

## Design of nonlinear controllers for active vehicle suspension with state constraints

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**Abstract:** Design of active vehicle suspension has tradeoffs between three main performance metrics (passenger comfort, suspension deflection and road holding ability). It has been known that each performance can be achieved by  $H_\infty$  controller and they can be gathered by LPV (Linear Parameter Varying) method. However, because the suspension deflection limit was not explicitly considered, this limit may be exceeded. In this paper, the authors propose a "reference shaping" based method in order to improve the control performance. In this approach, a "virtual reference signal" is imposed to the system such that the suspension deflection is kept small. The effectiveness of the approach is examined by numerical simulations.

### I. INTRODUCTION

Conventional passive suspensions that employ a spring and damper between the car body and wheel assembly, represent a tradeoff between conflicting performance metrics such as passenger comfort, road holding, and suspension deflection. There has been considerable research into the use of feedback to control active suspensions. This would be implemented by placing a hydraulic actuator, controlled by feedback, between the chassis and wheel assembly.

In previous research[1], linear controller was designed to improve either passenger comfort or suspension deflection, and two linear controllers were gathered single nonlinear controller by using LPV method [2], [3]. However, because previous research doesn't consider that constraint is assigned to system, there are cases that the constraint was not satisfied by initial condition or disturbance. In this paper, the authors propose control system considered constraints. But there are various methods for linear system that have constraints with state, in this paper, the authors use reference shaper[4], [5], [6]. The result is expected that over shot is reduced by giving suitable reference track for suspension deflection. The effectiveness of the approach is examined by numerical simulations.

### II. QUARTER CAR SUSPENSION MODEL

The quarter car model is shown in Figure 1. In Figure 1 the sprung mass,  $m_s$ , represents the car chassis, while the unsprung mass,  $m_{us}$ , represents the wheel assembly. The spring,  $k_s$ , and damper,  $b_s$ , represent a passive spring and shock absorber that are placed between the car body and the wheel assembly, while the spring,  $k_t$ , serves to model the compressibility of the

road disturbance respectively. The force  $f_s$  applied between the sprung and unsprung masses, is controlled by feedback and represents the active component of the suspension system. This force is generated by means of a hydraulic actuator placed between the two masses, but in this paper, the dynamics of the actuator is ignored, and assume that the control signal is the force  $f_s$ . The authors assume that  $x_s$  and  $x_{us}$  are measured from their static equilibrium positions and that the tyre remains in contact with the road at all times, and defining  $x_1 := x_s$ ,  $x_2 := \dot{x}_s$ ,  $x_3 := x_{us}$ ,  $x_4 := \dot{x}_{us}$ . The quarter model is shown following state-space equation.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{m_s} [k_s(x_1 - x_3) + b_s(x_2 - x_4) - f_s] \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{m_{us}} [k_s(x_1 - x_3) + b_s(x_2 - x_4) \\ &\quad - k_t(x_3 - r) - f_s] \end{aligned} \tag{1}$$

The component values is shown in table 1.

Table 1. The component values

	value	unit
$m_s$	290	Kg
$m_{us}$	59	Kg
$b_s$	1000	N/m/s
$k_s$	16812	N/m
$k_t$	190000	N/m

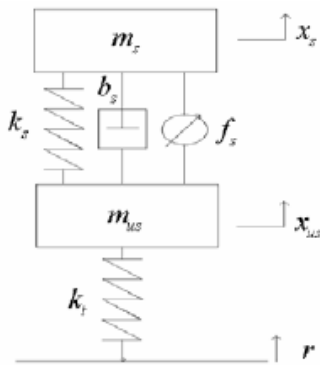


Figure.1. The quarter car model.

### III. LINEAR $H_\infty$ CONTROLLER DESIGN AND LPV EXTENSION

The linear controller aims that improving either passenger comfort or suspension deflection. In this paper, the suspension deflection is limited to 8cm. A controller that focuses on passenger comfort, i.e., keeps the acceleration transfer function small, does so at the expense of larger suspension deflection, and vice versa. The goal of the linear  $H_\infty$  controller design is to establish a baseline for the amount of reduction in vertical acceleration and suspension deflection that can be achieved, when these metrics are minimized independently. As is standard in the  $H_\infty$  framework, the performance objectives are achieved via minimizing weighted transfer function norms. A block diagram of the  $H_\infty$  control design interconnection is shown in Figure 2. In Figure 2, the weight functions are set as following

$$\begin{aligned}
 W_a &= 10^{-5} & W_{ref} &= 0.11 \\
 W_{act} &= \frac{0.5/1300(s+50)}{s+500} \\
 W_{x_1} &= \frac{\phi_a \cdot 2\pi 5}{s+2\pi 50} & W_{x_1-x_3} &= \frac{\phi_d \cdot 10}{s+5}
 \end{aligned} \tag{2}$$

Where  $\phi_a$  and  $\phi_d$  are gains that reflect the relative importance we place on the acceleration and deflection transfer functions in the  $H_\infty$  design. A large gain  $\phi_a$  and a small gain  $\phi_d$  correspond to a design that emphasizes passenger comfort. On the other hand, choosing  $\phi_a$  small and  $\phi_d$  large corresponds to a design that focuses on suspension deflection.

