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Grasping Control of Thumb-Index Finger Model: Lyapunov Stability Approach

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Abstract

This paper is concerned with the dynamics and control of grasping and regulating motion generated by a thumb index finger robot. Thumb index finger model is the optimized model to manipulate an object without joint redundancy. To manipulate an object, the overall motion of the finger needs to be restricted by the object states. Therefore, we derive the kinematics of model governed by the object states by using four constraints which are based on the nonslipping assumption between the rigid fingertips and the surface of an object. Then, the control input is derived via Lyapunov stability analysis including the dynamics of the overall system. Further, we propose a solution of the contact forces between the fingertips and an object via physical analysis, which can not be solved mathematically. Finally, computer simulations are presented to verify the effectiveness of the proposed concept and method.

1 Introduction

Since the beginning of robotics research, the fingered hand robots have been designed to mimic human hand which has the capability of dexterous manipulations and elaborate operations. In the history of development of the fingered hand robots, various hand models with four or five fingers with two or three joints were reported [1]-[3]. They are so far used only in the open-loop control system which do not consider the relationship between the fingers and an object, because there is no way to estimate the forces between them.

In order to overcome the disadvantages of open-loop control system, Arimoto *et al.* [4]-[5] suggested a pair of robot fingers with hemispherical finger-ends using sensory motor coordination. The basic theory of this work is the passivity. The passivity means that the energy variation of the overall system, which is composed of the fingers and an object is caused by the torque generated from a joint motor. In this paper, we employ the thumb-index model which is optimized to manipulate an object such as shifting, rotating and changing contact position without joint redundancy. By the kinematic constraints of the posture of the object, The overall joint angles are dependent. Adding the dynamics of the finger model based on this properties, the control input can be calculated. The control input, however, compared to previous approach which is based on passivity, should accompany with calculating the contact force between the fingertips and an object. We propose a contact forces based on physical insight, which are impossible to be calculated mathematically. Using the kinematics and dynamics of the overall system and the contact forces, the control input is finally determined via Lyapunov stability analysis.

This paper is organized as follows. In Section II, a set of dynamics and kinematics of the fingers and an object is derived on the basis of Hamilton's principle. In Section III, the method for designing the control input is proposed and the contact forces are derived via physical analysis. Simulation results are presented to verify the effectiveness of the proposed method in Section IV.

2 Dynamics and Kinematics of Thumb Index Model

2.1 Dynamics

For the sake of physical simplicity, we assume that a 3-joint dual finger robot shown in Fig. 1 moves on a horizontal plane to ignore the gravitational force. Further, we only deal with a solid rectangular object with hard spherical fingertips.

Applying Hamilton's principle to the following

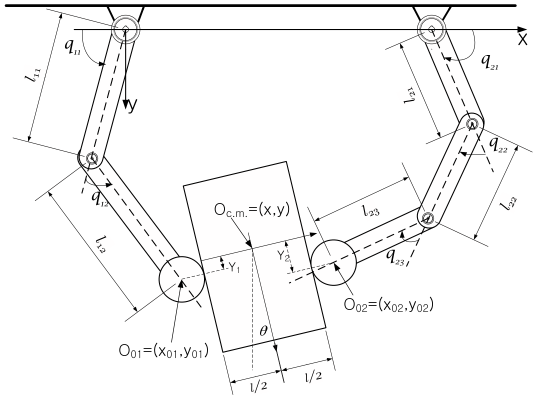

Figure 1: Thumb index finger robot system

equation,

$$\int_{t_0}^{t_1} \{\delta(K+Q+R) + u_1 \delta q_1 + u_2 \delta q_2\}\, dt = 0$$

we can obtain the dynamics of the fingers and an object described as follows:

$$H_i \ddot{\mathbf{q}}_i + \Gamma_i \dot{\mathbf{q}}_i + (-1)^i J_{0i}^T \begin{bmatrix} -\cos\theta \\ \sin\theta \end{bmatrix} f_i \\ - \left\{ J_{0i}^T \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} - r_i \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T \right\} \lambda_i = u_i, \quad i = 1, 2 \tag{1}$$

$$\begin{aligned} M\ddot{x} &= (f_1 - f_2)\cos\theta - (\lambda_1 + \lambda_2)\sin\theta, \\ M\ddot{y} &= -(f_1 - f_2)\sin\theta - (\lambda_1 + \lambda_2)\cos\theta, \\ I\ddot{\theta} &= f_1 Y_1 - f_2 Y_2 - \frac{l}{2}(\lambda_1 - \lambda_2), \end{aligned} \tag{2}$$

where Γ_1 and Γ_2 stand for the coefficient of $\dot{\mathbf{q}}_i$ including coriolis, centrifugal forces and differential functions of inertia moment. M and I are the mass and the inertia moment of an object, respectively. Further, f_i and λ_i stand for the normal and tangential contact forces, which are exerted on an object for secure grasp and dexterous movements, respectively.

