Observer-based fuzzy control for nonlinear networked control system with pack drop

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Abstract: In this paper, an observer-based fuzzy controller is proposed for the nonlinear networked control systems (NCSs) with packet drop. Using Takagi-Sugeno (T-S) fuzzy model, the nonlinear NCS is represented by a fuzzy system, and the observer-based controller is design in the fuzzy form. The stochastic stability condition of the closed-loop system is obtained by Lyapunov functional. Its sufficient condition is represented to the linear matrix inequality (LMI) form and the observer and control gains are obtained by LMI. An example is given to demonstrate the verification discussed throughout the paper.

Keywords: nonlinear networked control system, stochastic stability, fuzzy observer-based control, packet drop, linear matrix inequality

I. INTRODUCTION

During the recent years, due to increasing of the ubiquitous computing, the importance of the networked control system (NCS) is grown and many people have concerned about stability and stabilization of system. The NCS offers many advantages in the economy and efficiency, and has applications in wireless networked system, internet-based control, automation technology, communication technique and so on. However, it has some problem such as packet drop, time delay, and sampling. To solve these problems, many papers have been published[1-8].

The packet drop is one of the main problems in NCS. It needs a stochastic approach to solve the packet drop problem. Thus, traditional deterministic stabilization method is not useful in packet drop problem. Moreover, if NCS includes the nonlinearity, uncertainty, time delay, and so on, it is more difficult to obtain the stability condition.

In a few number of research concerned stability problem of NCS[4-8]. Wang[6] studied the robust control method for NCS with packet drop. Hu[7] developed the optimization problem of packet dropping margin. But they did not considered the nonlinearity problem in NCS. Zhang[8] designed the fuzzy controller for nonlinear systen with packet drop. This paper used the state feedback method, which needs the information of all states, but it is impossible in NCS.

In this paper, we aim to solve the problem of stability of a fuzzy NCS with output packet drop, input transmission failure and Gaussian noise. An observer-based controller is proposed for the stochastic stability. Using a Lyapunov functional, the sufficient condition for stability is offered in the linear matrix inequality (LMI) format. A numerical example is presented to demonstrate the effectiveness of the proposed controller and theorem.

II. PRELMINIRIES

Consider a fuzzy networked control system, in which the i th IF-THEN rule is represented as follows:

$$\begin{aligned}&\textit{Plant Rule } i:\\&\textit{IF } z_1 \textit{ is } \Gamma_1^i, \cdots, \textit{and } z_p \textit{ is } \Gamma_p^i\\&\textit{THEN} \begin{cases} x_{k+1} = A_i x_k + B_i u_k + D_i x_k \omega_k, \\ y_k = \alpha_k C x_k, \qquad (1 \le i \le r) \end{cases}\end{aligned} \tag{1}$$

where x_k is state variable, u_k is input, y_k is output, and ω_k is a scalar zero-mean Gaussian white noise with $\mathrm{E}\{\omega_k^2\} = \gamma^2$. Γ_q^i is a fuzzy set for $1 \le q \le p$, r is the number of fuzzy rules, A_i, B_i, C , D_i are nominal system matrices, α_k is a stochastic variable which determines the packet drop of output part. This stochastic variable is a Bernoulli process as the following probability:

$$\begin{aligned} \Pr ob\{\alpha_k = 1\} &= \mathrm{E}\{\alpha_k\} = \hat{\alpha} \\ \Pr ob\{\alpha_k = 0\} &= 1 - \mathrm{E}\{\alpha_k\} = 1 - \hat{\alpha} \end{aligned}$$

Using the center-average defuzzification, product inference, and singleton fuzzifier, the T-S fuzzy system (1) is inferred as follows:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} \theta_i (A_i x_k + B_i u_k + D_i x_k \omega_k) \\ y_k &= \alpha_k C x_k \end{aligned} \tag{2}$$

where

$$\theta_i = \left(\prod_{q=1}^{p} \Gamma_q^i(z_q)\right) \Bigg/ \left(\sum_{i=1}^{r}\left(\prod_{q=1}^{p} \Gamma_q^i(z_q)\right)\right)$$

and $\Gamma_q^i(z_q)$ is the degree of the membership function.