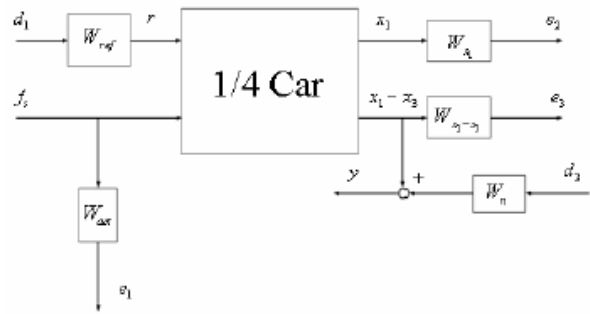


Figure.2.  $H_\infty$  control design interconnection.

#### III.1 Linear $H_\infty$ design controller

In design 1, we focus on keeping the body travel small, without regard for the suspension deflection. Therefore the author choose  $\phi_a = 25$  and set  $\phi_d = 0$ .

In design 2, we focus on keeping the suspension deflection. Therefore the author choose  $\phi_a = 0$  and set  $\phi_d = 200$ .

The time responses of these two controllers are shown in figure 3 and 4. Two responses correspond to the road disturbance  $d(t)$ :

$$d(t) = \begin{cases} a(1 - \cos 8\pi t), & 0.5 \leq t \leq 0.75 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where  $a = 0.055$  correspond to road bump of peak magnitude 11cm. From Figure 3 and 4, two design is shown that design 1 is suited improving passenger comfort and design 2 is suited improving suspension deflection.

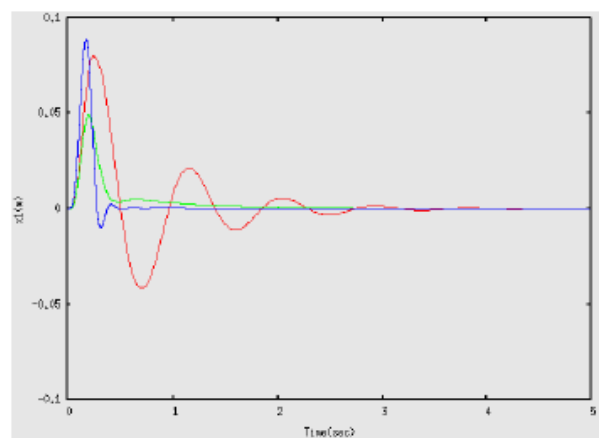


Figure.3. Body travel, Passive(red), Design1(green), Design2(blue).

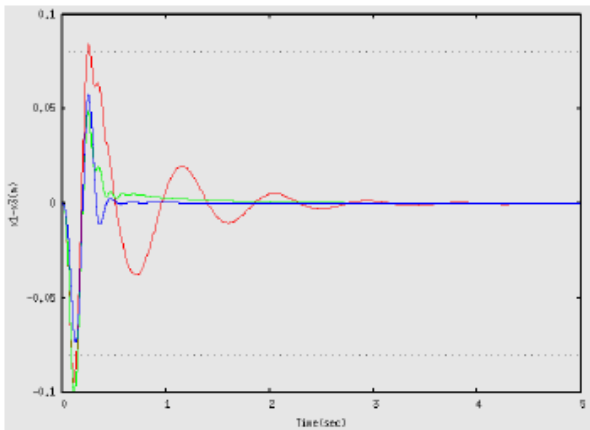


Figure.4. Suspension deflection,  
Passive(red), Design2(green), Design2(blue).

### III.2 LPV design controller

In LPV design controller, the two  $H_\infty$  controllers designed above are combined into a single nonlinear controller by using linear parameter varying (LPV) method. The LPV controller schedules on suspension deflection i.e.,  $\rho = x_1 - x_3$ , and focuses on minimizing either the vertical acceleration or suspension deflection response, depending on the magnitude of the suspension deflection. Therefore, in order to achieve the shift in focus from vertical acceleration to suspension deflection, the weighting function  $W_{x_1}$  and  $W_{x_1-x_3}$  are chosen to be parameter-dependent, i.e., function of  $\rho$ . The parameter-dependent weight functions are shown following.

