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Formation Control of Mobile Robots with Disturbances

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Abstract

This paper proposes a formation control method for nonholonomic mobile robots in the presence of disturbances. We use the kinematic model based on the leader-following approach for the formation control of multiple robots. However, unlike many researches considering only the kinematic model, we also consider the dynamic model to obtain the torque input because it is more realistic to use the torque as the input than the velocity. Moreover, the sliding mode control method is used to deal with disturbances acting on the mobile robots. The system stability and the convergence of tracking errors are proven using Lyapunov stability theory.

1 Introduction

Over the past decade, the formation control of multiple robots has been studied by many researchers because of its usefulness in many applications, such as automated transportation, spacecraft interferometry, mitigation of natural and man-made disasters, surveillance, mapping, and border patrol [1]. Various approaches have been proposed for the formation control of multiple robots in the literature. These are roughly categorized as behavior-based [2], virtual structure [3], and leader-following [4].

In the leader-following approach, the referenced robot, called a leader, tracks the predefined trajectory, and the other robots, the followers, maintain the desired distance and angle with respect to the leader. This approach has been adopted by many researchers because of the simplicity, scalability, and reliability. Desai et al. [5] presented a feedback linearization control method for the formation of multiple mobile robots. Li et al. [6] proposed a kinematic model using Cartesian coordinates and applied a backstepping technique to the formation control of multiple mobile robots. Shao et al. [7] introduced a virtual robot to keep the relative position between the leader and followers. However, these methods only consider the kinematic model of mobile robots and do not consider the disturbances acting on the robots. The control input of the controller for the kinematic model is the velocity, but it is more realistic that the input is a torque [8]. In addition, in practice, we have to deal with unstructured uncertainties, noise, and disturbances. To solve these problems, Sanchez and Fierro [9] proposed a sliding mode controller for the leader-following robot formation. However, since the control law requires the derivative of the absolute value, it has a drawback that it is difficult to obtain the control input.

Motivated by these observations, we propose a sliding mode formation control method based on the leader-following approach.

2 Problem Statement

2.1 Kinematics and Dynamics of Mobile Robots

We consider the two-wheeled mobile robot. The posture of the mobile robot can be described by three parameters (x, y, θ) where x and y are position variables, θ is a heading direction angle. It is assumed that the driving wheels of the mobile robot purely roll and do not slip. This nonholonomic constraint can be expressed as $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$. Then, the kinematic equation in Cartesian coordinates is derived as follows:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \quad (1)$$

where v and ω are the linear and angular velocities of the mobile robot.

The dynamic equation of the mobile robot with nonholonomic constraints can be described by Euler-

Lagrange formulation as

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = B(q)\tau - A^T(q)\lambda \quad (2)$$

where $q \in \mathbb{R}^n$ is generalized coordinates, $\tau \in \mathbb{R}^r$ is a control input vector, $\lambda \in \mathbb{R}^m$ is a vector of constraint forces, $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric, positive definite inertia matrix, $V(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the centripetal and coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravitational vector, $B(q) \in \mathbb{R}^{n\times r}$ is an input transformation matrix, and $A(q) \in \mathbb{R}^{m\times n}$ is a matrix related with nonholonomic constraints.

Using $A(q)\dot{q} = 0$ and $A(q)J(q) = 0$ obtained by the nonholonomic constraints, an r-dimensional vector z exists such that $\dot{q} = J(q)z$ where $J(q) \in \mathbb{R}^{n\times r}$ consists of linearly independent vectors in the null space of $A(q)$.

