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A Design of Algorithms for Real-Time Generation of Linear-Recursive Sequences on Cellular Automata

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Abstract

A model of cellular automata (CA) is considered to be a non-linear model of complex systems in which an infinite one-dimensional array of finite state machines (cells) updates itself in a synchronous manner according to a uniform local rule. We study a sequence generation problem on the CA and propose several state-efficient real-time sequence generation algorithms for non-regular sequences. We show that Fibonacci sequence can be generated in real-time by a CA with 5 states. We also study infinite linear-recursive sequences, such as tribonacci, tetranacci and pell sequences generated on the CA.

1 Introduction

A model of cellular automata (CA) was devised originally for studying self-reproduction by John von Neumann. It is now studied in many fields such as complex systems. We study a sequence generation problem on the CA. Arisawa[1], Fischer[2], Korec[3] and Kamikawa and Umeo[5], [6] studied generation of a class of natural numbers on CA. In this paper, we show that Fibonacci sequence can be generated in real-time by a CA with 5 states. We also study infinite linear-recursive sequences, such as tribonacci sequence, tetranacci sequence and pell sequence generated on the CA. We show a design of algorithm for real-time generation of linear-recursive sequences on CA.

2 Real-time sequence generation problem on CA

A cellular automaton consists of an infinite array of identical finite state automata, each located at a positive integer point (See Figure 1).

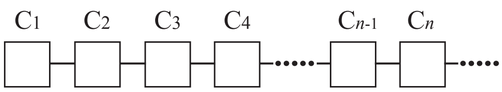

Figure 1: Cellular automaton.

Each automaton is referred to as a cell. A cell at point i is denoted by C_i, where $i \geq 1$. Each C_i, except for C_1, is connected to its left- and right-neighbor cells via a communication link. Each cell can know state of its left- and right-neighbor cells via communication link. One distinguished leftmost cell C_1, the communication cell, is connected to the outside world. A cellular automaton (abbreviated by CA) consists of an infinite array of finite state automata $A = (Q, \delta, F)$, where

1. Q is a finite set of internal states.
2. δ is a function defining the next state of a cell, such that δ: $Q \times Q \times Q \rightarrow Q$, where $\delta(\mathtt{p}, \mathtt{q}, \mathtt{r}) = \mathtt{s}$, $\mathtt{p}$, $\mathtt{q}$, $\mathtt{r}$, $\mathtt{s} \in Q$ has the following meaning: We assume that at step t the cell C_i is in state $\mathtt{q}$, the left cell C_{i-1} is in state $\mathtt{p}$ and the right cell C_{i+1} is in state $\mathtt{r}$. Then, at the next step $t+1$, C_i assumes state $\mathtt{s}$. The leftmost cell C_1 is connected to the outside world. The outside world is expressed by $*$. A quiescent state $\mathtt{q} \in Q$ has a property such that $\delta(\mathtt{q}, \mathtt{q}, \mathtt{q}) = \mathtt{q}$.
3. $F \subseteq Q$ is a special subset of Q. The set F is used to specify a designated state of C_1 in the definition of sequence generation.

We now define the **sequence generation problem** on CA. Let M be a CA and $\{t_n | n = 1, 2, 3, ...\}$ be an infinite monotonically increasing positive integer sequence defined natural numbers, such that $t_n \geq n$ for any $n \geq 1$. We then have a semi-infinite array of cells, as shown in Figure 1, and all cells, except for C_1, are in the quiescent state at time $t = 0$. The communication cell C_1 assumes a special state $\mathtt{r}$ in Q for initiation of the sequence generator. We say that M generates a sequence $\{t_n | n = 1, 2, 3, ...\}$ in k *linear-time* if and only if the leftmost end cell of M falls into a special state in $F \subseteq Q$ at time $t = k \cdot t_n$, where k is a positive integer. We call M a *real-time* generator when $k = 1$.

3 Generation Algorithms of Linear-Recursive Sequences

In this section, we propose generation algorithm of linear-recursive sequences. First, we show a design of algorithm for real-time generation of linear-recursive sequences on CA. Next, we show that Fibonacci sequence can be generated on a CA with 5 states.

3.1 A Design of Algorithm

Let m be any natural number, such that $m \geq 1$. Let k be natural number given, such that $k \geq 1$, $k < m$. Let $b_1, b_2, \ldots, b_k, c_1, c_2, \ldots, c_k$ be natural number given, such that $b_1, b_2, \cdots, b_k \geq 1$, $c_1 < c_2 < \cdots < c_k$. Let a_m be kth order linear-recursive sequences, such that $a_m = b_1 \cdot a_{m-1} + b_2 \cdot a_{m-2} + \cdots + b_k \cdot a_{m-k}$, $a_1 = c_1$, $a_2 = c_2$, $\ldots$, $a_k = c_k$. We show a design of algorithm for real-time generation of sequence a_m on CA.

