Linear Frequency Estimator for Motor Application with Quadratic Constrained Condition

Ga Hyung Choi*, Tae Sung Yoon** and Jin Bae Park*

*Department of Electrical Engineering, Yonsei Unversity, Seoul, Korea
(Tel : 82-2-2123-2773)
(E-mail: choigh99@yonsei.ac.kr, jbpark@yonsei.ac.kr)
**Department of Electrical Engineering, Changwon National University, Changwon, Korea
(Tel : +82-55-213-3633)
(E-mail: tsyoon@changwon.ac.kr)

Abstract: Conventional linear prediction algorithm with sinusoidal signal for the estimation of motor's speed has a limitation in the range of low speed. If an estimator can get additional information then its performance is able to be improved. A sinusoidal signal has the natural property which is quadratic equation so called Pythagorean identity. However, since the equation was nonlinear form, it needs change to a linear constrained condition. Adding it to the measurement equation, it is possible to derive a linear state space equation which has more information without additional sensor. The experimental results and the computer simulations show that the performance of the proposed algorithm in this study is better than that of the conventional algorithm. It supports that the additional constrained condition can improve the estimator's performance.

Keywords: Linear Hall sensor, Linear estimator, Motor speed, Frequency estimation, Constrained condition**.**

I. INTRODUCTION

Angular velocity is a necessary information for motor control. If a system is under the condition which is a varying velocity or repeated running and stopping, it is more important for motor control to get the velocity. Motor's angular velocity can be obtained by various sensors like tachometer, latch type Hall sensor, linear type Hall sensor, current sensor and shunt resistor [1]. Among these sensors, the shunt resistor has the best cost advantage. However, since an estimator using the shunt resistor can be derived from the motor parameters, it is not suitable for the common device. One of the methods not using the motor parameters is to use the linear Hall sensor. Its output signal represents the motor's rotation. The signal is a continuous sinusoidal wave. Therefore, an estimator using the linear Hall sensor can estimate motor's speed precisely. The linear Hall sensor's output is represented by a sinusoidal signal and an additive noise in [2]. And using linear prediction, a linear state space equation is derived. With this equation, one can estimate motor's speed and it shows good performance as tachometer but not in the low speed range. The motor can be operated at various velocities. And it is able to repeat running and stopping. Therefore, the estimator needs additional information to complement the error at low speed.

The additional information like a constrained condition can improve the accuracy of the estimator [3]. The sinusoidal signal has natural constrained condition. When one has two sinusoidal waves which are delayed 90 degree in phase each other, the square of the sine plus the square of the cosine is always 1 which is called Pythagorean identity. With this quadratic constrained condition, the estimator in [2] can enhance the accuracy at low speed. To adopt this condition into the linear state space equation, it needs the change from the quadratic form to a linear form. The linear constrained condition can be used like additional measurement information as in [3].

In this paper, we set sinusoidal signal with additive white noise to measurement data and estimate the frequency of these data. To solve the low speed problem, $\cos(\alpha)$ is multiplied to the measurement data so that state space equation has $\cos(w_k)$ and $\sin(w_k)$ in state values. Since each state has 90 degree phase delay, these two states must satisfy the Pythagorean identity. This gives another information without additional sensor. To change Pythagorean identity to a linear form constraint, we use the past estimated value. The derived linear constraint is added into the measurement equation of the linear state space equation. The linear constraint is not the same as the original constraint. It causes error in the measurement matrix. Therefore, we use the Robust Least Square (RLS) algorithm to compensate these measurement uncertainties [4]. To prove the performance of the proposed algorithm, the motor installed in a car window system is used and its rotation is obtained from the linear Hall sensor. With the sensor data, we estimated motor's speed and compared the results with those of [2]. The proposed algorithm shows enhanced performance not only in the low speed range but also the high range. When the motor was stopped, the proposed algorithm estimated the speed with lower error than [2]. We also simulated the proposed algorithm under varying frequencies. The analysis by the mean of estimation error and the root mean square error shows that the additional constrained condition can improve the performance of the estimation.

