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Nonlinear Adaptive Control System Design and Experiment for a 3-DOF Model Helicopter

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Abstract

This paper deals with model following control of a model helicopter with three degree-of-freedom. Since the model dynamics are described linearly by unknown system parameters, a parameter estimation technique is introduced. The integral type of estimation model is proposed here since the use of the derivative type of model cannot obtain the desired estimation result. In addition, we introduce extra terms into the equations of motion that express model uncertainties and external disturbances. The experimental results show the effectiveness of the proposed method.

keywords: model helicopters, nonlinear control, model following control, parameter identification

1 Introduction

Interests in designing feedback controllers for helicopters have increased over the last ten years or so due to the important potential applications of this area of research. The main difficulties in designing stable feedback controllers for helicopters arise from the nonlinearities and couplings of the dynamics of these aircraft. To date, various efforts have been directed to the development of effective nonlinear control strategies for helicopters [1]-[3]. Most of the existing results have concerned flight regulation. In this paper, the flight tracking control problem of a 3-DOF model helicopter is considered, and a nonlinear adaptive model following control method is proposed. Experimental results are presented to demonstrate the performance of the designed controller.

2 System description

The system dynamics are expressed by the following highly nonlinear and coupled state variable equations

$$\dot{\boldsymbol{x}}_p = f(\boldsymbol{x}_p) + [g_1(\boldsymbol{x}_p)\ g_2(\boldsymbol{x}_p)]\boldsymbol{u}_p \tag{1}$$

where

$$\begin{aligned}
\boldsymbol{x}_p &= [x_{p1}\ x_{p2}\ x_{p3}\ x_{p4}\ x_{p5}\ x_{p6}]^T \\
&= [\varepsilon\ \dot{\varepsilon}\ \theta\ \dot{\theta}\ \phi\ \dot{\phi}]^T,\ \boldsymbol{u}_p = [u_{p1}\ u_{p2}]^T \\
f(\boldsymbol{x}_p) &= \begin{bmatrix} \dot{\varepsilon} \\ p_1\cos\varepsilon + p_2\sin\varepsilon + p_3\dot{\varepsilon} \\ \dot{\theta} \\ p_5\cos\theta + p_6\sin\theta + p_7\dot{\theta} \\ \dot{\phi} \\ p_9\dot{\phi} \end{bmatrix} \\
g_1(\boldsymbol{x}_p) &= [0\quad p_4\cos\theta\quad 0\quad 0\quad 0\quad p_{10}\sin\theta]^T \\
g_2(\boldsymbol{x}_p) &= [0\quad 0\quad 0\quad p_8\quad 0\quad 0]^T
\end{aligned} \tag{2}$$

It may be noted that all the parameters $p_i\ (i = 1, \cdots, 10)$ are constants[4].

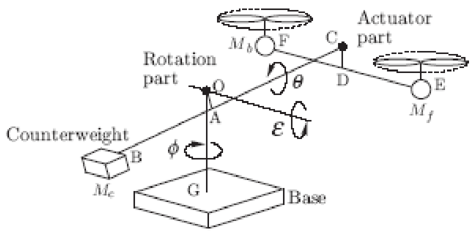

Fig. 1: Overview of model helicopter

3 Control system design

In this section, a nonlinear model reference control system is designed for the 3-DOF model helicopter described in the previous section.

The reference model is given as

$$\begin{aligned}
&\dot{\boldsymbol{x}}_M = A_M\boldsymbol{x}_M + B_M\boldsymbol{u}_M,\ \boldsymbol{y}_M = C_M\boldsymbol{x}_M \\
&\boldsymbol{x}_M = [x_{M1},\ x_{M2},\ x_{M3},\ x_{M4},\ x_{M5},\ x_{M6},\ x_{M7},\ x_{M8}]^T \\
&\boldsymbol{y}_M = [\ \varepsilon_M \quad \phi_M\ ]^T,\quad \boldsymbol{u}_M = [\ u_{M1} \quad u_{M2}\ ]^T
\end{aligned} \tag{3}$$

where

$$A_M = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}, B_M = \begin{bmatrix} \boldsymbol{i}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{i}_1 \end{bmatrix}, C_M = \begin{bmatrix} \boldsymbol{i}_2^T & \boldsymbol{0}^T \\ \boldsymbol{0}^T & \boldsymbol{i}_2^T \end{bmatrix}$$

$$K_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ k_{i1} & k_{i2} & k_{i3} & k_{i4} \end{bmatrix}, \boldsymbol{i}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \boldsymbol{i}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If the outputs are chosen $\boldsymbol{y}_p = [\varepsilon,\ \phi]^T$,the decoupling matrix is obviously singular. Hence, we apply a nonlinear structure algolithm to design a model reference controller[5]. Using this algorithm, The input vector is given by

$$\boldsymbol{u}_p = R(\boldsymbol{x}) + S(\boldsymbol{x})\boldsymbol{u}_M,\ \boldsymbol{x} = [\boldsymbol{x}_p^T\ \ \boldsymbol{x}_M^T]^T \tag{4}$$