3.2 First Order Linear-Recursive Sequences

We propose the generation algorithm for $k = 1$. It is approved that $a_m = b_1 \cdot a_{m-1}$, $a_1 = c_1$. However, it is limited to $b_1 \geq 2$. Because all terms take c_1 for $b_1 = 1$, and a_m is not an infinite monotonically increasing positive integer sequence. Figure 2 shows a time-space diagram for generation of the term a_m, when the term a_{m-1} is an even number.

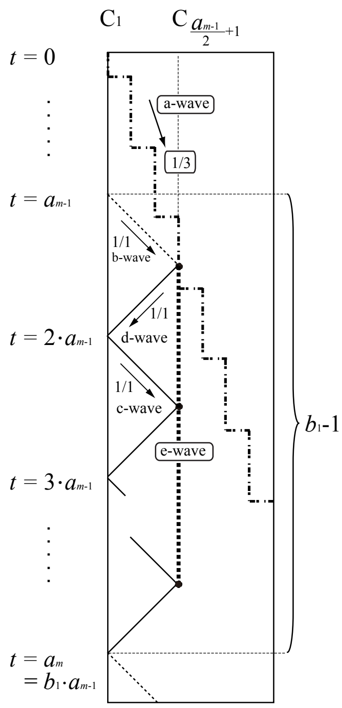

Figure 2: Time-space diagram for generation of the term a_m (when the term $a_{m\;1}$ is an even number).

	1	2	3	4	5	6	7	8	9
0	A3	Q	Q	Q	Q	Q	Q	Q	Q
1	W1	A1	Q	Q	Q	Q	Q	Q	Q
2	W2	A2	Q	Q	Q	Q	Q	Q	Q
3	W3	A3	Q	Q	Q	Q	Q	Q	Q
4	W4	R	A1	Q	Q	Q	Q	Q	Q
5	W5	R	A2	Q	Q	Q	Q	Q	Q
6	G	R	A3	Q	Q	Q	Q	Q	Q
7	Q	B	R	A1	Q	Q	Q	Q	Q
8	Q	R	B	A2	Q	Q	Q	Q	Q
9	Q	R	R	E1	Q	Q	Q	Q	Q
10	Q	R	D	E2	A1	Q	Q	Q	Q
11	Q	D	R	E2	A2	Q	Q	Q	Q
12	t1	R	R	E2	A3	Q	Q	Q	Q
13	t2	C	R	E2	R	A1	Q	Q	Q
14	t2	R	C	E2	R	A2	Q	Q	Q
15	t2	R	R	D	R	A3	Q	Q	Q
16	t2	R	D	R	R	R	A1	Q	Q
17	t2	D	R	R	R	R	A2	Q	Q
18	G	R	R	R	R	R	A3	Q	Q
19	Q	B	R	R	R	R	R	A1	Q
20	Q	R	B	R	R	R	R	A2	Q

Figure 3: A con guration of generation of the term a_m (when the term $a_{m\;1}$ is an even number).

