The Thirteenth International Symposium on Artificial Life and Robotics 2008(AROB 13th '08),
B-Con Plaza, Beppu, Oita, Japan, January 31-February 2, 2008

Marker versus Inkdot over Three-Dimensional Patterns

Takao ITO[1], Makoto SAKAMOTO[1], Hidenobu OKABE[2], Hiroshi FURUTANI[1], Michio KONO[1], and Satoshi IKEDA[1]

[1] Dept. of Computer Science and Systems Engineering, University of Miyazaki, Miyazaki 889-2192, JAPAN
[2] System Solution Division, NS-ELEX Co., Ltd. 104-0033, JAPAN

Abstract

A multi-marker automaton is a finite automaton which keeps marks as *pebbles* in the finite control, and cannot rewrite any input symbols but can make marks on its input with the restriction that only a bounded number of these marks can exist at any given time. An improvement of picture recognizability of the finite automaton is the reason why the multi-marker automaton was introduced. On the other hand, a multi-inkdot automaton is a conventional automaton capable of dropping an inkdot on a given input tape for a landmark, but unable to further pick it up. This paper deals with marker versus inkdot over three-dimensional input tapes, and investigates some properties.

Key Words **:** finite automaton, inkdot, marker, recognizability, three-dimension

1 Introduction and Preliminaries

A multi-marker automaton is a finite automaton which keeps marks as *pebbles* in the finite control, and cannot rewrite any input symbols but can make marks on its input with the restriction that only a bounded number of these marks can exist at any given time. An improvement of picture recognizability of the finite automaton is the reason why the marker automaton was introduced. That is, a two-dimensional multi-marker automaton can recognize connected pictures [1].

On the other hand, as is the well-known open problems in computational complexity, there is the historical open question whether or not the separation exists between deterministic and nondeterministic space (especially hard-level) complexity classes. Related to the open question, D. Ranjan et al. introduced a slightly modified Turing machine model, called a one-inkdot Turing machine [6]. An inkdot machine is a conventional Turing machine capable of dropping an inkdot on a given input tape for landmark, but unable to further pick it up. Against an earlier expectation, it was proved that nondeterministic inkdot Turing machines are more powerful than nondeterministic ordinary Turing machines for sublogarithmic space bounds. As is well-known result in the case of two-

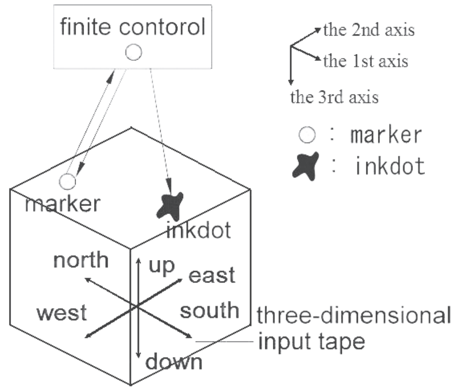

Fig. 1: Image of Marker or Inkdot on Three-dimensional Input Tape.

dimensional input tapes, there is a set of square tapes accepted by a nondeterministic finite automaton, but not by any deterministic Turing machine with sublogarithmic space bounds. Thus, it makes no sense to ask the same question whether the separation exists between deterministic and nondeterministic complexity classes for the two-dimensional Turing machines. However, there is an other important aspect in the inkdot mechanism : we can see a two-dimensional finite automaton with inkdot as a weak recognizer of the inherent properties of digital pictures. By this motivation, a two-dimensional multi-inkdot automaton was introduced [5,6].

By the way, the question of whether processing three-dimensional digital patterns is much more difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the research of three-dimensional automata as the computational model of three-dimensional pattern processing has been meaningful. From this viewpoint, we investigated a multi-marker automaton and a multi-inkdot automaton on three-dimensional input tapes [7-12].

This paper deals with a relationship between marker and inkdot for three-dimensional automata, and shows some properties (see Fig.1).