Since the controller requires the angular velocity signals $\dot{\varepsilon}$, $\dot{\theta}$ and $\dot{\phi}$, in the experiment , these signals are calculated numerically from the measured angular positions by a discretized differentiator with the first-order filter. Hence, for example, we have

$$\dot{\varepsilon}(k) \approx \frac{1}{\alpha T_s + 1}\left[\dot{\varepsilon}\,(k-1) + \alpha\left\{\varepsilon\,(k) - \varepsilon\,(k-1)\right\}\right] \tag{5}$$

where, T_s is the sampling period and the design parameter α is a positive constant.

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4 Parameter identification based on the differential equations

4.1 Parameter identification algorithm

It is difficult to obtain the desired control performance by applying the above algorithm directly to the experimental system, since there are parameter uncertainties in the model dynamics. However, it is straightforward to see that the system dynamics (1) are linear with respect to unknown parameters, even though the equations are nonlinear. It is therefore possible to introduce a parameter identification scheme in the feedback control loop. Henceforth, we explain only axis ε due to the limitations of space. Axes θ and ϕ are the same way as axis ε.

In the present study, the parameter identification scheme is designed in discrete-time form using measured discrete-time signals. Hence, the estimated parameters are calculated recursively at every instant kT, where T is the updating period of the parameters and k is a nonnegative integer. Henceforth we omit T for simplicity. Then, the dynamics of the model helicopter given by equation (1) can be re-expressed as

$$w_1(k) \equiv \ddot{\varepsilon}(k) = \boldsymbol{\zeta}_1^T \boldsymbol{v}_1(k) \qquad (6)$$

where

$$\begin{aligned} \boldsymbol{\zeta}_1 &= \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}^T \\ \boldsymbol{v}_1(k) &= \begin{bmatrix} \cos\varepsilon & \sin\varepsilon & \dot{\varepsilon} & u_1\cos\theta \end{bmatrix}^T \end{aligned}$$

the estimated values of $w_1(k)$are obtained as

$$\widehat{w}_1(k) = \widehat{\boldsymbol{\zeta}}_1^T(k)\boldsymbol{v}_1(k) \qquad (7)$$

where $\widehat{\zeta}_1(k)$ is the estimated parameter vectors. Along with the angular velocities, the angular accelerations $w_1(k) = \ddot{\varepsilon}(k)$is also obtained by numerical calculation using a discretized differentiator.

The parameters are estimated using a recursive least squares algorithm as follows

$$\widehat{\boldsymbol{\zeta}}_1(k) = \widehat{\boldsymbol{\zeta}}_1(k-1) + \frac{P_1(k-1)\boldsymbol{v}_1(k-1)\left[w_1(k-1) - \widehat{w}_1(k-1)\right]}{\lambda_1 + \boldsymbol{v}_1^T(k-1)P_1(k-1)\boldsymbol{v}_1(k-1)} \qquad (8)$$

$$\begin{aligned} P_1^{-1}(k) &= \lambda_1 P_1^{-1}(k-1) + \boldsymbol{v}_1(k-1)\boldsymbol{v}_1^T(k-1) \\ & P_1^{-1}(0) > 0, \quad 0 < \lambda_1 \leq 1 \end{aligned} \qquad (9)$$

Then, the tracking of the two outputs is achieved under the persistent excitation of the signals v.

4.2 Experimental studies

The estimation and control algorithm described above were applied to the experimental system. The design parameters are given as follows: $T_s = 2$ [ms], $T = 2$ [ms], $\alpha = 100$. $\lambda_1 = 0.999$, $\lambda_2 = 0.9999$, $\lambda_3 = 0.999$ and $P_1{}^{-1}(0) = P_2{}^{-1}(0) = 10^4 I_4$, $P_3{}^{-1}(0) = 10^4 I_2$

$$\begin{aligned} u_{M1} &= \begin{cases} 0.3 & 45k - 52.5 \leq t < 45k - 30 \\ -0.1 & 45k - 30 \leq t < 45k - 7.5 \end{cases} \\ u_{M2} &= \begin{cases} 0 & 0 \leq t < 7.5 \\ 0.4 & 45k - 37.5 \leq t < 45k - 22.5 \\ -0.4 & 45k - 22.5 \leq t < 45k \end{cases} \\ & \qquad k = 1, 2, 3, \cdots \end{aligned} \qquad (10)$$

The outputs of the experimental results are shown in Fig. 2 and Fig.3. The tracking is incomplete because the neither of the output errors of ε or ϕ converge. Figure 4 display the estimated parameter $\widehat{p}_2$.The estimated parameters move to the limiting values of the variation range.