II. LINEAR STATE SPACE MODEL

The sinusoidal signal obtained from the linear Hall sensor which has stationary DC offset can be represented by linear prediction method [5].

$$d_k = A_k \cos(w_k k) + v_k \tag{1}$$

$$d_k + d_{k-2} = 2\cos(w_k)\{d_{k-1} - v_{k-1}\} + v_k + v_{k-2} \tag{2}$$

where A_k is amplitude of the Hall sensor, w_k is angular velocity and v_k is white noise having the zero mean and the variance of R_k. With multiplying $\cos(\alpha)$ to (2), one can derive the following equation contained sine and cosine function.

$$\begin{aligned}(d_k + d_{k-2})\cos(\alpha) &= 2\cos(w_k)\cos(\alpha)\{d_{k-1} - v_{k-1}\} \\ &\quad + (v_k + v_{k-2})\cos(\alpha) \\ &= 2\{d_{k-1} - v_{k-1}\}\{\cos(w_k + \alpha) + \sin(w_k)\sin(\alpha)\} \\ &\quad + (v_k + v_{k-2})\cos(\alpha)\end{aligned} \tag{3}$$

(2) and (3) represent linear state space equation which the measurement matrix has uncertainty,

$$\begin{aligned}x_{k+1} &= x_k + w_k \\ y_k &= (\tilde{H}_k - \Delta H_k)x_k + \bar{v}_k\end{aligned} \tag{4}$$

where

$$x_k = \begin{bmatrix}\cos(w_k) \\ \cos(w_k + \alpha) \\ \sin(w_k)\end{bmatrix}$$
$$y_k = \begin{bmatrix} d_k + d_{k-2} \\ (d_k + d_{k-2})\cos(\alpha)\end{bmatrix}$$
$$\tilde{H}_k = \begin{bmatrix} 2d_{k-1} & 0 & 0 \\ 0 & 2d_{k-1} & 2d_{k-1}\sin(\alpha)\end{bmatrix} \tag{5}$$
$$\Delta H_k = \begin{bmatrix} 2v_{k-1} & 0 & 0 \\ 0 & 2v_{k-1} & 2v_{k-1}\sin(\alpha)\end{bmatrix}$$
$$\bar{v}_k = \begin{bmatrix} v_k + v_{k-2} \\ (v_k + v_{k-2})\cos(\alpha)\end{bmatrix}$$

and w_k is modeling error under the assumptions of white and zero mean. (5) shows the state variable x_k that has two sinusoid signals which are delayed 90 degree in phase each other.

III. QUADRATIC CONSTRAINED CONDITION

The two state variables, $\cos(w_k)$ and $\sin(w_k)$, have following constraint naturally in (5).

$$\cos^2(w_k) + \sin^2(w_k) = 1 \tag{6}$$

(6) can be considered another measurement information without additional sensor. However, since (6) is quadratic form, it is not suitable for the linear state equation. To change (6) to linear form, one can use a past estimation result as follows:

$$\begin{aligned}1 = &\left\{\cos(\hat{w}_{k-1}) + \varepsilon_{c_k}\right\}\cos(w_k) \\ &+ \left\{\sin(\hat{w}_{k-1}) + \varepsilon_{s_k}\right\}\sin(w_k)\end{aligned} \tag{7}$$

With (7), the quadratic constraint becomes another measurement. With augmenting (7) to (4), (4) can be re-written

$$\begin{aligned}x_{k+1} &= x_k + w_k \\ y_{a_k} &= (\tilde{H}_{a_k} - \Delta H_{a_k})x_k + \bar{v}_{a_k}\end{aligned} \tag{8}$$

where

$$y_{a_k} = \begin{bmatrix} d_k + d_{k-2} \\ (d_k + d_{k-2})\cos(\alpha) \\ 1\end{bmatrix}$$
$$\tilde{H}_{a_k} = \begin{bmatrix} 2d_{k-1} & 0 & 0 \\ 0 & 2d_{k-1} & 2d_{k-1}\sin(\alpha) \\ \cos(\hat{w}_{k-1}) & 0 & \sin(\hat{w}_{k-1})\end{bmatrix} \tag{9}$$
$$\Delta H_{a_k} = \begin{bmatrix} 2v_{k-1} & 0 & 0 \\ 0 & 2v_{k-1} & 2v_{k-1}\sin(\alpha) \\ -\varepsilon_{c_{k-1}} & 0 & -\varepsilon_{s_{k-1}}\end{bmatrix}$$
$$\bar{v}_{a_k} = \begin{bmatrix} v_k + v_{k-2} \\ (v_k + v_{k-2})\cos(\alpha) \\ 0\end{bmatrix}$$

The augmented equation (8) has also uncertainty in measurement matrix. To obtain a compensated estimation results from these uncertainties, we use the RLS algorithm in [4].