Let Σ be a finite set of symbols, a *three-dimensional tape* over Σ is a three-dimensional rectangular array of elements of Σ. The set of all three-dimensional tapes over Σ is denoted by $\Sigma^{(3)}$. Given a tape x in $\Sigma^{(3)}$, for each integer $j(1 \leq j \leq 3)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x)=n_1$, $l_2(x)=n_2$ and $l_3(x)=n_3$ is denoted by $\Sigma^{(n_1\ n_2\ n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq 3)$, let $x(i_1, i_2, i_3)$ denote the symbol in x with coordinates (i_1, i_2, i_3). Furthermore, we define

$$x[(i_1, i_2, i_3),(i'_1, i'_2, i'_3)],$$

when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 3)$, as the three-dimensional input tape y satisfying the following conditions:

(i) for each $j(1 \leq j \leq 3)$, $l_j(y)=i'_j - i_j+1$;

(ii) for each $r_1,r_2,r_3(1 \leq r_1 \leq l_1(y),1 \leq r_2 \leq l_2(y),1 \leq r_3 \leq l_3(y)),y(r_1,r_2,r_3)=x(r_1+i_1-1,r_2+i_2-1,r_3+i_3-1)$.(We call $x[(i_1, i_2, i_3),(i'_1, i'_2, i'_3)]$ the $[(i_1, i_2, i_3),(i'_1,i'_2, i'_3)]$-*segment* of x.)

A three-dimensional finite automaton (3-FA), which can be considered as a natural extension of the two-dimensional finite automaton to three dimensions, consists of read-only three-dimensional input tape, a finite control, and an input head which can move east, west, south, north, up, or down [1]. By 3-AFA (resp., 3-NFA, 3-DFA, 3-AM_1, 3-NM_1, 3-DM_1, 3-AI_k, 3-NI_k, 3-DI_k), we denote alternating (resp., nondeterministic, deterministic, alternating 1-marker, nondeterministic 1-marker, deterministic 1-marker, alternating k-inkdot, nondeterministic k-inkdot, deterministic k-inkdot) 3-FA. Furthermore, by 3-UFA (resp., 3-UM_1, 3-UI_k), we denote alternating (resp., alternating 1-marker, alternating k-inkdot) 3-FA with only universal states.

A *configuration* of an alternating 1-marker 3-FA M on a three-dimensional input tape x is the form $((i_1, i_2, i_3)$, *marker-position*, $q)$, where (i_1, i_2, i_3) is the input head position, *marker-position* is the position of the marker on x (let *marker-position* be 'no' if the marker is not placed on the input tape x), and component q represents a state of the finite control. For each input tape x, we write $c \vdash_{M\,x} c'$, and say that c' is an *immediate successor* of c (of M on x), if configuration c' is derived from configuration c in one step of M on x according to the *next-move relation*. A configuration with no immediate successor is called a *halting configuration*. Let M be an alternating 1-marker (k-inkdot) 3-FA, and x be an input tape. A sequence of configurations $c_1c_2......c_m$ $(m \geq 1)$ is called a *computation* of M on x if $c_1 \vdash_{M\,x} c_2 \vdash_{M\,x} \cdots \vdash_{M\,x} c_m$.

For each $X \in \{D, N, U, A\}$, we denote by $\mathcal{L}$[3-XFA] the class of sets of all three-dimensional tapes accepted by 3-XFA's. That is,

$\mathcal{L}$[3-XFA]$=\{T \mid T=T(M)$ *for some* 3-XFA $M\}$,

where $T(M)$ is the set of all three-dimensional tapes accepted by M. $\mathcal{L}$[3-XM_1] and $\mathcal{L}$[3-XI_k] are defined similarly. For any family of three-dimensional automata M's, $\mathcal{L}[M^c]$ denotes the class of sets of cubic tapes accepted by M's. For a set T of three-dimensional tapes, the *complementation* of T is denoted by $\hat{T}$. Define co$-\mathcal{L}=\{T \mid T \in \mathcal{L}\}$.

