Development for on-board use anti-tilting table with Horizontally slider type parallel mechanism —Analysis on Inverse Kinematics and Work-space of the Mechanism

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Abstract: In this paper, the authors propose a new type of parallel mechanism, i.e. 6-STRT parallel mechanisms, and its applications. This mechanism is consisted of 6-sliders of horizontal motion attached on a base plate and 6-links of which lower ends are connected to sliders and upper ends to end plate via universal joints, so the end plate can be moved in 6 degree of freedom. The authors are intending to apply this mechanism making perfectly stable stage on the ship regardless all kind of ship motion such as rolling, pitching or heaving.

In this paper, the authors introduce the specialty and kinematics of this mechanism with the matrix-vector method as the first step of our research. The searching algorithm of work-space boundary in this research is characterized with simplicity, wide range of application, clearness of physical meaning, preciseness of mathematical logic, and easiness of programming method and it is thought to be very practical method on the robot design. The work-space and configuration of the parallel robot is analyze in terms of the relationship between work-space and structural parameter of parallel mechanism.

Keywords: Parallel mechanism, Inverse kinematics, Work-space.

I. INTRODUCTION

The initial application of parallel robot can be traced back to 1930s. Really arouse interests in the field of kinematics of machinery was the 6 degrees of freedom parallel mechanism i.e. Stewart-platform^[1] proposed by Stewart in 1965. Until 1978, the Professor Hunt used parallel mechanism as the robot structure for the first time which opened the prelude to the study of parallel robot^[2]. With light structure, stiffness non-cumulative error and other unique nature, parallel robot has become a hot spot in robotic research since the 1990s. There are many types of parallel mechanism, such a stretch type represented in Stewart platform, such a rotating structure represented in Delta mechanism^[3] or linear motion mechanism^[4]. Those mechanisms have many merits, but many shortages for the application to ship. Then the authors have developed a new type of parallel robot, as shown in Fig.1.

This paper introduces a new type of parallel robot developed by the authors, see Fig.1. The motion of 6 degrees of freedom of end plate is driven by six links, which are driven by the movement of six sliders installed in the base plate. The merits of the structure are as follows.

1. The distance between base plate and end plate become minimum in the parallel mechanism.

2. The stiffness of this mechanism is better than stretch type and rotary type.

3. As a result of screw driver, this mechanism has higher output power and better safety.

4. As the actuators installed in the base plate, the mass and inertia of moving parts is smaller and the output response is faster.

The following is the structure sketch of this parallel mechanism.



Fig.1. the 6-STRT Parallel mechanism

II. MODELING OF KINEMATICS

1. Modeling of Inverse Kinematics

The authors propose to use a minimal set of parame-

ters in order to derive the inverse kinematic models: the main feature of this set is the definition of two coordinate systems:

 $\Sigma_b(o_b - x_b y_b z_b)$: The coordinate system is fixed on the base plate and the origin is coincident with the center point of the base plate, as shown in Fig.2.

 $\Sigma_t (o_t - x_t y_t z_t)$: The coordinate system is fixed on the end plate and the origin of coordinate is coincident with the center point of the end plate, as shown in Fig.3. Here we define some parameters used in this paper as follows:

x, y, z: position parameter of the end plate.

 α , β , γ : posture parameter of the end plate.

E: length of link.

Lij: displacement of sliders.

 ${}^{A}_{B}R$: transformation matrix.

d: distance of joint to its symmetry center line.

h: distance of joint to center point of end plate.

e: distance of slider to its symmetry center line.

 δ_i : allocation angle of the end plate.

 ϕ_i : allocation angle of the base plate.

Here, we show two numbers of slider sets with i (=1,2,3) and divide each slider with j (=1,2).



Fig.2 Kinematics model of base plate



Fig.3 Kinematics model of end plate

Let us take out one motion chain, which represents the relationship between the relative position of two coordinate systems (i.e. Σ_b and Σ_t).