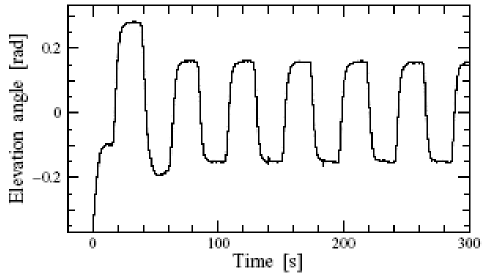

Fig. 2: The time evolution of angle ε(—) and reference output $\varepsilon_M(\cdots)$.

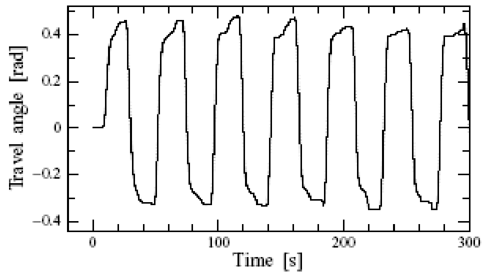

Fig. 3: The time evolution of angle ϕ(—) and reference output $\phi_M(\cdots)$.

5 Parameter identification based on the integral form of the model equations

5.1 The model equations

5.1.1 Parameter identification algorithm

The main reason why the experimental results exhibit the poor tracking performance described in the previous subsection 4.2 lies in the fact that the parameter identification is unsatisfactory due to the inaccuracy of the estimation of the velocity and the acceleration signals. To overcome this problem in this subsection, a parameter estimation scheme is designed for modified dynamics equations obtained by applying integral operators to the differential equation expressing the system dynamics (6). Neither velocities nor accelerations appear in these modified equations.

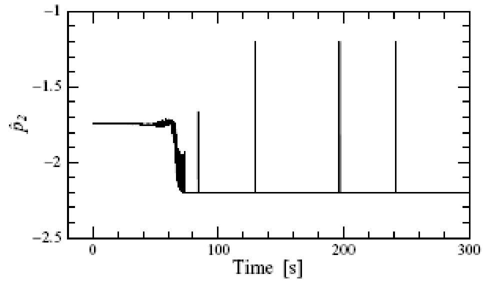

Fig. 4: Time evolution of the estimated parameter $\widehat{p}_2$. The dotted lines represent the limiting values.

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Define $z_1(k)$ by the following double integral

$$z_1(k) \equiv \int_{kT-nT}^{kT} \int_{\tau-nT}^{\tau} \ddot{\varepsilon}(\sigma) d\sigma d\tau \quad (11)$$

Then, the direct calculation of the right-hand side of equation (11) leads to

$$\begin{aligned} z_1(k) &= \int_{kT-nT}^{kT} (\dot{\varepsilon}(\tau) - \dot{\varepsilon}(\tau - nT))\, d\tau \\ &= \varepsilon(kT) - 2\varepsilon(kT - nT) + \varepsilon(kT - 2nT) \quad (12) \end{aligned}$$

Next, discretizing the double integral of the right-hand side of equation (6) yields

$$\begin{aligned} & p_1 \int_{kT-nT}^{kT} \int_{\tau-nT}^{\tau} \cos\varepsilon(\sigma) d\sigma d\tau + \cdots \\ & + p_3 \int_{kT-nT}^{kT} \{\varepsilon(\tau) - \varepsilon(\tau - nT)\}\, d\tau + \cdots \\ \approx\ & p_1 T^2 \sum_{l=k-(n-1)}^{k} \sum_{i=l-(n-1)}^{l} \cos\varepsilon(i) + \cdots \\ & + p_3 T \sum_{l=k-(n-1)}^{k} \{\varepsilon(l) - \varepsilon(l-(n-1))\} + \cdots \quad (13) \end{aligned}$$