2 Known Results and Related Results

This section surveys known results and related results in [7-12] concerning k-inkdot and 1-marker 3-FA's. The following result shows a relationship among the accepting powers of 3-FA's k-inkdot 3-FA's, and 1-marker 3-FA's.

Theorem 2.1[7-12]**.**
(1) $\mathcal{L}$[3-DFA] $=$ $\mathcal{L}$[3-DI_k] $\subsetneq$ $\mathcal{L}$[3-DM_1],
(2) $\mathcal{L}$[3-NFA] $\subsetneq$ $\mathcal{L}$[3-NI_k] $\subsetneq$ $\mathcal{L}$[3-NM_1],
(3) $\mathcal{L}$[3-UFA] $\subsetneq$ $\mathcal{L}$[3-UI_k] $\subsetneq$ $\mathcal{L}$[3-UM_1], *and*
(4) $\mathcal{L}$[3-AFA] $\subsetneq$ $\mathcal{L}$[3-AI_k] $\subseteq$ $\mathcal{L}$[3-AM_1].

It is unknown whether $\mathcal{L}$[3-AI_k] $\subsetneq$ $\mathcal{L}$[3-AM_1]. What are the relationships between $\mathcal{L}$[3-NI_k] and $\mathcal{L}$[3-DM_1], between $\mathcal{L}$[3-UI_k] and $\mathcal{L}$[3-DM_1], and between $\mathcal{L}$[3-AI_k] and $\mathcal{L}$[3-NM_1]? The following theorem answer this question.

Theorem 2.2.
(1) $\mathcal{L}$[3-NI_k] *is incomparable with* $\mathcal{L}$[3-DM_1],
(2) $\mathcal{L}$[3-UI_k] *is incomparable with* $\mathcal{L}$[3-DM_1], *and*
(3) $\mathcal{L}$[3-NM_1] $\subsetneq$ $\mathcal{L}$[3-AI_k].

Proof: Let $T_1=\{x \in \{0,1\}^{(3)} \mid \exists n \geq 1\ [l_1(x)=l_2(x)=l_3(x)=2n \wedge$ (the top half of x is the same as the bottom half of x)$]\}$, and $T_2=\{x \in \{0,1\}^{(3)} \mid \exists n \geq 2\ [l_1(x)=l_2(x)=l_3(x)=\mathrm{n} \wedge \exists\ i(2 \leq i \leq n)$ $\{$ the first plane of x is the same as the ith plane of x $\}]\}$.

By using the same technique as in the proof of Lemma 5.1, Corollary 5.1, Theorem 6.4 in [5], we can show that the complement of T_1 is in $\mathcal{L}$[3-NI_k], $T_1 \in \mathcal{L}$[3-UI_k], and $T_2 \notin \mathcal{L}$[3-NI_k]$\cup\mathcal{L}$[3-UI_k]. Furthermore, we can easily prove that $T_2 \in \mathcal{L}$[3-DM_1] [2,4].

By using the same technique as in the proof of Theorem 4.1 in [2], we can show that the complement of T_1 is not in $\mathcal{L}$[3-DM_1]. From these observation, (1) and (2) of the theorem follow. (3) of the theorem can be proved by using the same idea of Ref. [3,4].

The following result in [7-12] shows a relationship among the accepting powers of determinism, nondeterminism, alternation with only universal states, and alternation for k-inkdot 3-FA's.

Theorem 2.3[7-12]**.** (1) $\mathcal{L}$[3-DI_k] $\subsetneq$ $\mathcal{L}$[3-NI_k] $\subsetneq$ $\mathcal{L}$[3-AI_k], *and* (2) $\mathcal{L}$[3-DI_k] $\subsetneq$ $\mathcal{L}$[3-UI_k] $\subsetneq$ $\mathcal{L}$[3-AI_k].

A relationship between $\mathcal{L}$[3-NI_k] and $\mathcal{L}$[3-UI_k] is shown in the following theorem.

