d &= \left[ \begin{array}{cc} e^{A_c T} & e^{A_c(T-L_d)} \int_0^{L_d} e^{A_c \tau} B_c d\tau \\ 0 & 0 \end{array} \right] \\ B_d &= \left[ \begin{array}{c} \int_0^{T-L_d} e^{A_c \tau} B_c d\tau \\ 1 \end{array} \right] \end{aligned}$$

Now, the PWM amplifier with the following specifications 1-3 is designed and is manufactured by constituting digital control systems to the PWM amplifier (controlled object) at inductive load. Such specification is demanded in Magnetic-Field Current Tracking Control in MRI[?] etc..

1. The band-width of control systems is about 1[kHz] to inductive load, where $0 \leq L_L < 5$[mH].
2. Against the range of inductance load of spec.1, an over-shoot is not allowable in a step response.
3. The specs. 1 and 2 are satisfied regardless of change in large direct-current power supply.

The load changes for the controlled object and the direct-current power supply change are considered as parameter changes in eq.(2). Such parameter changes can be transformed to equivalent disturbances. Therefore, what is necessary is just to constitute the control systems whose pulse transfer functions from equivalent disturbances and to the output become as small as possible in their amplitudes.

## 3 Design method of approximate 2-degree-of-freedom digital integral-type control system

First, the transfer function between the reference input $r$ and the output $y$ is specified as

$$\begin{aligned} W_{ry} &= \frac{(1+H_1)(1+H_2)(1+H_3)}{(1-n_1)(1-n_2)(1-n_3)} \\ &\times \frac{(1+H_4)(z-n_1)(z-n_2)(z-n_3)}{(z+H_1)(z+H_2)(z+H_3)(z+H_4)} \end{aligned} \quad (3)$$

where $n_1$ and $n_2$ are the zeros for discrete-time controlled object (2). It shall be specified that $H_1$ and $H_2$, $H_3$ satisfy the relations $H_1 \gg H_2, H_3, H_4 > 0$. Then $W_{ry}(z)$ can be approximated by the following:

$$W_{ry}(z) \approx W_m(z) = \frac{1+H_1}{z+H_1} \quad (4)$$

We constitute a state feedback system using $u = Fx_d + Gr$ to the controlled object(2), and decide $F = [F(1,1) \ F(1,2) \ F(1,3) \ F(1,4)]$ and $G$ so that $W_{ry}(z)$ becomes eq.(3). The equivalent disturbance is defined as $Q = [q_u \ \ q_{\bar{y}} \ \ q_y]^T$ and the pulse transfer function between $Q$ and the output $y$ of the state feedback system is defined as $W_{Qy}(z)$. The system with an inverse system and a filter added to the state feedback

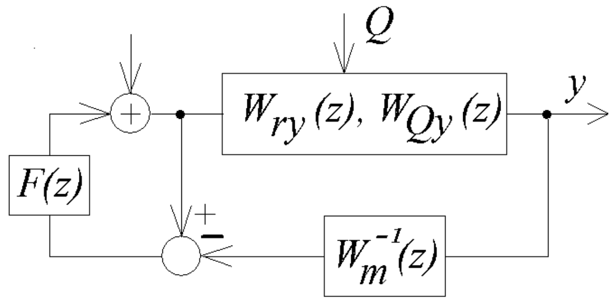


Figure 2: System reconstituted with an inverse system and a filter

system is constituted as shown in **Fig.2**, in which the pulse transfer function $F(z)$ is given by

$$F(z) = \frac{k_z}{z - 1 + k_z} \tag{5}$$

The transfer functions between $r$ and $y$, $Q$ and $y$ of the system in **Fig.4** are written approximately as

$$y \approx \frac{1 + H_1}{z + H_1} r \tag{6}$$

$$y \approx \frac{z - 1}{z - 1 + k_z} W_{Qy}(z) Q \tag{7}$$

It turns out from eqs.(8) and (9) that the characteristic $r - y$ can be specified with $H_1$ and the characteristic $Q - y$ can be independently specified with $k_z$. That is, the systems of **Fig.2** are of approximate 2-degree-of-freedom, and their sensitivity against the disturbance, i.e., load changes becomes lower with the increase of $k_z$.

Now, if an equivalent conversion of the controller in **Fig.2** is carried out introducing steady state gain $g$ between $r - y$, an approximate 2-degree-of-freedom digital integral-type control systems will be obtained as shown in **Fig.3**.

## 4 Inductance estimating and controller parameter bumpless switching method

### 4.1 Inductance estimating method

Load inductance can be estimated from the continuous-time controlled object of eq.(1). If it is $e_o(t) \gg (R_1 + R_l)i$, at an inductive load, the inductance is computed by the voltage of capacity and the

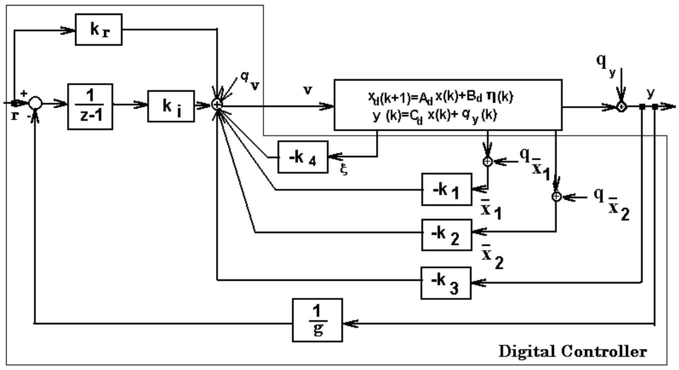


Figure 3: Approximate 2-degree-of-freedom digital integral type control system

derivative of current of load coil such that

$$L_1 + L_L \approx \frac{e_o(t)}{\dot{i}_1(t)} \tag{8}$$

The inductance can be estimated from eq.(11), which is expressed in the discrete approximation form of

$$L_1 + L_L \approx \frac{e_o(k) + e_o(k-1)}{2(i_1(k) - i_1(k-1))} T \tag{9}$$

Eqs.(12) and the filter are implemented by a DSP. The holding function is added.

