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Motion Analysis of Tripod Parallel Mechanism

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Abstract: The Tripod Parallel Mechanism consists of three links of fixed length and a rigid platform, and they are connected by revolute joints. The platform can be achieved six-degree-of freedom (6-DOF) motion by the movement of the bottom ends of the three links on the horizontal plane. This mechanism has advantages over the common six extendible parallel manipulators. It has a much larger work space, simple structure and so on.

In this paper, it is shown that the vector analysis for this Tripod Parallel Mechanism and derive the positions of the bottom ends of three links for the attitude of platform by the inverse kinematics and the conditions of geometrically-constraint. And then, the trajectories of bottom ends of three links by numerical simulation are shown.

Keywords: tripod parallel mechanism, six degree of freedom, inverse kinematics, geometrically constraint

I. INTRODUCTION

The Stewart type Parallel Mechanism which achieves 6-DOF motion by coordinated motion of six actuators has many advantages compared with the conventional serial link mechanism [1]. They are:

- higher payload-to-weight ratio since the payload is carried by six cylinders in parallel,
- higher accuracy due to non-cumulative joint error,
- higher structural rigidity,
- simpler solution of the inverse kinematics equations.

On the other hand, this type manipulator has some weaknesses that are:

- small work space,
- complex structure

A different type of parallel mechanism is proposed in [2]. It consists of three links of fixed length and a rigid platform, and they are connected by revolute joints. The platform can be achieved 6-DOF motion by the coordinated movement of the bottom ends of the three links of fixed length on the horizontal plane. This mechanism has a much larger work space and simpler configuration than the Stewart type parallel mechanism.

Figure1 shows the same type of above parallel mechanism which is produced experimentally by us. We call this mechanism, "The tripod parallel mechanism". For our parallel mechanism, the planer motion of the bottom end of three links is actualized by the X-Y unit which consists of two linear drive actuators. From the experimental results, it was confirmed that the tripod parallel mechanism has much larger work space than the Stewart type parallel mechanism.

Fig.1. Tripod parallel mechanism

In this paper, as the next step of the research, it is reported that a vector analysis of the tripod parallel mechanism and a derivation method of the positions of bottom ends of three fixed links for the attitude of platform by the inverse kinematics and conditions of geometrically-constraint. And then, the trajectories of bottom ends of three links by numerical simulation are shown.

II. MOTION ANALYSIS

Manipulation tasks are usually given as a set of positions and orientations in the world reference frame of the platform trajectory. To achieve these tasks it is necessary to transform the platform trajectory into the motion of bottom ends of fixed length links. This transformation is known as the inverse kinematic problem and in this case, it is the calculation of the position of the three bottom ends from a given position and orientation of the movable platform.

1. Structure of the tripod parallel mechanism

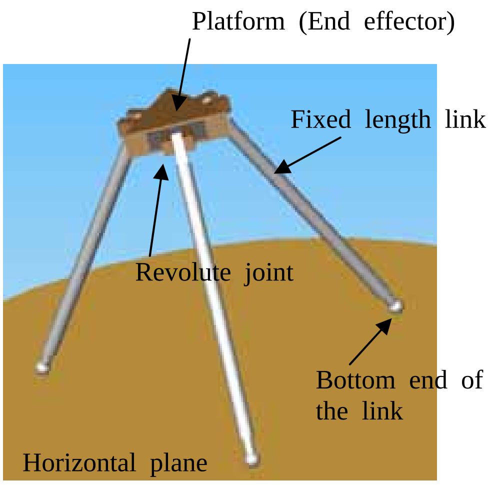

Fig.2. Simplified diagram of Tripod mechanism

As the preparation for motion analysis, we describe the structure of tripod parallel mechanism. As shown in Fig.2, the tripod parallel mechanism consists of a platform (end-effector) and three fixed length links, and they are connected by revolute joints. The bottom ends of three links are given two degree of freedom motion on the horizontal plane with any way. In the case of our experimental setup, this motion can be actualized by the pair of linear actuators.

Coordinated motion of three bottom ends of links is converted through a spatial mechanism with fixed length links, into a six-degree-of-freedom motion of the platform.

2. Vector analysis

Figure3 shows the vector diagram of the tripod parallel mechanism. Upper triangle plate "a_1, a_2, a_3" is platform and the sphere "◯" is bottom end of link. These three bottom ends are constrained on the Horizontal plane. This system has two reference frames, $(\mathbf{i},\mathbf{j},\mathbf{k})^T$ and $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)^T$. (Using this notation that $\mathbf{i},\mathbf{j},\mathbf{k}$ and $\mathbf{e}_i$ denote the unit vectors along the respective coordinate axes for each of the two frames). The one is the motion frame which is located at the centroid of the platform. The other is the world reference frame. Sign $\mathbf{O}'$ denotes the origin of the motion frame and sign $\mathbf{O}$ denotes the origin of the world reference frame.