Substituting the equation $\dot{q} = J(q)z$ into (2) yields

$$H(q)\dot{z} + F(q,z) = \tau \quad (3)$$

where $H(q) = (J^T(q)B(q))^{-1}J^T(q)M(q)J(q)$ and $F(q,z) = (J^T(q)B(q))^{-1}J^T(q)(M(q)\dot{J}(q) + V(q,\dot{q})J(q))z$. If the bounded disturbance τ_d exists in the mobile robot, the actual dynamic equation of the mobile robot can be rewritten as

$$H(q)\dot{z} + F(q,z) + \tau_d = \tau \quad (4)$$

where $\tau_d = H(q)f$, $f = [f_1 \quad f_2]^T$, $|f_i| \le f_{mi}$, $i = 1,2$, and $\tau = [\tau_l \quad \tau_r]^T$ is a torque vector applied to the left and right driving wheels.

Property 1 *The matrix $H(q)$ is bounded and invertible.*

2.2 Formation Model Based on the Leader-Following Approach

To design the sliding mode formation controller, we use the leader-following model. For the follower, we consider a virtual follower R_h which locates a distance D_f from the center of the follower R_f, where $D_f = \rho^d \cos\varphi^d$, ρ^d is a desired distance, and φ^d is a desired angle. The virtual follower R_h is defined as follows [7]:

$$\begin{aligned} x_h &= x_f + D_f \cos\theta_f \\ y_h &= y_f + D_f \sin\theta_f \\ \theta_h &= \theta_f \end{aligned} \quad (5)$$

where (x_f, y_f, θ_f) denotes the position and the orientation of the follower. Then, on the basis of (1), the derivative of (5) is calculated as

$$\begin{bmatrix} \dot{x}_h \\ \dot{y}_h \\ \dot{\theta}_h \end{bmatrix} = \begin{bmatrix} \cos\theta_f & -D_f \sin\theta_f \\ \sin\theta_f & D_f\cos\theta_f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_f \\ \omega_f \end{bmatrix} \quad (6)$$

where v_f and ω_f are the linear and angular velocities of the follower.

We also assume that there exists a virtual leader R_v, which locates a distance D_l from the center of the leader R_l, where $D_l = \rho^d \sin\varphi^d$. The virtual leader R_v is defined as follows:

$$\begin{aligned} x_v &= x_l + D_l\cos(\theta_l - \frac{\pi}{2}) = x_l + D_l \sin\theta_l \\ y_v &= y_l + D_l \sin(\theta_l - \frac{\pi}{2}) = y_l - D_l\cos\theta_l \\ \theta_v &= \theta_l \end{aligned} \quad (7)$$

where (x_l, y_l, θ_l) denotes the position and the orientation of the leader. The derivative of (7) can be obtained as

$$\begin{bmatrix} \dot{x}_v \\ \dot{y}_v \\ \dot{\theta}_v \end{bmatrix} = \begin{bmatrix} \cos\theta_l & D_l\cos\theta_l \\ \sin\theta_l & D_l \sin\theta_l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_l \\ \omega_l \end{bmatrix} \quad (8)$$

where v_l and ω_l are the linear and angular velocities of the leader.

Assumption 1 *The linear velocity v_l of the leader is not zero.*

3 Main Results

The objective of the formation control problem is to keep a relative pose (ρ^d, φ^d) between the leader and the follower. For this, we use the virtual follower R_h and the virtual leader R_v. That is, our control objective is to steer the virtual follower R_h so as to track the virtual leader R_v. If the virtual follower can track the virtual leader exactly, then the follower can maintain the relative pose (ρ^d, φ^d) with respect to its leader.

The tracking errors between R_h and R_v are chosen as

$$x_e = x_h - x_v, \quad y_e = y_h - y_v, \quad \theta_e = \theta_h - \theta_v \quad (9)$$

To guarantee the convergence of (9), we define the following variables as

$$s_x = \dot{x}_e + k_1 x_e, \quad s_y = \dot{y}_e + k_2 y_e, \quad s_\theta = \dot{\theta}_e + k_3\theta_e \quad (10)$$

where k_i is a positive constant, $i = 1,2,3$.