IV. LINEAR FREQUENCY ESTIMATOR

The RLS algorithm in [4] can be summarized as follows:

Measurement update:

$$\begin{aligned}P_{k|k}^{-1} &= \lambda P_{k|k-1}^{-1} + \tilde{H}_{a_k}^T \tilde{H}_{a_k} - W_k \\ \hat{x}_{k|k} &= (I + P_{k|k}W_k)\hat{x}_{k|k-1} + P_{k|k}\tilde{H}_k^T(y_{a_k} - \tilde{H}_{a_k}\hat{x}_{k|k-1})\end{aligned} \tag{10}$$

Time update:

$$\begin{aligned} P_{k+1|k} &= F_k P_{k|k} F_k^T \\ \hat{x}_{k+1|k} &= F_k \hat{x}_{k|k} \end{aligned} \tag{11}$$

where λ is weight parameter which is able to adjust along the system characteristic and W_k is stochastic property of the measurement uncertainty, ΔH_{a_k}. The stochastic property of ΔH_{a_k} can be represented by definition in (8) as follows:

$$\begin{aligned} W_k &= E\left[\Delta H_{a_k}^T \Delta H_{a_k}\right] \\ &= \begin{bmatrix} 4R_k + A_k & 0 & B_k \\ 0 & 4R_k & 4R_k \sin(\alpha) \\ B_k & 4R_k \sin(\alpha) & 4R_k \sin^2(\alpha) + C_k \end{bmatrix} \end{aligned} \tag{12}$$

where

$$E\left(\begin{bmatrix} \varepsilon_{c_{k-1}} \\ \varepsilon_{s_{k-1}} \end{bmatrix}\begin{bmatrix} \varepsilon_{c_{k-1}} \\ \varepsilon_{s_{k-1}} \end{bmatrix}^T\right) = \begin{bmatrix} A_k & B_k \\ B_k & C_k \end{bmatrix} \tag{13}$$

(13) is covariance of error which caused by transforming quadratic equation (6) to linear constraint (7). The error covariance matrix is also tuning parameter like λ.

V. EXPERIMENTAL RESULTS

To prove the performance of the proposed algorithm, the motor installed in a car window system is used and its rotation is obtained from the linear Hall sensor. With the sensor data, we estimated motor's speed and compared the results with those of [2]. Since the algorithm in [2] showed good performance as tachometer, we did not compare with tachometer in these experiments. Parameters of the estimator are as follows:

$$\begin{aligned} &\lambda = 0.95 \\ &R_k = 0.000049339[V^2] \\ &P_{0|-1} = \begin{bmatrix} 10^3 & 0 & 0 \\ 0 & 10^3 & 0 \\ 0 & 0 & 10^3 \end{bmatrix} \\ &\begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} 10^{-12} & 0 \\ 0 & 10^{-12} \end{bmatrix} \\ &\cos(\alpha) = 0.809, \quad \alpha = \frac{\pi}{5}[rad] \end{aligned} \tag{14}$$

The linear Hall sensor output is represented in Fig. 1. At the around 3000 step and the around 6000 step, there was a speed change. The motor was stopped at around 7200 step. The estimated motor's speed obtained from the linear Hall sensor data was shown in Fig. 2. The proposed algorithm had the lower error than [2] at both the low and the high speed range. However, in stopped condition, proposed algorithm showed bias error. The approximation of the linear constrained condition from quadratic equation might cause these errors.

The computer simulation results along varying frequencies were represented in Fig. 3 and Fig. 4. The mean of the estimation error in Fig. 3 showed both two algorithms increased bias error as frequency decreased. Root means square of estimation error in Fig. 4 showed also both two algorithms decrease performance as frequency come down. However, in both two cases, the proposed algorithm was better than [2]. Therefore, the results supported that the additional constrained condition can improve the estimator's performance.