As a result, the integral form of the dynamics is obtained as

$$z_1(k) = \zeta_1^T \bar{v}_1(k) \quad (14)$$

where

$$\begin{aligned} \bar{v}_1(k) &= \begin{bmatrix} q_{11}(k) & q_{12}(k) & q_{13}(k) & q_{14}(k) \end{bmatrix}^T \\ q_{11}(k) &= T^2 \sum_{l=k-(n-1),}^{k} \sum_{i=l-(n-1)}^{l} \cos\varepsilon(i) \\ q_{12}(k) &= T^2 \sum_{l=k-(n-1)}^{k} \sum_{i=l-(n-1)}^{l} \sin\varepsilon(i) \\ q_{13}(k) &= T \sum_{l=k-(n-1)}^{k} \{\varepsilon(l) - \varepsilon(l-(n-1))\} \\ q_{14}(k) &= T^2 \sum_{l=k-(n-1)}^{k} \sum_{i=l-(n-1)}^{l} u_{p1}(i) \cos\theta(i) \end{aligned}$$

Hence, the estimate model for (14) is given by

$$\widehat{z}_1(k) = \widehat{\zeta}_1^T(k) \bar{v}_1(k) \quad (15)$$

and the system parameters $\widehat{\zeta}_1^T(k)$ can be identified from expression (15) without use of the velocities or accelerations of ε, θ, ϕ.

5.1.2 Experimental studies

The design parameters are given as follows: $n = 100$, $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}_3 = 0.9999$ and $P_1{}^{-1}(0) = P_2{}^{-1}(0) = 10^3 I_4$, $P_3{}^{-1}(0) = 10^3 I_2$. The other parameters are the same as those of the previous section.

The outputs are shown in Figures 5 and 6. The tracking performance of both the outputs ε and ϕ is improved in comparison with the previous section. However, there remains a tracking error. The estimated parameter plotted in Figure 7 change slowly, and the variations of the estimated parameter is smaller than those of the corresponding values shown in Figure 4.

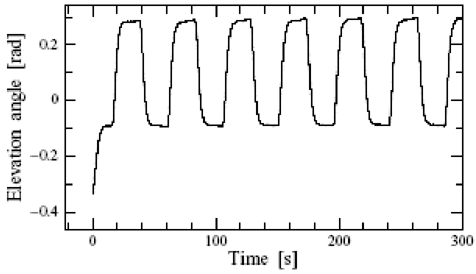

Fig. 5: The time evolution of angle ε(—) and reference output $\varepsilon_M(\cdots)$.

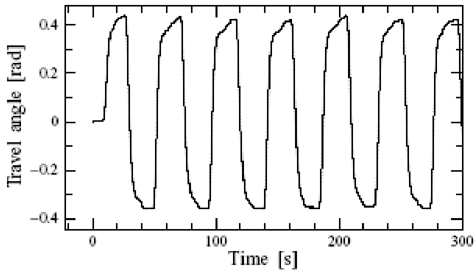

Fig. 6: The time evolution of angle ϕ(—) and reference output $\phi_M(\cdots)$.

5.2 The model equations with model uncertainties and external disturbances

5.2.1 Parameter identification algorithm

Although the use of the integral form of the dynamics has improved the tracking performance of both the outputs ε and θ, tracking errors still remain. On the basis that these errors are caused by model uncertainties and external disturbances, for example, motor dynamics or friction (other than viscous friction), we add the extra terms p_{11}, p_{12} and p_{13}into equation (1) to represent model uncertainties and external disturbances. For simplicity, here we assume these terms are constant. Then, the system dynamics are expressed as

$$w_1(k) \equiv \ddot{\varepsilon}(k) = \zeta_1^T v_1(k) \quad (16)$$

where

$$\begin{aligned} \zeta_1 &= \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_{11} \end{bmatrix}^T \\ v_1(k) &= \begin{bmatrix} \cos\varepsilon & \sin\varepsilon & \dot{\varepsilon} & u_1\cos\theta & 1 \end{bmatrix}^T \end{aligned}$$

It is worth noting that all the parameters p_i $(i = 1\ldots,4,11)$ of the equations are constant. The integral

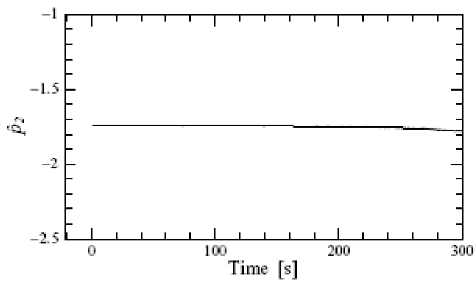

Fig. 7: Time evolution of the estimated parameter $\widehat{p}_2$. The dotted lines represent the limiting values.