The control mode is switched with an estimated inductance. If the estimated value is changed due to noise etc. when the load inductance is close to a threshold of change, the problem of frequent mode switching will occur. Therefore, the hysteresis characteristic with mode switching is established. The inductances of $TH - L$ and $TH - H$ used as the width of hysteresis are set up experimentally so that a change does not occur beyond necessity. As for the parameter, Low(L)-mode is chosen at the start of an inductance estimation program. Even when an estimated value exceeds $TH - L$, it does not switch to the High(H)-mode. Only when it exceeds $TH - H$, it switches to the H-mode. At the time of being conversely less than $TH - L$ a switch from the H-mode to the L-mode is performed.

### 4.2 Bumpless mode switching method

The control mode is switched with an estimated inductance. If the estimated value is changed due to noise etc. when the load inductance is close to a threshold of change, the problem of frequent mode switching will occur. Therefore, the hysteresis characteristic with mode switching is established. If the control variable changes rapidly when the controller switches the control mode, a bump will arise in an output. In order to suppress the bump, auxiliary feedback functions are implemented by a combination of H-mode controller and L-mode controller.

## 5 Experimental studies

A single-chip DSP from Texas Imstruments(TI) is used, which realizes a digital controller as in **Fig.3**. We will design a control system so that all the specifications are satisfied within $0 \leq L_L \leq 3$[mH] and $3 \leq L_L \leq 5$[mH] at L-mode and H-mode, respectively. First of all, in order to set the band-width to about 1[kHz], $H_1, H_2$ and $H_3$ are specified as

$$H_1 = -0.89 \quad H_2 = -0.05 \quad H_3 = -0.5 \quad H_4 = -0.4 \tag{10}$$

If it sets up with

$$L_L = 1[\text{mH}] \quad k_z = 0.09 \tag{11}$$

then a specification can be satisfied at L-mode. The parameters of controller become as

$$\begin{aligned} k_{1l} &= -0.04280 \quad k_{2l} = -0.43702 \quad k_{3l} = 0.61873 \\ k_{4l} &= -0.13447 \quad k_{il} = -0.012601 \quad Gg = 0 \end{aligned} \tag{12}$$

Next, $H_1, H_2$ and $H_3$ are specified as

$$H_1 = -0.89 \quad H_2 = -0.05 \quad H_3 = -0.01 \quad H_4 = -0.5 \tag{13}$$

If it is set up with

$$L_L = 5[\text{mH}] \quad k_z = 0.45 \tag{14}$$

then a specification can be satisfied at H-mode. The parameters of controller become as

$$\begin{aligned} k_{1h} &= -0.054912 \quad k_{2h} = -0.078368 \quad k_{3h} = 3.962700 \\ k_{4h} &= -3.76630 \quad k_{ih} = -0.31917 \quad Gg = 0 \end{aligned} \tag{15}$$

According to the estimated value, the mode is switched from L to H. When $L_L = 4$[mH] at L-mode, the parameters are switched from L-mode to H-mode. If it switches from L-mode to H-mode, the range of $L_L$ is extended to

$$0 \leq L_L \leq 5[\text{mH}] \tag{16}$$

and then all the specifications are satisfied. **Fig.4** shows a step responses of $i_1$ and $e_i$ when performing the DDC using the automatic bumpless mode switching from L-mode to H-mode. L and H in **Fig.4** show that L-mode and H-mode controllers are operating, respectively. This experimental result shows that the bumpless mode switching can spread the range of the value of inductive load. Fig.14 Experimental result of step response at inductive
load ($L_L = 5$

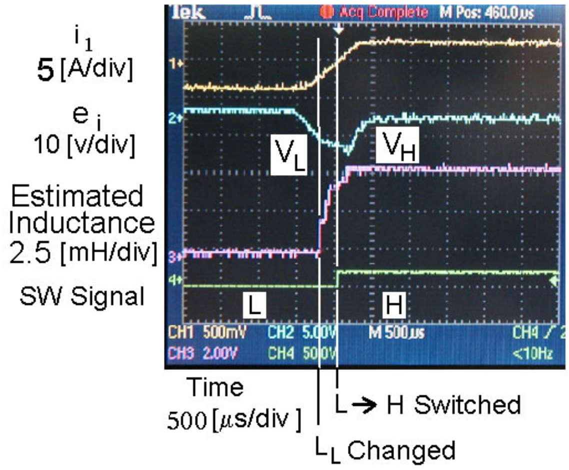


Figure 4: Experimental result of step response at inductive load ($L_L = 5$[mH]) using automatic bumpless switching from L-mode to H-mode

## 6 Conclusion

In this paper, the concept of controller with bumpless mode switching to attain the robust control of the current of PWM power amplifier against extensive load changes was given. The proposed digital controller with bumpless mode switching was realized by using a DSP implemented to the controlled object. It was shown from an experiment that a sufficiently robust digital controller is realizable. The range of inductive load can be extended by performing the bumpless mode switching.

## References

[1] K.Higuchi, K.Nakano, T.Kajikawa, K.Araki and F.Chino, "Robust Digital Control of PWM Power Amplifier by Approximate 2-Degree-of-Freedom System with Bumpless Mode Switching," *IEEE CDC-2003*, (2003.12).