Hence, we consider a observer-based fuzzy controller for fuzzy networked control system in the following form:

$$\begin{aligned} &Controller \quad Rule \quad i: \\ &IF \quad z_1 \quad is \quad \Gamma_1^i, \cdots, and \quad z_p \quad is \quad \Gamma_p^i \\ &THEN \begin{cases} \hat{x}_{k+1} = A_i \hat{x}_k + B_i u_k + L_i (y_k - \hat{y}_k), \\ \hat{y}_k = \hat{\alpha} C \hat{x}_k \\ u_k = \beta_k K_i \hat{x}_k \qquad (1 \le i \le r) \end{cases} \end{aligned} \tag{3}$$

where $\hat{x}_k$ is an estimation of state variable, L_i and K_i are observer and controller gains, respectively. β_k is a stochastic variable which determines the transmission failure of input part. This stochastic variable is a Bernoulli process as the following probability:

$$\begin{aligned} \Pr ob\{\beta_k = 1\} &= \mathrm{E}\{\beta_k\} = \hat{\beta} \\ \Pr ob\{\beta_k = 0\} &= 1 - \mathrm{E}\{\beta_k\} = 1 - \hat{\beta} \end{aligned}$$

The input-output form of the controller is then

$$\begin{aligned} \hat{x}_{k+1} &= \sum_{i=1}^{r} \theta_i (A_i \hat{x}_k + B_i u_k + L_i (y_k - \hat{y}_k)) \\ \hat{y}_k &= \hat{\alpha} C x_k \\ u_k &= \sum_{i=1}^{r} \theta_i \beta_k K_i \hat{x}_k \end{aligned} \tag{4}$$

Assumption 1. The stochastic variable of the output packet drop α_k, the stochastic variable of the input transmission failure β_k, and the Gaussian noise ω_k are independent.

$$\mathrm{E}\{\alpha_k \beta_k \omega_k\} = \mathrm{E}\{\alpha_k\}\,\mathrm{E}\{\beta_k\}\,\mathrm{E}\{\omega_k\}$$

Assumption 2. The output matrix C has full row rank, i.e., there exists the inverse of CC^T.

Consider the estimation error as follows

$$e_k = x_k - \hat{x}_k \tag{5}$$

Substituting (4) into (2) and (5), we obtain the following closed-loop system:

$$\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \sum_{i=1}^{r}\sum_{i=1}^{r} \theta_i \theta_j (\Phi_{ij} + \varepsilon_k \Lambda_{ij} + \hat{D}_i \omega_k) \begin{bmatrix} x_k \\ e_k \end{bmatrix} \tag{6}$$

where

$$\Phi_{ij} = \begin{bmatrix} A_i + \hat{\beta} B_i K_j & -\hat{\beta} B_i K_j \\ 0 & A_i + \hat{\alpha} L_i C \end{bmatrix},$$

$$\varepsilon_k = \begin{bmatrix} (\beta_k - \hat{\beta}) I & 0 \\ 0 & (\alpha_k - \hat{\alpha}) I \end{bmatrix},$$

$$\Lambda_{ij} = \begin{bmatrix} B_i K_j & -B_i K_j \\ -L_i C & 0 \end{bmatrix}, \qquad \hat{D}_i = \begin{bmatrix} D_i & 0 \\ D_i & 0 \end{bmatrix}$$

The objective of this paper is to obtain the stability condition of the closed-loop system (6). But system (6) includes the stochastic variable, it is not impossible to guarantee the traditional deterministic stable condition. We solve this problem in next section.

III. MAIN RESULTS

For the fuzzy networked control system with stochastic variable, we need to define the notion of stochastic stability.

Definition 1. The equilibrium point $x_k = 0$ of the closed-loop networked control system (6) is said to be stochastically stable if, for each $\mu > 0$, there is $\delta = \delta(\mu) > 0$ such that

$$\|x_0\| < \delta \Rightarrow \|x_k\| < \mu, \quad \forall k \ge 0$$

In order to derive the stability condition in the LMI form, we need the following lemmas.