VI. CONCLUSION

To estimate motor's speed from the sinusoidal signal obtained by the linear Hall sensor, we established linear state pace equation reflecting the constraint and used the RLS algorithm to cope with the measurement uncertainty. Since a conventional linear prediction algorithm has a limitation of the range in the low speed, we gave the additional information to estimator which is quadratic equation so called Pythagorean identity that is natural property in sinusoidal signal. However, since the equation is a nonlinear form, we change it to a linear constrained condition. Adding it to the measurement equation, it is possible to derive another linear state space equation which has more information without additional sensor. The experimental results and the computer simulations show that the performance of the proposed algorithm is better than that of the conventional algorithm. Moreover, it supports that the additional constrained condition can improve the estimator's performance

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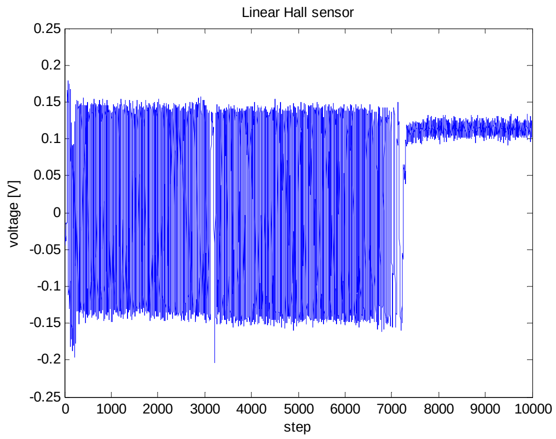

Fig 1. Linear Hall sensor output

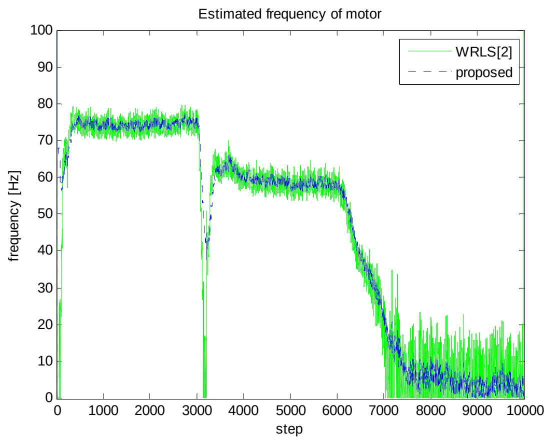

Fig 2. Frequency estimation results from linear Hall sensor

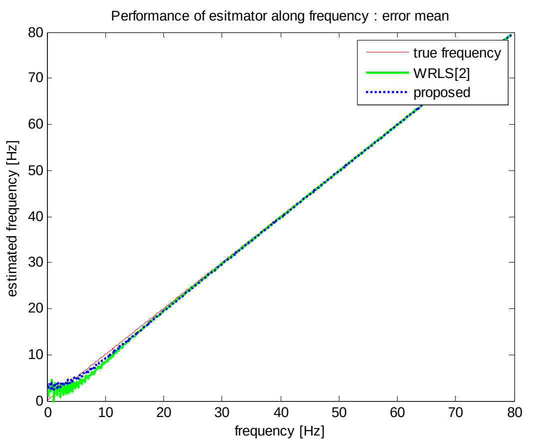

Fig 3. Performance of estimator along frequency : mean of estimation error

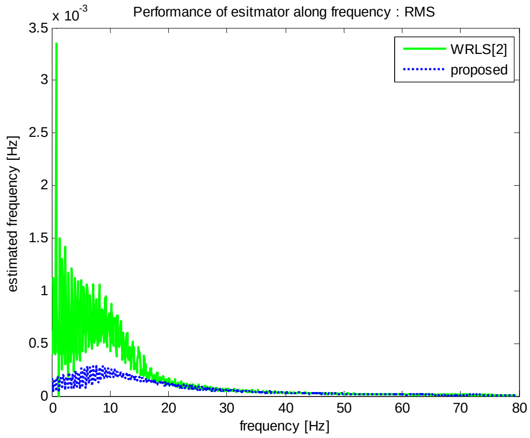

Fig 4. Performance of estimator along frequency : root mean square of estimation error