Fig.4 Model of a motion chain

According to Fig.4, the relations of each vector are as follows:

$${}^{b}P_{ij} = {}^{b}P_{ot} + {}^{A}_{B}R {}^{i}P_{ij}$$

$$\tag{1}$$

$${}^{b}P_{ot} = \begin{bmatrix} x & y & z \end{bmatrix}^{T}$$
(2)

$$E_{ij} = {}^{b}P_{ot} + {}^{A}_{B}R {}^{i}P_{ij} - S_{ij}$$
(3)

$${}^{A}_{B}R = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

with: $c\alpha = \cos \alpha \quad s\alpha = \sin \alpha \quad c\beta = \cos \beta$ $s\beta = \sin \beta \quad c\gamma = \cos \gamma \quad s\gamma = \sin \gamma$

The authors first express the vectors of each joint on the end plate in the frame Σ_t , which can be written as:

$${}^{t}P_{ij} = \begin{bmatrix} \cos \delta_{i} & -\sin \delta_{i} & 0\\ \sin \delta_{i} & \cos \delta_{i} & 0\\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h\\ (-1)^{j}d\\ 0 \end{bmatrix}$$
(4)
with: $\delta_{i} = 2\pi (i-1)/3$ $i=1,2,3$

Besides, we can write the vectors of each slider on the base plate in the frame Σ_b :

$$S_{ij} = \begin{bmatrix} \cos \phi_i & -\sin \phi_i & 0\\ \sin \phi_i & \cos \phi_i & 0\\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_{ij} \\ (-1)^j e\\ 0 \end{bmatrix}$$
(5)
with: $\phi_i = 2\pi (i-1)/3$ $i = 1, 2, 3$

Let us note coordinates of each slider in the frame Σ_b : (x_{ij}, y_{ij}, z_{ij}), besides, coordinates of each joint in the frame Σ_b :(X_{ij}, Y_{ij}, Z_{ij}), then following equation stands:

$$(X_{ij} - x_{ij})^2 + (Y_{ij} - y_{ij})^2 + Z_{ij}^2 = E^2$$
(6)

The previous calculations give the solution to the inverse kinematics model of the parallel robot:

$$L_{ij} = B_{ij} + \sqrt{B_{ij}^2 - C_{ij}}$$
(7)

where:

$$B_{ij} = X_{ij} \cos \phi_i + Y_{ij} \sin \phi_i$$

$$C_{ij} = 2(-1)^j e(X_{ij} \sin \phi_i - Y_{ij} \cos \phi_i) + X_{ij}^2$$

$$+ Y_{ij}^2 + Z_{ij}^2 + e^2 - E^2$$

2. Velocity and Acceleration of the Mechanism

From equation (7), the displacement of each slider L_{ij} is the function of the parameters with regard to position and posture of end plate, which can be written as follows:

$$L_{ii} = f_{ii}(x, y, z, \alpha, \beta, \gamma)$$
(8)

When we partial differentiate both sides by t(time), then we get the velocity of the each slider.

$$V_{i,j} = \begin{bmatrix} \frac{\partial f_{ij}}{\partial x} & \frac{\partial f_{ij}}{\partial y} & \cdots & \frac{\partial f_{ij}}{\partial \gamma} \end{bmatrix} \begin{bmatrix} v_p \\ \omega_p \end{bmatrix}$$

$$= J_a \begin{bmatrix} v_p \\ \omega_p \end{bmatrix}$$
(9)

where: J_a denotes Jacobin matrix.

 v_p , ω_p : Velocity and angular velocity of the end plate.

If Jacobin matrix J_a is none-singular matrix, then we can get the velocity of end plate from velocity of each slider.

As the same manner, with differentiating equation (9) by t(time) we get the acceleration of each slider.

$$a_{ij} = A_{ij} \begin{bmatrix} v_p \\ \omega_p \end{bmatrix} + J_a \begin{bmatrix} a_p \\ \varepsilon_p \end{bmatrix}$$
(10)
ith: $A_{ij} = \begin{bmatrix} \frac{\partial^2 f_{ij}}{\partial x^2} & \frac{\partial^2 f_{ij}}{\partial y^2} & \cdots & \frac{\partial^2 f_{ij}}{\partial \gamma^2} \end{bmatrix}$

 a_p, ε_p : acceleration and angle acceleration of the end plate.

Similarly, if J_a is none-singular matrix, we can get the acceleration of the end plate from acceleration of each slider.