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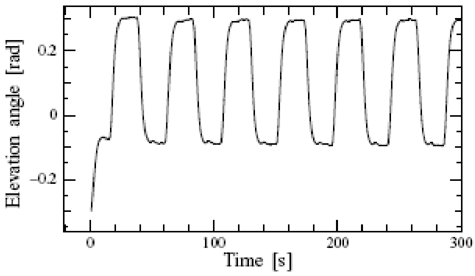

Fig. 8: The time evolution of angle ε(—) and reference output $\varepsilon_M(\cdots)$.

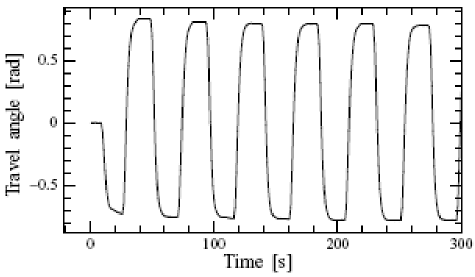

Fig. 9: The time evolution of angle ϕ(—) and reference output $\phi_M(\cdots)$.

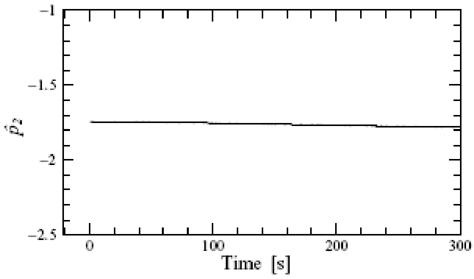

Fig. 10: Time evolution of the estimated parameter $\widehat{p}_2$. The dotted lines represent the limiting values.

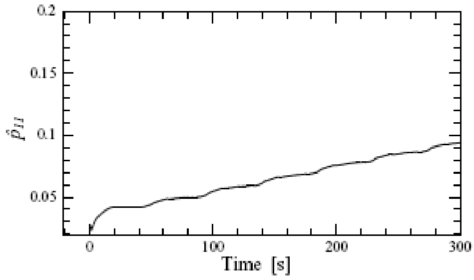

Fig. 11: Time evolution of the estimated parameter $\widehat{p}_{11}$. The dotted lines represent the limiting values.

form of the dynamics is obtained as

$$z_1(k) = \boldsymbol{\zeta}_1^T \widetilde{\boldsymbol{v}}_1(k) \tag{17}$$

where

$$\begin{aligned} \widetilde{\boldsymbol{v}}_1(k) &= \left[\begin{array}{cc} \bar{\boldsymbol{v}}_1^T(k) & q_{15}(k) \end{array} \right]^T \\ q_{15}(k) &= T^2 \sum_{l=k-(n-1)}^{k} \sum_{i=l-(n-1)}^{l} 1 \\ &= T^2(n-1)^2 \end{aligned}$$

Hence, the estimate model for expression (17) is given by

$$\widehat{z}_1(k) = \widehat{\zeta}_1^T(k) \widetilde{\boldsymbol{v}}_1(k) \tag{18}$$

and the system parameters $\widehat{\zeta}_1(k)$ can be identified by a recursive least squares algorithm based on expression (18) that is similar to that given by equations (8) and (9).

5.2.2 Experimental studies

The weighting factor of the least squares algorithm is given by $\widetilde{\lambda} = 0.9995 + 0.0005\exp(-5\sqrt{e_1^2 + e_2^2})$, while the updating period of the parameters, T, is $T = 10$ $[ms]$. The gain of the input u_{M2} is chenged to ± 0.8 from ± 0.4. The other design parameters are the same as those of the previous section.

The outputs are depicted in Figures 8 and 9, while the estimated parameters are shown in Figures 10 and 11. The tracking performance of both of the outputs ε and ϕ has been further improved by the inclusion of the uncertainties.

6 Summary

This paper considers the nonlinear adaptive model following control of a 3-DOF model helicopter. Two parameter identification schemes are discussed: The first scheme is based on the differential equation model. This scheme is unable to obtain a good tracking control performance due to the inaccuracy of the estimated velocity and acceleration signals. The second scheme is designed for a dynamics model derived by applying integral operators to the differential equations expressing the system dynamics. Hence, this identification algorithm requires neither velocity nor acceleration signals. The experimental results show that the second method yields a better tracking objective, although tracking errors still remain. Finally, we introduce extra terms into the equations of motion to express model uncertainties and external disturbances. With reference to experimental results, this modification is shown to further improve the control performance of the identification algorithm.

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