Generation of the term a_m is described in terms of 6 waves: *a-wave*, *b-wave*, *c-wave*, *d-wave*, *e-wave* and *o-wave*. The a-wave is generated on C_1 at time $t = 0$. Figure 3 shows a number of snapshots of the cell configuration at the propagation of the a-wave shown in Figure 2. In Figure 3, state `A1`, `A2` and `A3` advance toward the right at speed 1-cell/3-step in cell space. Therefore, state `A1`, `A2` and `A3` which propagate in cell space is called a-wave. A sequence generation algorithm is designed geometrical by using wave which propagates in cell space. The a-wave propagates in the right direction at 1/3 speed. Figure 3 shows a number of snapshots of the cell configuration at the propagation of the a-wave. The a-wave moves to cell C_2 at time $t = 1$. Afterwards, the a-wave moves by one cell every 3 steps. When we assume $P_a(t)$ to be a function whitch shows the position of the a-wave at time t, it is approved that $P_a(t) = \lceil \frac{t}{3} \rceil + 1$. At time $t = a_{m-1}$, cell C_1 is in a state included F and the b-wave is generated on C_1. The b-wave propagates in the right direction at 1/1 speed, and the b-wave reaches the a-wave. When the a-wave collides with the b-wave, the a-wave keeps propagating, the b-wave is eliminated, the d-wave is generated and the e-wave is generated. When a_{m-1} is an even number, the b-wave collides with the second state of 3 states to compose the a-wave and the e-wave is generated (See Figure 3). Let r be natural number. When the cell which collides the a-wave with the b-wave is assumed to be cell C_r, it is approved that $r = P_a(a_{m-1} + r - 1) = \frac{a_{m-1}}{2} + 1$. Therefore, the e-wave is generated on cell $C_{\frac{a_{m-1}}{2}+1}$ at time $t = a_{m-1} + \frac{a_{m-1}}{2}$. The e-wave keeps staying on cell $C_{\frac{a_{m-1}}{2}+1}$. The d-wave propagates in the left direction at 1/1 speed, and the d-wave reaches the leftmost cell C_1 at time $t = 2 \cdot a_{m-1}$. When the d-wave collides with the leftmost cell C_1, the d-wave is eliminated and the c-wave is generated. The c-wave propagates in the right direction at 1/1 speed, and the c-wave reaches the e-wave at time $t = 2 \ldots a_{m-1} + \frac{a_{m-1}}{2}$. When the c-wave collides with the e-wave, the b-wave is eliminated and the d-wave is generated. The d-wave propagates in the left direction at 1/1 speed. The d-wave reaches cell C_1 at time $t = 3 \cdot a_{m-1}$. Therefore, Time where the b-, c- and d-waves reciprocate between the leftmost cell C_1 and the e-wave is a_{m-1} steps. The b-, c- and d-waves reciprocate $b_1 - 1$ times between the leftmost cell C_1 and the e-wave (See Figure 2). When the d-wave of times $b_1 - 1$ reaches the cell C_1, a state of C_1 changes to a state included F at time $t = b_1 \cdot a_{m-1}$.

Figure 4 shows a time-space diagram for generation of the term a_m, when the term a_{m-1} is an odd number. When a_{m-1} is an odd number, the o-wave is generated by the collision of the a-wave and the b-wave. The b-wave collides with the third state of 3 states to compose the a-wave and the o-wave is generated (See Figure 5). When the cell which collides the a-wave with the b-wave is assumed to be cell C_r, it is approved that $r = P_a(a_{m-1}+r-1) = \lfloor \frac{a_{m-1}}{2} \rfloor + 1$. Because a_{m-1} is an odd number, it is approved that $r = \frac{a_{m-1}-1}{2} + 1$. Therefore, the o-wave is generated on cell $C_{\frac{a_{m-1}-1}{2}+1}$. he c-wave propagates in the right direction at 1/1 speed, and the c-wave reaches the o-wave at time $t = a_{m-1} + \frac{a_{m-1}-1}{2}$. When the c-wave collides with the o-wave, the b-wave is eliminated. The d-wave is generated after 1 step. The d-wave propagates in the left direction at 1/1 speed. The d-wave reaches cell C_1 at time $t = a_{m-1} + \frac{a_{m-1}-1}{2} + 1 + \frac{a_{m-1}-1}{2} = 2 \cdot a_{m-1}$. Therefore, Time where the b-, c- and d-waves reciprocate between the leftmost cell C_1 and the o-wave is a_{m-1} steps. The b-, c- and d-waves reciprocate $b_1 - 1$ times between the leftmost cell C_1 and the o-wave (See Figure 4). When the d-wave of times $b_1 - 1$ reaches the cell C_1, a state of C_1 changes to a state included F at time $t = b_1 \cdot a_{m-1}$.

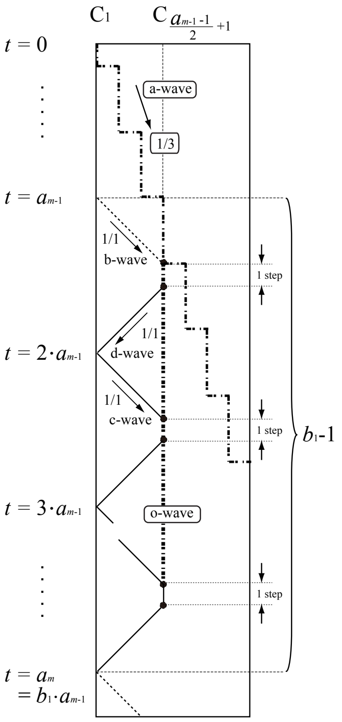

Figure 4: Time-space diagram for generation of the term a_m (when the term a_{m-1} is an odd number).