2.2 Kinematics

The transformed equations from kinematic constraints are represented as follows:

$$l_{12}\cos\phi_{12} + l_{11}\cos\phi_{11} \\ = -x + (\frac{l}{2} + r_1)\cos\theta - Y_1 \sin\theta, \tag{3}$$

$$l_{12}\sin\phi_{12} + l_{11}\sin\phi_{11} \\ = y + (\frac{l}{2} + r_1)\sin\theta + Y_1 \cos\theta, \tag{4}$$

$$\phi_{12} = \frac{Y_1 - Y_1(0)}{r_1} + \theta, \tag{5}$$

$$l_{23}\cos\phi_{23} + l_{22}\cos\phi_{22} + l_{21}\cos\phi_{21} \\ = x + (\frac{l}{2} + r_2)\cos\theta + Y_2 \sin\theta - L, \tag{6}$$

$$l_{23}\sin\phi_{23} + l_{22}\sin\phi_{22} + l_{21}\sin\phi_{21} \\ = y - (\frac{l}{2} + r_2)\sin\theta + Y_2 \cos\theta, \tag{7}$$

$$\phi_{23} = \frac{Y_2 - Y_2(0)}{r_2} - \theta, \tag{8}$$

where,

$$\begin{aligned} \phi_{i1} &= q_{i1}, \\ \phi_{i2} &= q_{i2} + q_{i3}, \\ \phi_{i3} &= q_{i1} + q_{i2} + q_{i3}, \quad i = 1, 2. \end{aligned}$$

Numerically, we can find a solution of $\mathbf{q}_1$ and $\mathbf{q}_2$ by solving a set of nonlinear equations as from (3) to (8). However, we need the differential forms to derive the control input of the overall dynamic system. The differential form of the kinematics of the fingers can be derived as follow:

$$A_i \ddot{\Phi}_i = B_i \ddot{\mathbf{z}} + C_i \dot{\Phi}_i + D_i \quad i = 1, 2, \tag{9}$$

where,

$$\Phi_i = \begin{bmatrix} \phi_{i1} & \phi_{i2} & \phi_{i3} \end{bmatrix}^T, \qquad \mathbf{z} = \begin{bmatrix} \mathbf{x}^T & \mathbf{y}^T \end{bmatrix}^T,$$

$$A_1 = \begin{bmatrix} l_{11}\cos\phi_{11} & l_{12}\cos\phi_{12} - r_1\cos\theta \\ l_{11}\sin\phi_{11} & l_{12}\sin\phi_{12} - r_1\sin\theta \end{bmatrix},$$

$$A_2 = \begin{bmatrix} l_{i1}\cos\phi_{21} & l_{22}\cos\phi_{22} & l_{23}\cos\phi_{23} \\ l_{21}\sin\phi_{21} & l_{22}\sin\phi_{22} & l_{23}\sin\phi_{23} \\ 0 & 0 & r_2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 1 & \left(\frac{l}{2} + r_1\right)\cos\theta - Y_1\sin\theta & \cos\theta & 0 \\ 1 & 0 & \left(\frac{l}{2} + r_1\right)\sin\theta - Y_1\cos\theta & \sin\theta & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 1 & -\left(\frac{l}{2} + r_2\right)\cos\theta - Y_2\sin\theta & 0 & \cos\theta \\ -1 & 0 & \left(\frac{l}{2} + r_2\right)\sin\theta - Y_2\cos\theta & 0 & -\sin\theta \\ 0 & 0 & -r_2 & 0 & 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} l_{11}\sin\phi_{11}\ \dot{\phi}_{11} & l_{12}\sin\phi_{12}\dot{\phi}_{12} \\ -l_{11}\cos\phi_{11}\ \dot{\phi}_{11} & -l_{12}\cos\phi_{12}\dot{\phi}_{12} \end{bmatrix},$$