Theorem 2.4. *$\mathcal{L}$[3-NI_k] is incomparable with $\mathcal{L}$[3-UI_k].*

Proof: Let T_1 and T_2 be sets described in the proof of Theorem 2.2. We can easily prove that $T_1 \in \mathcal{L}$[3-UI_k]$-\mathcal{L}$[3-NI_k] [5], and the complement of T_2 is in $\mathcal{L}$[3-NI_k], but not in $\mathcal{L}$[3-UI_k]. From this fact, the theorem follows.

For 1-marker 3-FA's, we can easily get the following result [2,4]. That is, alternation is better than nondeterminism, which is better than determinism.

Theorem 2.5. $\mathcal{L}$[3-DM_1] $\subsetneq$ $\mathcal{L}$[3-NM_1] $\subsetneq$ $\mathcal{L}$[3-AM_1].

3 Main Results

This section investigates an open problem. That is, a relationship between $\mathcal{L}$[3-UMI_1] and $\mathcal{L}$[3-AI_k].

Here is some preliminaries. Let $c_1c_2......c_m(m \geq 1)$ be a computation of M on an input tape x. Then, this computation is called:

· a *halting computation* of M on x if c_m is a halting configuration other than any accepting configuration,

· a *double-looping computation* of M on x if there exist some $i(1 \leq i \leq m-2)$ and some (possibly empty) sequence of configurations s such that (i) $c_j \neq c_k$ for each $1 \leq j \leq k \leq i$, (ii) $c_1c_2......c_m = c_1c_2......c_{i-1}c_isc_isc_i$, and (iii) each configuration in c_is is different from each other, and different from each $c_r(1 \leq r \leq i)$, and

· a *rejecting computation* of M on x if the sequence $c_1c_2......c_m$ is a halting, or double-looping computation.

Theorem 3.1. $\mathcal{L}$[3-AI_k]$-\mathcal{L}$[3-UM_1] $\neq \phi$.

Proof: Let $V(m)$=$\{x_1c_1x_2c_2......x_mc_m \mid \forall i(1 \leq i \leq m)$ $\{x_i \in \{0,1\}^{(m\ m\ m)} \wedge c_i \in \{2\}^{(m\ 1\ m)}\}\}$, and T_3=$\{xy \mid$ $\exists m \geq 1\{x,y \in V(m)\} \wedge x \neq y\}$, where for any two three-dimensional tapes x and y with $l_3(x)$=$l_3(y)$, we denote by xy the three-dimensional tape obtained by concatenating y to the east of x. To prove the theorem, we below show that (1) $T_3 \in \mathcal{L}$[3-AI_k], and (2) $T_3 \notin \mathcal{L}$ [3-UM_1]. It is obvious that Part (1) of the theorem holds. Here we only prove (2). We suppose to the contrary that there is a 3-UM_1 M which accepts T_3. Let Q be the set of states of the finite control of M. We divide Q into two disjoint subsets Q^+ and Q^- which correspond to the sets of states when M holds and does not hold the marker in the finite control, respectively. M starts from the initial state in Q^+ with the input head on the upper-northwestmost symbol of an input tape. We assume without loss of generality that M satisfies the following condition (A): ' M does not go out of the boundary symbols #'s. (Of course, M does not go into the input tape from the outside of the boundary symbols #'s.)' For each $M \geq 1$, let $W(m)$=$\{xy \mid x,y \in V(m)\}$. Below we shall again consider the computations of M on tapes in $W(m)$ for large $m \geq 1$. Let x be any tape in $V(m)$ that is supposed to be an east or west half on an input tape (in $W(m)$) to M, and let #'sx (resp., x#'s) be the tape obtained from x by attaching the boundary symbols #'s to the west, south, north, upper, and lower (resp., east, south, north, upper,and lower) sides. Note that, from the above condition (A), both the entrance points to #'sx (resp., x#'s) and the exit points from #'sx (resp., x#'s) are the east (resp., west) side of #'sx (resp., x#'s). Let $PT(m)$ be the set of these entrance (or exit) points. Clearly, $|PT(m)| = (m+2)^2$. Suppose that the marker of M is not placed on the #'sx (resp., x#'s). Then, we define a mapping M_x^w(resp., M_x^e), which depends on M and x, from $Q \times PT(m)$ to the power set of $(Q \times PT(m)) \cup Q_{stop} \cup \{loop\}$ as follows (where Q_{stop} is the set of halting states other than accepting states, and *loop* is a new symbol):