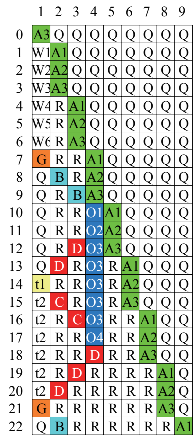

Figure 5: A con guration of generation of the term a_m (when the term a_{m-1} is an odd number).

The first term $a_1 = c_1$ is generated in an internal state. A state of the leftmost cell C_1 is changed a state included F at time $t = c_1$ by counting the $c_1 - 1$ step by using an internal state. And the b-wave is generated. Therefore, 1st Order Linear-Recursive Sequences can be generated on CA in real-time. In Figure 6, we show a number of snapshots of the configuration for $b_1 = 3$ and $c_1 = 3$ from $t = 0$ to 29.

3.3 Second Order Linear-Recursive Sequences

Next, we consider the case of $k = 2$. We propose the generation algorithm of second order linear-recursive sequences which enhance the algorithm described in section 3.2. It is approved that $a_m = b_1 \cdot a_{m-1} + b_2 \cdot a_{m-2}$, $a_1 = c_1$, $a_2 = c_2$. Figure 7 shows a time-space diagram for generation of a second order linear-recursive sequence. The a-wave is generated on C_1 at time $t = 0$. We assume that the e- or o-wave is generated on cell $C_{\lfloor \frac{a_{m-2}}{2} \rfloor + 1}$. At time $t = a_{m-1}$, cell C_1 is in a state included F and the b-wave is generated on C_1. The b-wave propagates in the right direction at 1/1 speed, and the b-wave reaches the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-2}}{2} \rfloor + 1}$. When the e- or o-wave collides with the b-wave, the b-wave keeps propagating and the d-wave is generated. The b-wave propagates, and the b-wave reaches the a-wave. When the a-wave collides with the b-wave, the b-wave is eliminated and the e- or o-wave is generated on cell $C_{\lfloor \frac{a_{m-1}}{2} \rfloor + 1}$. The d-wave propagates in the left direction at 1/1 speed, and the d-wave reaches the leftmost cell C_1 at time $t = a_{m-1} + a_{m-2}$. The b-, c- and d-waves reciprocate b_2 times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-2}}{2} \rfloor + 1}$. When the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-2}}{2} \rfloor + 1}$ collides with the b- or c-wave b_2 times, the e- or o-wave is eliminated. At the next, The c- and d-waves reciprocate $b_1 - 1$ times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-1}}{2} \rfloor + 1}$. When The b-, c- and d-waves reciprocate b_2 times between cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-2}}{2} \rfloor + 1}$ and reciprocate $b_1 - 1$ times between cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-1}}{2} \rfloor + 1}$, a state of the leftmost cell C_1 changes a state included F. The first some terms and some e- and o-waves are generated in an internal state. For example, Figure 8 shows generation of pell sequence ($a_m = 2 \cdot a_{m-1} + a_{m-2}, a_1 = 1, a_2 = 2$).

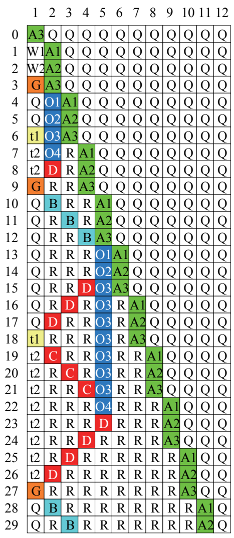

Figure 6: A con guration of real-time generation of a rst order linear-recursive sequence ($a_m = 3 \cdot a_{m-1}, a_1 = 3$).

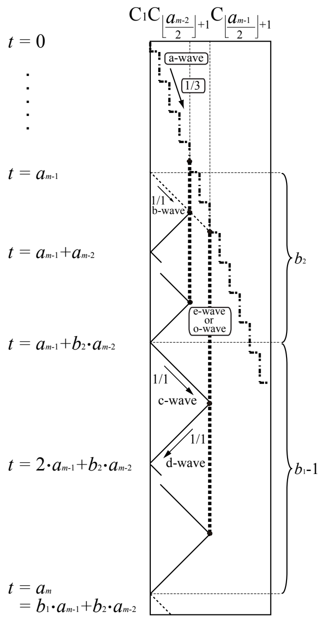

Figure 7: Time-space diagram for generation of a second order linear-recursive sequence.