$$C_2 = \begin{bmatrix} l_{21}\sin\phi_{21}\dot{\phi}_{21} & l_{22}\sin\phi_{22}\dot{\phi}_{22} & l_{23}\sin\phi_{23}\dot{\phi}_{23} \\ -l_{21}\cos\phi_{21}\dot{\phi}_{21} & -l_{22}\cos\phi_{22}\dot{\phi}_{22} & -l_{23}\cos\phi_{23}\dot{\phi}_{23} \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_i = \begin{bmatrix} -2\dot{Y}_i\sin\theta\dot{\theta} - \dot{\theta}^2\Big(\big(\frac{l}{2}+r_i\big)\sin\theta + Y_i\cos\theta\Big) \\ 2\dot{Y}_i\cos\theta\dot{\theta} + \dot{\theta}^2\Big(\big(\frac{l}{2}+r_i\big)\cos\theta + (-1)^i Y_i\sin\theta\Big) \\ 0 \end{bmatrix}.$$

3 Design of Control Input

3.1 Control Input

The control input should consist of the states of an object because the objective of control is to manipulate an object using the dynamics of fingers.

Substituting (9) into (1), we can obtain

$$H_iT^{-1}A_i^{-1}B_i\ddot{\mathbf{z}} + H_iT^{-1}A_i^{-1}(C_iT\dot{q}_i + D_i) + \Gamma_i\dot{q}_i + (-1)^i J_{0i}^T\begin{bmatrix}-\cos\theta \\ \sin\theta\end{bmatrix} f_i - \left\{J_{0i}^T\begin{bmatrix}\sin\theta \\ \cos\theta\end{bmatrix} - r_i\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T\right\}\lambda_i = u_i, \qquad i=1,2. \tag{10}$$

For regulating the posture and position of an object, the states $\mathbf{z}$ should be guaranteed to converge to the desired states $\mathbf{z_d}$.

Theorem 1 *Assume that the control input is formulated as follows:*

$$\begin{aligned} u_i =& \Gamma_i\dot{q}_i + (-1)^i J_{0i}^T\begin{bmatrix}-\cos\theta \\ \sin\theta\end{bmatrix} f_i \\ &- \left\{J_{0i}^T\begin{bmatrix}\sin\theta \\ \cos\theta\end{bmatrix} - r_i\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T\right\}\lambda_i \\ &+ H_iT^{-1}A_i^{-1} \\ &\times\Big[C_iT\dot{q}_i + D_i - B_i\{(P_{\mathbf{z}}+1)\dot{\mathbf{z}} + P_{\mathbf{z}}(\mathbf{z}-\mathbf{z}_d)\}\Big], \end{aligned} \tag{11}$$

where, $P_{\mathbf{z}} = diag[P_x, P_y, P_\theta, P_{Y_1}, P_{Y_2}]$, and P_x, P_y, P_θ, P_{Y_1}, P_{Y_2} are strictly positive constant. Then, the states $\mathbf{z}$ are guaranteed to converge to the desired states $\mathbf{z_d}$.

3.2 Contact Forces

We have designed the control input u_i for $i=1,2$. However, the control input u_i is imperfect to control the overall system because of the lack of information about the normal and tangential contact forces such as f_1, f_2, λ_1, and λ_2. Before obtaining the contact forces, defining of $\ddot{\mathbf{x}}$ should be preceded so as to obtain those from (2). It is also necessary to define Y_1 and Y_2.

We need to redefine the Lyapunov candidate function to define the state space of $\mathbf{z}$.

Theorem 2 *If the state equation of $\mathbf{z}$ is formulated as follows:*

$$\ddot{\mathbf{z}} = -(P_{\mathbf{z}}+I)\dot{\mathbf{z}} - P_{\mathbf{z}}(\mathbf{z}-\mathbf{z}_d), \tag{12}$$

Then, these states are asymptotic stable.

Even though we have calculated (12) and we still should find one more condition to get the contact forces from (2), because (2) is not enough to obtain the contact forces. We can find this condition from the physical meaning of an object motion. To grasp an object securely, the desired normal force f_d should be exerted continuously. Only the additional force $\Delta f = M\times(\ddot{x}\cos\theta - \ddot{y}\sin\theta)$ is added to accelerate and decelerate an object.