Redefine (6) and (8) as follows:

$$M_1(\theta_h, D_f)\dot{R}_h = z \quad (11)$$

$$M_2(\theta_v, D_l)\dot{R}_v = z_l \quad (12)$$

Table 1: Initial positions for four mobile robots

	$[x(0),\ y(0),\ \theta(0)]$
Leader	[0, 0, 0]
Follower 1	[0, -0.5, 0]
Follower 2	[1, 0.5, 0]

where

$$M_1(\theta_h, D_f) = \begin{bmatrix} \cos\theta_h & \sin\theta_h & 0 \\ -\alpha\sin\theta_h & \alpha\cos\theta_h & 1-\alpha D_f \end{bmatrix},$$
$$M_2(\theta_v, D_l) = \begin{bmatrix} \cos\theta_v & \sin\theta_v & -D_l \\ 0 & 0 & 1 \end{bmatrix},$$
$$z = [v_f\ \omega_f]^T,\ z_l = [v_l\ \omega_l]^T,\ \alpha = \tanh(v_l/\kappa_1). \tag{13}$$

Here, κ_1 is a positive constant. Based on (13), we propose the sliding surface S as follows:

$$S = M_1(\theta_h, D_f) \times \begin{bmatrix} s_x + (1-\alpha D_f)\sin\theta_h s_\theta \\ s_y - (1-\alpha D_f)\cos\theta_h s_\theta + \tanh(y_e s_\theta)s_\theta \\ \alpha s_\theta \end{bmatrix} \tag{14}$$

where $S = [s_1 \quad s_2]^T$.

By using the computed-torque method, we can choose the torque control input as follows:

$$\tau = H(q)\dot{z}_l + F(q,z) + H(q)u \tag{15}$$

where $u = [u_1 \quad u_2]^T$ is a control law. Substituting (15) into (4), we have

$$\dot{z} + f = \dot{z}_l + u. \tag{16}$$

In this paper, we propose the control law u as follows:

$$u = \Upsilon(R_h, R_v) - LS - P\mathrm{sgn}(S) \tag{17}$$

where $L = \mathrm{diag}[l_1, l_2] > 0$, $P = \mathrm{diag}[p_1, p_2] > 0$, and $\mathrm{sgn}(S) = [\mathrm{sgn}(s_1), \mathrm{sgn}(s_2)]^T$. $\Upsilon(R_h, R_v)$ is represented as follows:

$$\begin{aligned} &\Upsilon(R_h, R_v) = \\ &\dot{M}_1(\theta_h, D_f) \begin{bmatrix} -k_1 x_e + \dot{x}_v \\ -k_2 y_e + \dot{y}_v \\ \dot{\theta}_e - \alpha s_\theta + \dot{\theta}_v \end{bmatrix} \\ &+ M_1(\theta_h, D_f) \begin{bmatrix} -k_1\dot{x}_e + \ddot{x}_v \\ -k_2\dot{y}_e + \ddot{y}_v \\ \ddot{\theta}_e - \dot{\alpha}s_\theta - \alpha\dot{s}_\theta + \ddot{\theta}_v \end{bmatrix} \\ &- \dot{M}_2(\theta_v, D_l) \begin{bmatrix} \dot{x}_v \\ \dot{y}_v \\ \dot{\theta}_v \end{bmatrix} - M_2(\theta_v, D_l) \begin{bmatrix} \ddot{x}_v \\ \ddot{y}_v \\ \ddot{\theta}_v \end{bmatrix} \\ &- \begin{bmatrix} \frac{d}{dt}\{\sin\theta_h \tanh(y_e s_\theta)s_\theta\} \\ \frac{d}{dt}\{-\alpha(1-\alpha D_f)s_\theta + \alpha\cos\theta_h\tanh(y_e s_\theta)s_\theta\} \end{bmatrix}. \end{aligned} \tag{18}$$

Theorem 1 *Consider the dynamic model* (4) *of the mobile robot with disturbances controlled by the proposed control law* (17). *Under Property 1 and Assumption 1, the control law* (17) *stabilizes the sliding surface S. Then, x_e, y_e, and θ_e converge to zero.*