	1	2	3	4	5	6	7	8	9	10	11	12
0	A3	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
1	G	A1	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
2	G	A2	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
3	W3	E3	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
4	W4	E3	A1	Q	Q	Q	Q	Q	Q	Q	Q	Q
5	G	E3	A2	Q	Q	Q	Q	Q	Q	Q	Q	Q
6	Q	E4	A3	Q	Q	Q	Q	Q	Q	Q	Q	Q
7	t1	R	O1	A1	Q	Q	Q	Q	Q	Q	Q	Q
8	t2	C	O1	A2	Q	Q	Q	Q	Q	Q	Q	Q
9	t2	R	O2	A3	Q	Q	Q	Q	Q	Q	Q	Q
10	t2	R	O3	R	A1	Q	Q	Q	Q	Q	Q	Q
11	t2	D	O4	R	A2	Q	Q	Q	Q	Q	Q	Q
12	G	R	O4	R	A3	Q	Q	Q	Q	Q	Q	Q
13	Q	C	O4	R	R	A1	Q	Q	Q	Q	Q	Q
14	Q	R	O5	R	R	A2	Q	Q	Q	Q	Q	Q
15	Q	R	D	B	R	A3	Q	Q	Q	Q	Q	Q
16	Q	D	R	R	B	R	A1	Q	Q	Q	Q	Q
17	t1	R	R	R	R	B	A2	Q	Q	Q	Q	Q
18	t2	C	R	R	R	R	E1	Q	Q	Q	Q	Q
19	t2	R	C	R	R	R	E1	A1	Q	Q	Q	Q
20	t2	R	R	C	R	R	E1	A2	Q	Q	Q	Q
21	t2	R	R	R	C	R	E1	A3	Q	Q	Q	Q
22	t2	R	R	R	R	C	E1	R	A1	Q	Q	Q
23	t2	R	R	R	R	R	E2	R	A2	Q	Q	Q
24	t2	R	R	R	R	D	E3	R	A3	Q	Q	Q
25	t2	R	R	R	D	R	E3	R	R	A1	Q	Q
26	t2	R	R	D	R	R	E3	R	R	A2	Q	Q
27	t2	R	D	R	R	R	E3	R	R	A3	Q	Q
28	t2	D	R	R	R	R	E3	R	R	R	A1	Q
29	G	R	R	R	R	R	E3	R	R	R	A2	Q
30	Q	C	R	R	R	R	E3	R	R	R	A3	Q

Figure 8: A con guration of real-time generation of pell sequence ($a_m = 2\ \ a_{m\ \ 1} + a_{m\ \ 2}, a_1 = 1, a_2 = 2$).

3.4 kth Order Linear-Recursive Sequences

Next, we generalize the generation algorithm described in section 3.3, and propose the generation algorithm of kth Order Linear-Recursive Sequences. Figure 9 shows a time-space diagram for generation of a kth order linear-recursive sequence. The a-wave is generated on C_1 at time $t = 0$. We assume that the e- or o-waves are generated on cell $C_{\lfloor \frac{a_{m-k}}{2} \rfloor+1}$, $C_{\lfloor \frac{a_{m-k+1}}{2} \rfloor+1}$, $\cdots$, $C_{\lfloor \frac{a_{m-3}}{2} \rfloor+1}$ and $C_{\lfloor \frac{a_{m-2}}{2} \rfloor+1}$. At time $t = a_{m-1}$, cell C_1 is in a state included F and the b-wave is generated on C_1. The b-wave propagates in the right direction at 1/1 speed, and the b-wave reaches the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-k}}{2} \rfloor+1}$. When the e- or o-wave collides with the b-wave, the b-wave keeps propagating and the d-wave is generated. The b-wave propagates by passing the e- or o-waves generated on cell $C_{\lfloor \frac{a_{m-k+1}}{2} \rfloor+1}$, $\cdots$, $C_{\lfloor \frac{a_{m-3}}{2} \rfloor+1}$, and the b-wave reaches the a-wave. When the a-wave collides with the b-wave, the b-wave is eliminated and the e- or o-wave is generated on cell $C_{\lfloor \frac{a_{m-1}}{2} \rfloor+1}$. The d-wave propagates by passing the e- or o-waves in the left direction at 1/1 speed, and the d-wave reaches the leftmost cell C_1 at time $t = a_{m-1} + a_{m-k}$. The b-, c- and d-waves reciprocate b_k times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-k}}{2} \rfloor+1}$. When the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-k}}{2} \rfloor+1}$ collides with the b- or c-wave b_k times, the e- or o-wave is eliminated. At the next, The c- and d-