With the condition in Table I and (12), finally we can calculate the contact forces from (2) as follows:

$$f_1 = f_d + \frac{1}{2}\big\{\Delta f + \Delta f sgn(\Delta f)\big\}, \tag{13}$$

$$f_2 = f_d + \frac{1}{2}\big\{-\Delta f + \Delta f sgn(\Delta f)\big\}, \tag{14}$$

$$\lambda_1 = \Lambda_1\ddot{\mathbf{x}} + \frac{f_d}{l}(Y_1 - Y_2), \tag{15}$$

$$\lambda_2 = \Lambda_2\ddot{\mathbf{x}} - \frac{f_d}{l}(Y_1 - Y_2), \tag{16}$$

where,

$$\Lambda_1 = \begin{bmatrix} \frac{M}{l}\Big(\frac{Y_1+Y_2}{2} + \frac{Y_1-Y_2}{2}\mathbf{sgn}(\Delta f)\Big)\cos\theta - \frac{M}{2}\sin\theta \\ -\frac{M}{l}\Big(\frac{Y_1+Y_2}{2} + \frac{Y_1-Y_2}{2}\mathbf{sgn}(\Delta f)\Big)\sin\theta - \frac{M}{2}\cos\theta \\ -\frac{I}{l} \end{bmatrix}^T,$$

$$\Lambda_2 = \begin{bmatrix} -\frac{M}{l}\Big(\frac{Y_1+Y_2}{2} + \frac{Y_1-Y_2}{2}\mathbf{sgn}(\Delta f)\Big)\cos\theta - \frac{M}{2}\sin\theta \\ \frac{M}{l}\Big(\frac{Y_1+Y_2}{2} + \frac{Y_1-Y_2}{2}\mathbf{sgn}(\Delta f)\Big)\sin\theta - \frac{M}{2}\cos\theta \\ \frac{I}{l} \end{bmatrix}^T,$$

Table 1: normal contact forces

Condition	$\ddot{x}\cos\theta - \ddot{y}\sin\theta > 0$	$\ddot{x}\cos\theta - \ddot{y}\sin\theta < 0$
Normal Forces	$f_1 = f_d + \Delta f$ $f_1 = f_d$	$f_2 = f_d$ $f_2 = f_d - \Delta f$

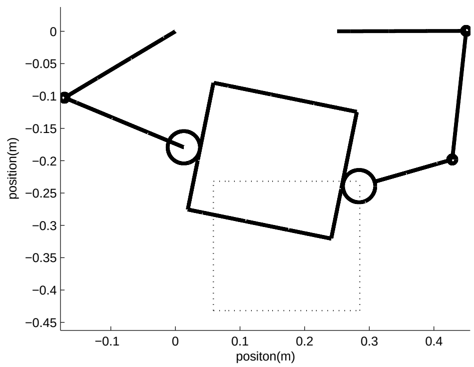

Figure 2: Final posture

Substituting (13) to (16) into (11), The overall system can be controlled by the control inputs $\mathbf{x_d}$, $\mathbf{Y}_d$ and f_d as shown in Fig. 2.

4 SIMULATION RESULTS

We carry out computer simulations in Matlab. From Fig. 2, we can confirm that the proposed control system performs secure grasp and manipulation such as shifting and changing the contact position simultaneously. From the result of Fig. 3, we can confirm that the normal contact forces f_1 and f_2, which accelerates and decelerates an object, respectively, are induced in order and converge to the desired force f_d eventually within a second. We can also confirm that the tangential contact forces λ_1 and λ_2 are induced to shift an object toward y-axis direction and eventually converge to zero. Thus, the simulation results reconfirm the effectiveness of the proposed control method.

5 Conclusion

This paper dealt with a thumb-index finger robot for grasping and regulating the posture and position of an object. We derived and analyzed the dynamics of a setup of the model with spherical fingertips pinching a rigid object. In order to calculate the kinematics of fingers, we used four geometric constraint based on the assumption of nonslipping condition between the fingertips and an object. To design the control input, we proposed the contact forces for manipulating an object, which were derived via physical insight. The computer simulation results verified the effectiveness of our proposed method.

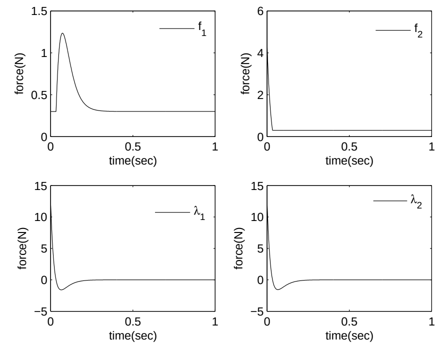

Figure 3: Normal and tangential contact forces by fingertips

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