Lemma 1[9]. If there exist a Lyapunov functional $V(x_k)$ and a nondecreasing convex function $a(\eta)$, such that $a(0)=0$ and $a(\eta)>0$ for $\eta>0$, satisfying the following three conditions:

1) $V(0)=0$
2) $a(\|x_k\|)\le V(x_k)$
3) $\mathrm{E}\{V(x_{k+1})\}-\mathrm{E}\{V(x_k)\}<0$

then the equilibrium point $x_k=0$ of the fuzzy networked control system (6) is stochastically stable.

Lemma 2[10]. For any real matrices X_i, Y_i for $1\le i\le n$, and $S\succ 0$ with appropriate dimensions, we have

$$2\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}h_ih_jh_kh_lX_{ij}^TSY_{kl}$$
$$\le\sum_{i=1}^{n}\sum_{j=1}^{n}h_ih_j\left(X_{ij}^TSX_{kl}+Y_{ij}^TSY_{kl}\right)$$

where $h_i\left(1\le i\le n\right)$ is defined as $h_i\ge 0$, $\sum_{i=1}^{n}h_i=1$

Lemma 2[13]. Given constant symmetric matrices N, O and L of appropriate dimensions, the following inequalities

$$O\succ 0,\qquad N+L^TOL\prec 0$$

are equivalent to the following inequality

$$\begin{bmatrix}N & L^T\\ L & -O^{-1}\end{bmatrix}\prec 0\quad\text{or}\quad\begin{bmatrix}-O^{-1} & L\\ L^T & N\end{bmatrix}\prec 0$$

The sufficient condition is provided for the stochastic stability of the closed-loop system (6) in the following theorem.

Theorem 1. If there exist some symmetric and positive definite matrix Q, some matrices W_i, N_i, such that the following LMIs are satisfied, then the networked closed-loop system (6) is stochastically stable.

$$\begin{bmatrix}-Q & * & * & * & * & * & *\\ 0 & -Q & * & * & * & * & *\\ \Omega_{ii} & -\hat{\beta}B_iW_i & -Q & * & * & * & *\\ 0 & \Psi_i & 0 & -Q & * & * & *\\ \tilde{\beta}B_iW_i & -\tilde{\beta}B_iW_i & 0 & 0 & -Q & * & *\\ -\tilde{\alpha}N_iC & 0 & 0 & 0 & 0 & -Q & *\\ \gamma D_i & 0 & 0 & 0 & 0 & 0 & -2Q\end{bmatrix}\prec 0 \tag{7}$$

$$\begin{bmatrix}-Q & * & * & * & * & * & *\\ 0 & -Q & * & * & * & * & *\\ \Omega_{ij}+\Omega_{ji} & -\hat{\beta}\tilde{W}_{ij} & -Q & * & * & * & *\\ 0 & \Psi_i+\Psi_j & 0 & -Q & * & * & *\\ \tilde{\beta}\tilde{W}_{ij} & -\tilde{\beta}\tilde{W}_{ij} & 0 & 0 & -Q & * & *\\ -\tilde{\alpha}\tilde{N}_{ij} & 0 & 0 & 0 & 0 & -Q & *\\ \gamma D_i & 0 & 0 & 0 & 0 & 0 & -2Q\end{bmatrix}\prec 0 \tag{8}$$

and

$$CQ=MC \tag{9}$$

for $1\le i,\ j\le r$ and $i\ne j$.

where

$$W_i=K_iQ,\qquad N_i=L_iM$$
$$\tilde{\alpha}=\sqrt{(1-\hat{\alpha})\hat{\alpha}},\qquad \tilde{\beta}=\sqrt{(1-\tilde{\beta})\tilde{\beta}}$$
$$\Omega_{ij}=A_iQ+\hat{\beta}B_iW_j,\qquad \Psi_i=A_iQ+\hat{\alpha}N_iC$$
$$\tilde{W}_{ij}=B_iW_j+B_jW_i,\qquad \tilde{N}_{ij}=N_iC+N_jC$$

and $*$ denotes the transposed element in symmetric position. The observer gain and control gain are obtain by the following equations:

$$K_i=W_iQ^{-1},$$
$$L_i=N_i\left\{CQC^T(CC^T)^{-1}\right\}^{-1}$$

Proof) It is omitted in this paper.