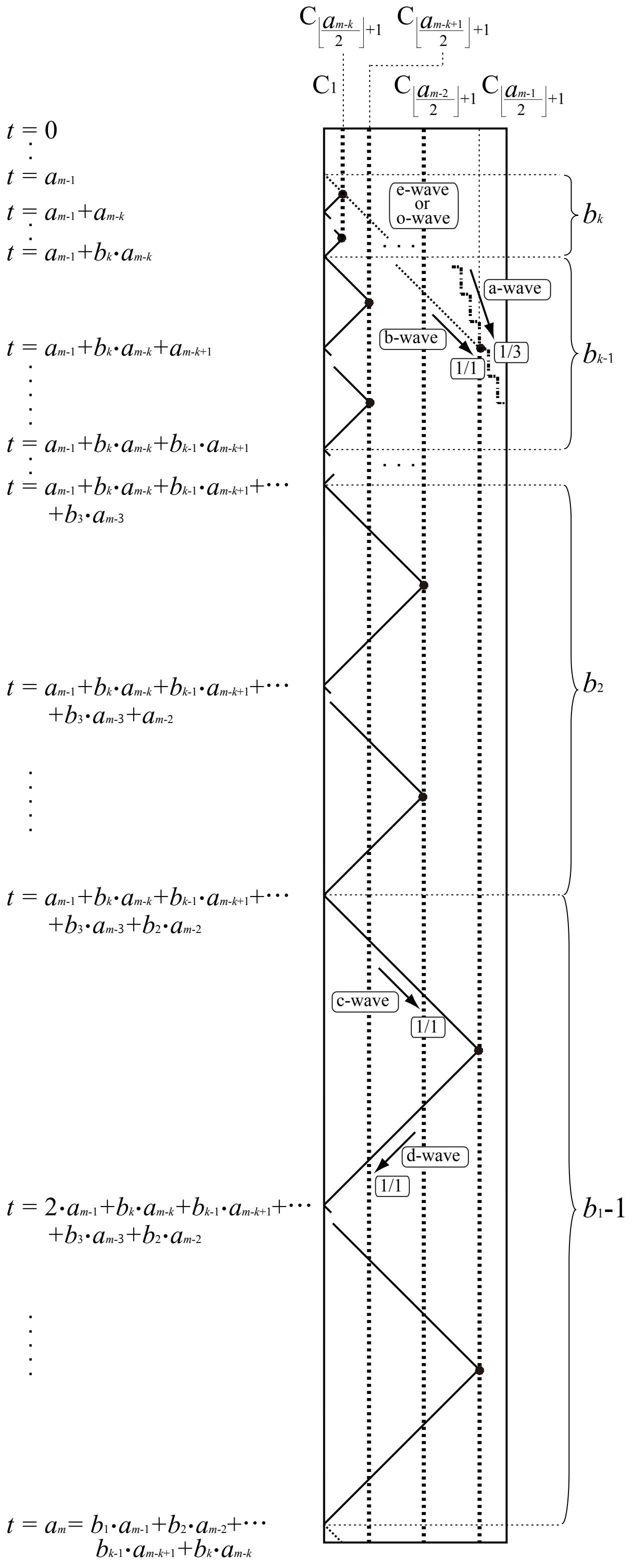

Figure 9: Time-space diagram for generation of a kth order linear-recursive sequence.

waves reciprocate b_{k-1} times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-k+1}}{2} \rfloor+1}$, reciprocate b_{k-2} times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-k+2}}{2} \rfloor+1}$, $\cdots$, reciprocate b_2 times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-2}}{2} \rfloor+1}$ and reciprocate $b_1 - 1$ times between the leftmost cell C_1 and the e- or o-wave generated on cell $C_{\lfloor \frac{a_{m-1}}{2} \rfloor+1}$. At time $t = a_m = b_1 \cdot a_{m-1} + b_2 \cdot a_{m-2} + \cdots + b_k \cdot a_{m-k}$, a state of the leftmost cell C_1 changes a state included F. The first some terms and some e- and o-waves are generated in an internal state. For example, Figure 10 shows generation of tetranacci sequence ($a_m = a_{m-1} + a_{m-2} + a_{m-3} + a_{m-4}, a_1 = 1, a_2 = 2, a_3 = 4, a_4 = 8$).

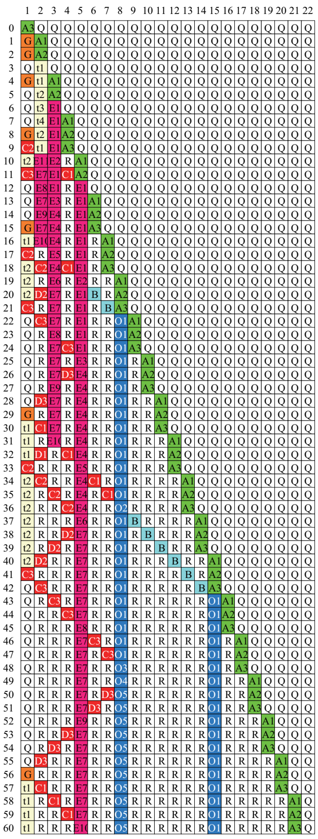

Figure 10: A con guration of real-time generation of tetranacci sequence ($a_m = a_m\ _1 + a_m\ _2 + a_m\ _3 + a_m\ _4, a_1 = 1, a_2 = 2, a_3 = 4, a_4 = 8$).