· for any (s,p), $(s^{'},p^{'}) \in Q^- \times PT(m)$, $(s^{'},p^{'}) \in M_x^w(s,p)$ (resp., $M_x^e(s,p)$) $\Leftrightarrow$ when M enters #'sx (resp., x#'s) in state s from entrance point p of the east (resp., west) edge of #'sx (resp., x#'s),there exists a computation of M in which M eventually exits #'sx (resp., x#'s) in state $s^{'}$ from exit point $p^{'}$ of the east(resp.,west) edge of #'sx (resp., x#'s),

· for any $(s,\ p) \in Q \times PT(m)$ and for any $q \in Q_{stop}$, $q \in M_x^w(s,\ p)$ (resp., $M_x^e(s,\ p)$) $\Leftrightarrow$ when M enters #'sx (resp., x#'s) in state s from entrance point p of the east (resp., west) edge of #'sx (resp., x#'s), there exists a computation of M in which M eventually enters state q in #'sx (resp., x#'s), and halts, and

· for any $(s,\ p) \in Q \times PT(m)$, $loop \in M_x^w(s,\ p)$ (resp., $M_x^e(s,\ p)$) $\Leftrightarrow$ when M enters #'sx (resp., x#'s) in state s from entrance point p of the east (resp., west) edge of #'sx (resp., x#'s), there exists a computation in which M enters a loop in #'s x (resp.,x #'s).

Let $x_1, x_2 \in V(m)$. We say that x_1 and x_2 are

· *M-equivalent* if two mappings $M_{x_1}^w$ and $M_{x_2}^w$ are equivalent, and two mappings $M_{x_1}^e$ and $M_{x_2}^e$ are equivalent, and

· *M-equivalent* if for any $(s,\ p)$, $(s^{'},\ p^{'}) \in Q^- \times PT(m)$, and for any $a \in \{w,\ e\}$, $(s^{'},\ p^{'}) \in M_{x_1}^a(s,\ p)$ if and only if $(s^{'},\ p^{'}) \in M_{x_2}^a(s,\ p)$.

(Note that if x_1 and x_2 are M-equivalent, then x_1 and x_2 are M-eqivalent.) Clearly, M-equivalence is an equivalence relation on $V(m)$. Clearly, there are at most $e(m) = (2^{|Q|(m+2)^2+d+1})^{|Q|(m+2)^2}$, where $d = |Q_{stop}|$, M-equivalence classes of $V(m)$. Let $P(m)$ be a largest M-equivalence classes of $V(m)$. Then, we have $|P(m)| \geq \frac{V(m)}{e(m)} = \frac{2^{m^4}}{e(m)}$.

Note that $|P(m)| \gg 1$ for large m. By using the same technique as in the proof of Theorem 6 in [4] and the well-known counting argument, finally, we can prove that $T_3 \notin \mathcal{L}$[3-UM_1].

4 Conclusion

We investigated about marker versus inkdot on three-dimensional input tapes, and showed some accepting properties of various three-dimensional automata with markers or inkdots.

We conclude this paper by giving the following open problems : (1) $\mathcal{L}[3\text{-}AI_k] \subsetneq \mathcal{L}[3\text{-}AM_1]$? (2) What are the relationships between $\mathcal{L}[3\text{-}NI_k]$ and $\mathcal{L}[3\text{-}UM_1]$ and between $\mathcal{L}[3\text{-}UI_k]$ and $\mathcal{L}[3\text{-}NM_1]$? (3) Is $\mathcal{L}[3\text{-}NM_1]$ incomparable with $\mathcal{L}[3\text{-}UM_1]$? (4) $\mathcal{L}[3\text{-}UM_1] \subsetneq \mathcal{L}[3\text{-}AI_k]$?

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