IV. SIMULATION

Consider a fuzzy system which is represented in the following form:

$$x_{k+1}=\sum_{i=1}^{2}\theta_i(A_ix_k+B_iu_k+D_ix_k\omega_k)$$
$$y_k=\alpha_kCx_k$$

where

$$x_k=\begin{bmatrix}x_{k1} & x_{k2}\end{bmatrix}^T,$$
$$A_1=\begin{bmatrix}0.8 & 0.6\\ 0.4 & 0.7\end{bmatrix},\qquad A_2=\begin{bmatrix}0.9 & 0.2\\ 0.3 & 0.8\end{bmatrix}$$
$$B_1=\begin{bmatrix}1\\ 0\end{bmatrix},\qquad B_2=\begin{bmatrix}1.2\\ 0\end{bmatrix},\qquad C=\begin{bmatrix}1 & 0\end{bmatrix}$$
$$D_1=\begin{bmatrix}0.2 & 0\\ 0 & 0.1\end{bmatrix},\qquad D_2=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}$$
$$\theta_1=\exp\left[-2x_{k1}^2\right],\qquad \theta_2=1-\theta_1$$

We assume the stochastic variables as follows:

$$\mathrm{E}\left\{\omega_k^2\right\}=1$$
$$\Pr ob\left\{\alpha_k=1\right\}=\Pr ob\left\{\beta_k=1\right\}=0.2$$

Using LMIs (7) and (8), The observer gain and control gain is obtained.

$$K_1 = \begin{bmatrix} -0.8657 & -0.6188 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.5657 & -0.3087 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.8399 \\ 0.6484 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.7059 \\ 0.3933 \end{bmatrix}$$

Time responses for each state are shown in Fig.1 and Fig.2 with the initial condition $x_k = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$.

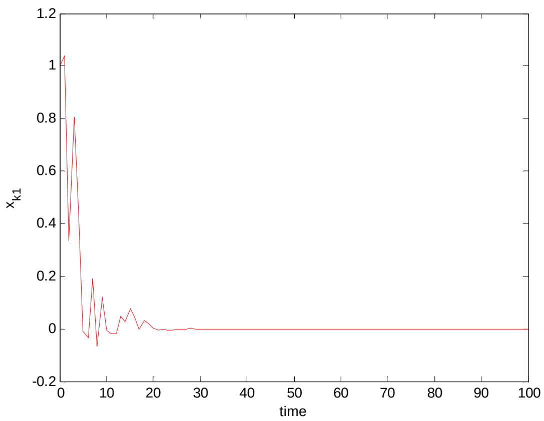

Fig.1. Time response of x_{k1}

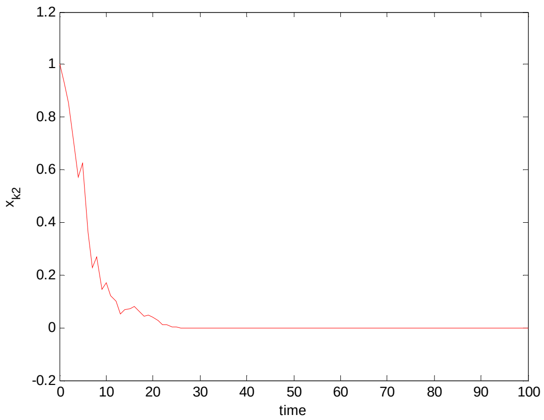

Fig.2. Time response of x_{k2}

V. CONCLUSION

In this paper, the observer-based fuzzy controller has been proposed for the nonlinear networked control systems with packet drop. The controller is designed in the T-S fuzzy system form and sufficient conditions for stochastic stabilization of the closed-loop system are designed in the LMI format. The numerical example has been shown to prove the advantage of the developed method.

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