3.5 Fibonacci Sequence

In this section, we show real-time generation algorithm of Fibonacci sequence. In a past research, Arisawa showed that Fibonacci sequence can be generated in 2 linear-time by a CA with 9 states. However, real-time generation algorithm of Fibonacci sequence on CA is not exist. A consists of an infinite array of finite state automata $A = (Q, \delta, F)$, where $Q = \{\mathtt{Q}, \mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}\}$, $F = \{\mathtt{A}\}$. We show that Fibonacci sequence can be generated in real-time by a CA with 5 states that is given in Table 1. In Figure 11, we show a time-space diagram for real-time generation of Fibonacci sequence.

Table 1: Transition rules for real-time generation of Fibonacci sequence.

Q (Left State \ Right State)	Q	A	B	C	D
Q	Q	Q	Q	C	Q
A	B		Q	D	
B	Q				
C	Q			D	
D	B				
*	Q	Q		A	Q

A (Left State \ Right State)	Q	A	B	C	D
Q		Q	Q		Q
A			A	C	
B				A	
C				D	
D	A	B	A	A	A
*	Q		A	A	Q

B (Left State \ Right State)	Q	A	B	C	D
Q				B	D
A	C	C	C	D	
B	C				
C	C		B		
D	C	B	B	B	B
*					

C (Left State \ Right State)	Q	A	B	C	D
Q	D	D	Q		D
A	D		Q	C	C
B	D			C	C
C	D		C	C	C
D	A		D	D	D
*					

D (Left State \ Right State)	Q	A	B	C	D
Q				C	C
A	C	A	A		A
B	D		C		
C	C			D	D
D	B	D	D	D	D
*	A				

Real-time generation of Fibonacci sequence is described in terms of 5 waves: *a-wave*, *b-wave*, *d-wave*, *we-wave*, *wo-wave*. Fibonacci sequence is 2nd linear-recursive sequence. Therefore, When each the we- or wo-wave collide with the b- or c-wave 1 time, the we- or wo-wave is eliminated. The initial configuration is the leftmost cell C_1 takes state D and other cells take a quiescent state Q. At time $t = 0$, the a-wave is generated on the leftmost cell C_1. The a-wave propagates in the right direction at 1/3 speed. State B, State C and State D are used for the propagation of the a-wave. At time $t = 0$, $\delta(\mathtt{C}, \mathtt{Q}, \mathtt{Q}) = \mathtt{B}$ are applied in cell C_2. At the next step, a state of C_2 changes to B. At time $t = 1$, $\delta(\mathtt{A}, \mathtt{B}, \mathtt{Q}) = \mathtt{C}$ are applied in cell C_2. At the next step, a state of C_2 changes to C. At time $t = 2$, $\delta(\mathtt{A}, \mathtt{C}, \mathtt{Q}) = \mathtt{D}$ are applied in cell C_2. At the next step, a state of C_2 changes to D. At time $t = 3$, $\delta(\mathtt{D}, \mathtt{Q}, \mathtt{Q}) = \mathtt{B}$ are applied in cell C_3. At the next step, a state of C_3 changes to B. The a-wave propagates by repeating the application of these transition rules. State A and State D are used for the propagation of the b-wave. State C is used for the propagation of the d-wave. The first 5 terms, the we-wave generated on cell C_5 and the d-wave to generate the 6th term are generated with an internal state. At time $t = 11$, the d-wave is generated on cell C_3. The d-wave propagates in the left direction at 1/1 speed, and reaches the leftmost cell C_1. When the d-wave reaches the leftmost cell C_1, $\delta(*, \mathtt{Q}, \mathtt{C}) = \mathtt{A}$ are applied in cell C_1. At time $t = 13$, a state of C_1 changes to A, and the b-wave is generated. The b-wave propagates in