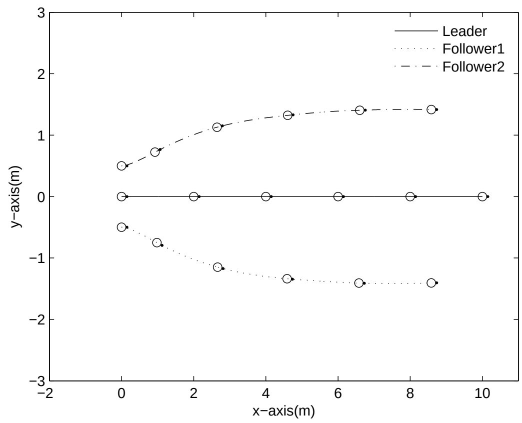
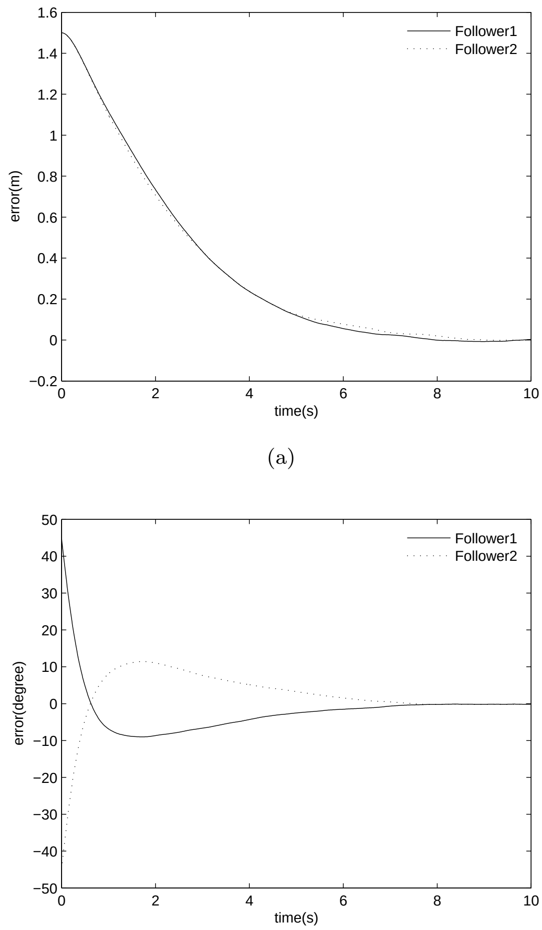


Figure 1: Trajectories of the leader and two followers for the straight line path case.

4 Simulation Results

In this section, we demonstrate the effectiveness of the proposed controller. Design parameters for the proposed controller are chosen as $k_1 = k_2 = 1$, $k_3 = 0.002$, $l_1 = l_2 = 0.5$, $\kappa_1 = 0.01$, and $p_1 = p_2 = 1$. The external disturbances are chosen to be normally distributed random noises with the upper bounds



(a)

(b)

Figure 2: Tracking errors (a) $\rho_d - \rho$ (b) $\varphi_d - \varphi$

$f_{m1} = f_{m2} = 0.5$. To simulate the formation control, we assume that there exist three mobile robots. The initial positions for three mobile robots are presented in Table I. The leader is moving on a straight line with velocities $[v_l,\ \omega_l]$=[1 m/s, 0 rad/s]. The desired relative poses of followers with respect to leader are $\rho^d_{1,2} = 2\ m$, $\varphi^d_1 = \pi/4\ rad$, and $\varphi^d_2 = -\pi/4\ rad$. Figs. 1 and 2 show the trajectories of the leader and followers and tracking errors, respectively.

5 Conclusions

In this paper, a new sliding mode formation controller for nonholonomic wheeled mobile robots with external disturbances has been proposed. The kinematic model based on the leader-following approach has been considered. The dynamic model has also been considered to obtain the control law at the torque level. The system stability and the convergence of tracking errors have been proven using Lyapunov stability theory.

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