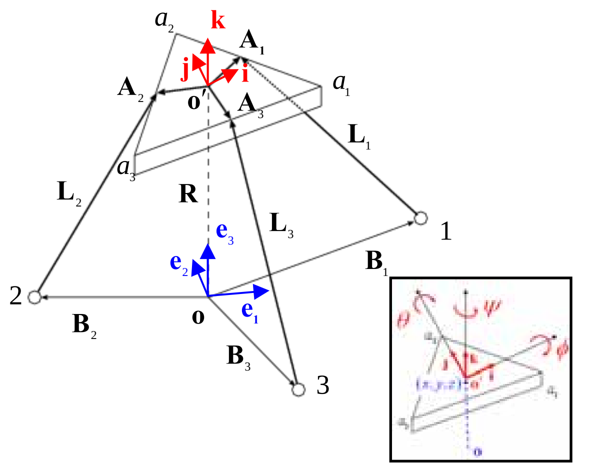

Fig.3. Vector diagram of Tripod parallel mechanism

Vector $\mathbf{A}_i = (a_{i1}, a_{i2}, a_{i3})(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)^T$ $(i=1,2,3)$ denotes the revolute joint which connects the platform and the fixed length link, $\mathbf{B}_i = (b_{i1}, b_{i2}, b_{i3})(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)^T$ $(i=1,2,3)$ is position vector of the bottom end of link and $\mathbf{R} = (x,y,z)(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)^T$ is the position vector of centroid $\mathbf{O}'$ with respect to the world reference frame. For the foregoing vectors and the fixed length link vector $\mathbf{L}_i = (l_{i1}, l_{i2}, l_{i3})(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)^T$ $(i=1,2,3)$, the following relation exists:

$$\mathbf{L}_i = \mathbf{A}_i + \mathbf{R} - \mathbf{B}_i \qquad (i=1,2,3)\ . \tag{1}$$

Using vector $\mathbf{A}_{im} = (a_{im1}, a_{im2}, a_{im3})(\mathbf{i},\mathbf{j},\mathbf{k})^T$ $(i=1,2,3)$ which denotes the revolute joint with respect to the motion frame and the coordinate transform matrix

$$\mathbf{T} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi - \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

, where ϕ , θ and ψ are "roll", "pitch" and "yaw" angles about $\mathbf{i},\mathbf{j},\mathbf{k}$ axes, respectively, link vector $\mathbf{L}_i$ can be rewrite as

$$\mathbf{L}_i = \mathbf{A}_{im}\mathbf{T} + \mathbf{R} - \mathbf{B}_i \qquad (i=1,2,3)\ . \tag{2}$$

Equation (2) denotes the relationship between 6-DOF attitude of platform (x,y,z,ϕ,θ,ψ) and link vector $\mathbf{L}_i$. This equation is called the inverse kinematic equation.

3. Derivation of bottom end position

Our purpose is to derive the bottom end of fixed length link vector $\mathbf{B}_i$ from an arbitrary six-degree-of freedom attitude (x,y,z,ϕ,θ,ψ) of platform. However, the inverse kinematic equation (2) has another unknown vector, that is link vector $\mathbf{L}_i$. Thus, vector $\mathbf{B}_i$ can't be calculated by using only equation (2).

Here, we note the geometrically-constraint of tripod parallel mechanism. There are three conditions of the constraint. First, the length of each link is fixed. This condition can be described as follows:

$$|\mathbf{L}_i|^2 = \mathbf{L}_i\mathbf{L}_i^T = l^2, \quad (i=1,2,3) \tag{3}$$

where l is the fixed length of link. Expanding equation (3), a quadratic vector equation

$$\begin{aligned}&\mathbf{A}_{im}\mathbf{A}_{im}^T + \mathbf{R}\mathbf{R}^T + \mathbf{B}_i\mathbf{B}_i^T \\ &+2\mathbf{R}\mathbf{T}^T\mathbf{A}_{im}^T - 2\mathbf{B}_i\mathbf{T}^T\mathbf{A}_{im}^T - 2\mathbf{R}\mathbf{B}_i^T - l^2 = 0 \quad (i=1,2,3)\end{aligned} \tag{4}$$