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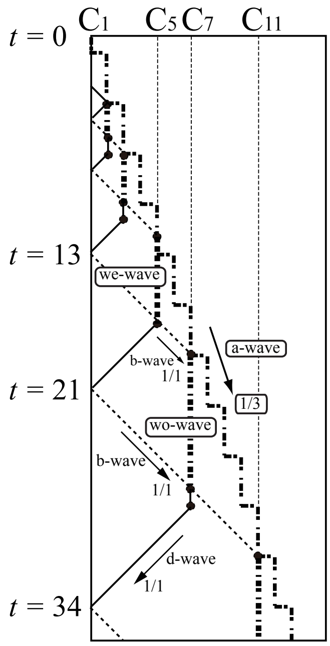

Figure 11: Time-space diagram for real-time generation of Fibonacci sequence.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	D	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
1	A	B	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
2	A	C	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
3	A	D	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
4	Q	C	B	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
5	A	Q	C	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
6	Q	D	D	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
7	Q	C	B	B	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
8	A	Q	B	C	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
9	Q	Q	B	D	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
10	Q	Q	D	D	B	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
11	Q	Q	C	D	C	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
12	Q	C	D	D	A	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
13	A	D	D	D	A	B	Q	Q	Q	Q	Q	Q	Q	Q	Q
14	Q	A	D	D	A	C	Q	Q	Q	Q	Q	Q	Q	Q	Q
15	Q	Q	A	D	A	D	Q	Q	Q	Q	Q	Q	Q	Q	Q
16	Q	Q	Q	A	A	C	B	Q	Q	Q	Q	Q	Q	Q	Q
17	Q	Q	Q	Q	C	Q	C	Q	Q	Q	Q	Q	Q	Q	Q
18	Q	Q	Q	C	D	D	D	Q	Q	Q	Q	Q	Q	Q	Q
19	Q	Q	C	D	D	D	B	B	Q	Q	Q	Q	Q	Q	Q
20	Q	C	D	D	D	D	B	C	Q	Q	Q	Q	Q	Q	Q
21	A	D	D	D	D	D	B	D	Q	Q	Q	Q	Q	Q	Q
22	Q	A	D	D	D	D	B	D	B	Q	Q	Q	Q	Q	Q
23	Q	Q	A	D	D	D	B	C	C	Q	Q	Q	Q	Q	Q
24	Q	Q	Q	A	D	D	B	C	D	Q	Q	Q	Q	Q	Q
25	Q	Q	Q	Q	A	D	B	C	C	B	Q	Q	Q	Q	Q
26	Q	Q	Q	Q	Q	A	B	C	C	C	Q	Q	Q	Q	Q
27	Q	Q	Q	Q	Q	Q	D	C	C	D	Q	Q	Q	Q	Q
28	Q	Q	Q	Q	Q	Q	C	D	C	C	B	Q	Q	Q	Q
29	Q	Q	Q	Q	Q	C	D	D	D	C	C	Q	Q	Q	Q
30	Q	Q	Q	Q	C	D	D	D	D	D	D	Q	Q	Q	Q
31	Q	Q	Q	C	D	D	D	D	D	D	B	B	Q	Q	Q
32	Q	Q	C	D	D	D	D	D	D	D	B	C	Q	Q	Q
33	Q	C	D	D	D	D	D	D	D	D	B	D	Q	Q	Q
34	A	D	D	D	D	D	D	D	D	D	B	D	B	Q	Q
35	Q	A	D	D	D	D	D	D	D	D	B	C	C	Q	Q
36	Q	Q	A	D	D	D	D	D	D	D	B	C	D	Q	Q

Figure 12: A con guration of real-time generation of Fibonacci sequence.

the left direction at 1/1 speed, and reaches the we-wave generated on cell C_5 at time $t = 17$. When the we-wave collides with the b-wave, the we-wave is eliminated, the d-wave is generated and the b-wave keeps propagating. The d-wave reaches the a-wave. When the a-wave collides with the b-wave, the b-wave is eliminated, the wo-wave is generated. The d-wave generated on cell C_5 reaches the leftmost cell C_1 at time $t = 21$. When the d-wave reaches the leftmost cell C_1, a state of C_1 changes to A, and the b-wave is generated. Therefore, Fibonacci sequence can be generated by repeating the propagation of 5 waves. We have implemented the algorithm on a computer. We have tested the validity of the rule set from $t = 0$ to $t = 20000$ steps. We obtain the following theorem. In Figure 12, we show a number of snapshots of the configuration from $t = 0$ to 36.

4 Conclusions

We have studied a sequence generation problem on CA. A design of algorithm for real-time generation of linear-recursive sequences on CA has been given. We have shown that Fibonacci sequence can be generated in real-time by a CA with 5 states. A future study in sequence generation problem on CA is to compare sequence generation power of CA and sequence generation power of 1-bit inter-cell-communication CA.

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