3. Constraints of Inverse Kinematics

According to Geometric relationship and material properties of the parallel mechanism, the constraint condition of motion of each slider is as follows.

$$L_{\min} \ge h$$

$$L_{\max} \le h + \sqrt{E^2 - d^2 - e^2 + 2ed}$$
(11)

III. ANALYSIS ON WORK-SPACE

Generally, the shape of work-space of parallel robot is thought to be very complicated and difficult. We would like to introduce a simple algorithm to search the work-space boundary in this paper.

1. Cylindrical Coordinate Search Method

As the motion of proposed parallel mechanism is concentrated in a cylindrical space, it is easy to imagine that its work-space should be searched under cylindrical coordinate. At first, we consider a cutting plane through *z*-axis and ρ axis. The ρ -axis exists on a *x*-*y* plane and makes an angle of $\theta(m)(=m \cdot \Delta \gamma)$ to *x*-axis as shown in Fig.5. Here $\Delta \gamma$ is a small angle divided cylindrical space with circular direction and *m* is a counted number of $\Delta \gamma$ from *x*-axis. Now, we will search for $\rho - z$ plane. We divide this plane with a strip of $\Delta \rho$ width along ρ -axis and Δz width along *z*-axis. Then the block position of s^{th} along ρ -axis and n^{th} along *z*axis in $\rho - z$ plane are shown as following.

$$\rho(s) = s \cdot \Delta \rho, \ z(n) = n \cdot \Delta z$$

The coordinate (x, y, z) in this space are presented with using (ρ, θ, z) .

$$x = x(s,m) = \rho(s)\cos(\theta(m))$$

$$y = y(s,m) = \rho(s)\sin(\theta(m))$$
 (12)

$$z = z(n) = z(n)$$

Here, $s, m, n \ge 0$

Now we confirm the existence of solution for all (s, m, n) with inverse kinematics.

The principle of the cylindrical coordinate search method is as follows:



Fig. 5 Principle of cylindrical coordinate search

2. Analysis on Work-space of 6-STRT Parallel Robot

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The authors analyze the work-space of 6-STRT type parallel robot of which link length are l_1 , l_2 and l_3 by cylindrical coordinate search method.

The Fig.6 is y-z section of work-space boundary of 6-STRT type parallel robot of which link length are l_1 , l_2 and l_3 .



Fig.6 y-z section of work-space boundary

The x-z section of work-space boundary is shown in Fig.7.



We can conclude to the above mentioned analysis, with the growth of the length of link, the center of work-space will move upward and the work-space will be gradually becoming smaller. This conclusion is significant for the optimal design of the parallel mechanism.

Certainly, there are some factors else will influence the work-space of parallel robot for example location of each joint, etc, but we will not discuss in this paper, as the effect is not large.

Finally, the authors show the three-dimensional workspace of the 6-STRT type parallel robot got with the cylindrical coordinate search method in Fig.8.



Fig.8 3D work-space of the parallel robot

IV. CONCLUSION

In this paper the authors propose a new type of the parallel mechanism; this mechanism consists of 6-sliders of horizontal motion and 6-links of which lower ends are connected to sliders and upper ends to end plate with universal joints. This mechanism can move the end plate in 6 degree of freedom, with higher power than conventional parallel mechanism, so the authors are thought to apply this mechanism to make a stable surface on ship. In the first of this research, the authors analyze inverse kinematics by the matrix-vector method and present the mathematical algorithm of work-space based on cylindrical coordinate of this mechanism for optimal design.

REFERENCES

[1] D. Stewart, A Platform six Degree of Freedom Proceedings of the Institute of mechanical Engineers. 1965, 180 (5) Part1: pp.371-386.

[2] K. H. Hunt, Kinematic Geometry of Mechanisms. Clarendon Press. Oxford. 1978, pp. 421-426.

[3] F.Pierrot, A. Fournier and P. Dauchez, Towards a Fully parallel 6-DOF robot for high-speed applic ations. IEEE Transactions on Robotics and Automation, Sacra mento, California(1991-4), France. pp.1289-1292.

[4] Takanori Masuta, Motoyoshi Fujiwara and Tatsuo Arai. Specific Kinematic Changes in a Linear-Actuated Parallel Mechanism According to Differences in Actuator Arrangement. JSME, Vol65-659(2001-7).