, which has one unknown vector $\mathbf{B}_i$ is obtained. At this moment, considering second condition that the position of the bottom end of link is constrained on the horizontal plane, the component b_{i3} of vector $\mathbf{B}_i$ is equal to zero, that is, $\mathbf{B}_i = (b_{i1}, b_{i2}, 0)$. Third condition is that the link vector $\mathbf{L}_i$ and the lateral side vectors of platform, which are represented by

$$\overrightarrow{a_1a_2} = (\alpha_1,\alpha_2,\alpha_3)(\mathbf{i},\mathbf{j},\mathbf{k})^T = \boldsymbol{\alpha}_1$$
$$\overrightarrow{a_2a_3} = \boldsymbol{\alpha}_2, \quad \overrightarrow{a_3a_1} = \boldsymbol{\alpha}_3$$

are consistently orthogonal. That is, inner product of both vectors is equal to zero as follows:

$$\boldsymbol{\alpha}_i\mathbf{T}\cdot\mathbf{L}_i = 0\,. \tag{5}$$

Equation (5) is rewrite as

$$(\alpha_{i1},\alpha_{i2},\alpha_{i3})\mathbf{T}\cdot(\mathbf{A}_{im}\mathbf{T}+\mathbf{R}-\mathbf{B}_i) = 0\,. \tag{6}$$

Equation (6) is a liner vector equation where $\mathbf{B}_i$ is the unknown vector. For equation (6), the component b_{i1} (or b_{i2}) is represented as

$$b_{i1} = P_{i1}b_{i2} + Q_{i1} \quad (\text{or} \quad b_{i2} = P_{i2}b_{i1} + Q_{i2}) \tag{7}$$

where P_{i1}, P_{i2}, Q_{i1} and Q_{i2} are coefficient vectors. Thus, the unknown vector $\mathbf{B}_i = (b_{i1}, b_{i2}, 0)$ can be derivative to solve the quadric equation (4) with equation (7). When the bottom end of link vector $\mathbf{B}_i$ can be derived, the link vector $\mathbf{L}_i$ can also be derived by equation (2).

4. Condition of solution

In the previous section, we presented the derivation of the position of bottom end of link. This method provides two solutions for each link since it is the derivation for the quadratic equation. This situation is shown in Fig.4.

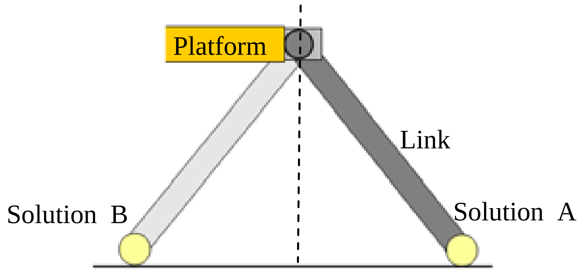

Fig.4. Existence of two candidates of solution

To avoid three links interfere with each other, and from the structural constraint that the link cannot revolve toward the platform from the vertical line through the upper revolute joint, we choose a solution which is outer region from the vertical line.

III. Numerical simulation

In this section, the numerical simulation results based on our analysis are shown. For this simulation, the length of each link is set to 250(mm) and three revolute joints are assigned from 30(deg) in the world reference frame at intervals of 120(deg) on the circumference of circle with radius 31.5(mm). The initial position of platform is set as [x_0, y_0, z_0] = [0.0, 0.0, 125] (mm) and initial orientation of platform is set as [ϕ_0, θ_0, ψ_0] =[0.0, 0.0, 0.0](mm). For the initial condition, we give three rotational motions. They are roll, pitch and yaw motion.

Figure 5 shows the trajectories of bottom ends of links for the roll motion of the platform. Where, the top is 3D graph where the vertical axis denotes the roll angle ϕ and sign B1, B2, B3 denote the bottom ends of 1st, 2nd and 3rd link, respectively. In this simulation, due to the condition of solution in the last chapter, the operating range of roll angle is restricted in -53.08 ~ 56.47(deg). This operating range is asymmetric since the assignments of links are asymmetric for roll axis.

Figure 6 and 7 show the results of pitch motion and yaw motion, respectively. As the simulation result for pitch motion, the operating range of pitch angle is restricted in -53.97 ~ 53.97(deg). This operating range is symmetric since the assignments of links are symmetric for pitch axis. For yaw motion, the operating area of each bottom end of link is not restricted. Thus, the operating range of yaw angle is not restricted.

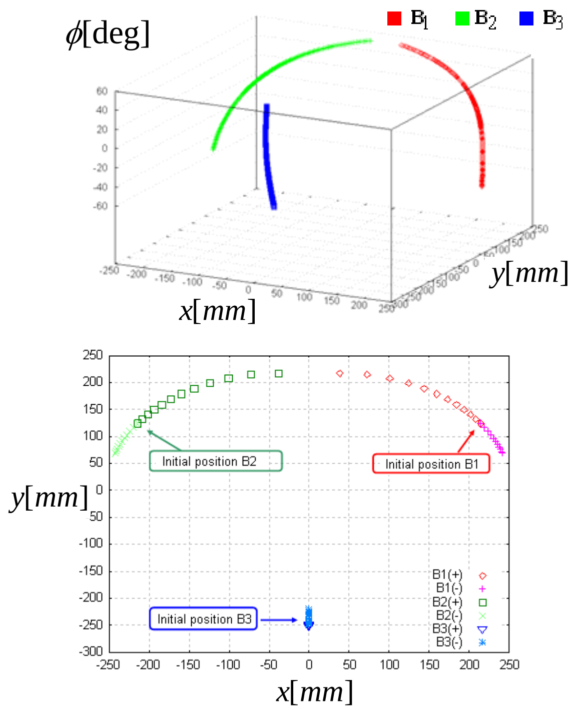

Fig.5 Result for roll motion

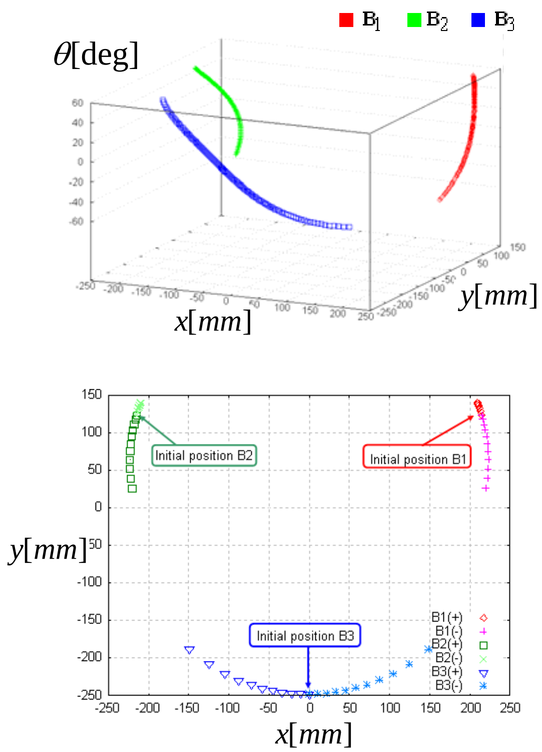

Fig.6 Result for pitch motion

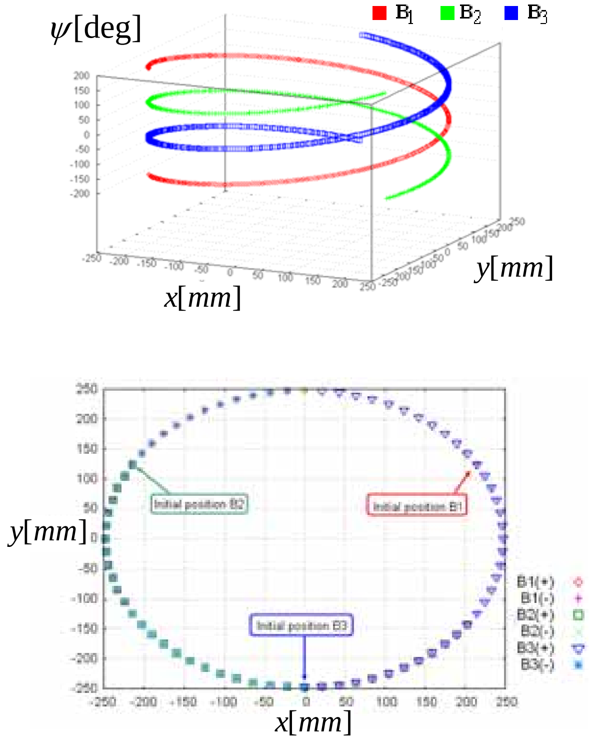

Fig.7 Result for yaw motion

Ⅳ. CONCLUSION

In this paper, we produced the vector analysis of the tripod parallel mechanism and derivation method of the position of bottom ends of three links for the attitude of platform by the inverse kinematics and conditions of geometrically-constraint. And then, we presented the numerical simulation results for our analysis.

In the future work, we'll confirm the analysis by a comparison between the simulation and the experiments for the actual tripod parallel mechanism.

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