$$W_{x_1}(\rho) = \frac{\phi_a(\rho) \cdot 2\pi 5}{s + 2\pi 50} \quad (4)$$

$$W_{x_1-x_3}(\rho) = \frac{\phi_d(\rho) \cdot 10}{s + 5}$$

The parameter dependence of the gains  $\phi_a(\rho)$  and  $\phi_d(\rho)$  is characterized by means of two constant  $\rho_1$  and  $\rho_2$ . For this design  $\rho_1$  and  $\rho_2$  are chosen  $\rho_1 = 0.0575$  and  $\rho_2 = 0.08$ . The LPV controller is implemented by linearly interpolating between these two controller points as follows,

$$K[\rho] = (A_1, B_1, C_1, D_1), \text{ if } \rho \in [-\rho_1, \rho_1], \quad (5)$$

$$= (A_2, B_2, C_2, D_2), \text{ if } \rho \in (-\infty, -\rho_2] \text{ or } [\rho_2, \infty),$$

$$= (A(\rho), B(\rho), C(\rho), D(\rho)), \text{ if } \rho \in (\rho_1, \rho_2),$$

$$= (A(-\rho), B(-\rho), C(-\rho), D(-\rho)), \text{ if } \rho \in (-\rho_2, -\rho_1)$$

where

$$A(\rho) := \frac{A_1 - A_2}{\rho_1 - \rho_2}(\rho - \rho_1) + A_1, \quad B(\rho) := \frac{B_1 - B_2}{\rho_1 - \rho_2}(\rho - \rho_1) + B_1,$$

$$C(\rho) := \frac{C_1 - C_2}{\rho_1 - \rho_2}(\rho - \rho_1) + C_1, \quad D(\rho) := \frac{D_1 - D_2}{\rho_1 - \rho_2}(\rho - \rho_1) + D_1,$$

Like the  $H_\infty$  design, disturbance is chosen road bump of peak magnitude 11cm. The time responses of these LPV controller is shown in figure 5 and 6. From figure 5 and 6, passenger comfort was improved, but suspension deflection was not improved. This result is thought that there is for considering state constraints.

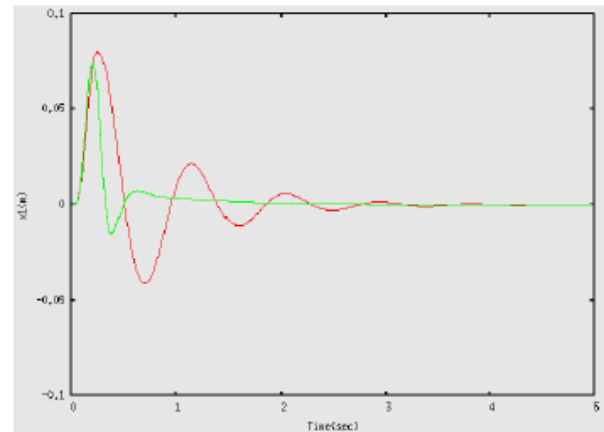


Figure.5. Body travel,  
Passive(red), LPV(green).

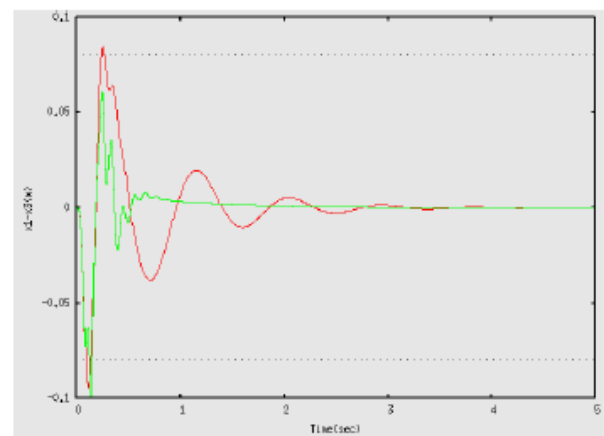


Figure.6. Suspension deflection,  
Passive(red), LPV(green).

## IV. REFERENCE SHAPER METHOD

Most real plants have some constraints on their state or input such as actuator saturation and amplitude limitation of certain state of systems. When constraints are disregarded, this design causes deterioration control performance and destabilization of control system. Therefore it is necessary to consider constraints so that control safely. The goal is solution problem by to input control system into reference signal. A block diagram 1 is shown in Figure 7.

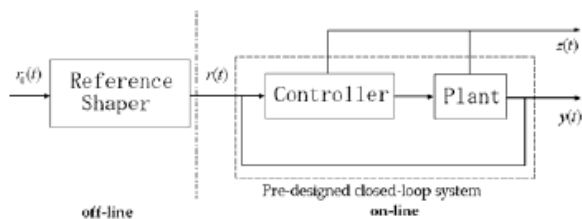


Figure.7. Reference shaping.

In general, there does not exist an analytical method to design the reference signal which improves the control performance. In this paper, the reference signal is chosen similar to the reversal of the blue line in Figure 4 such that the overshoot at the initial time interval is made small. The graph of the reference signal and is depicted as Figure 8.

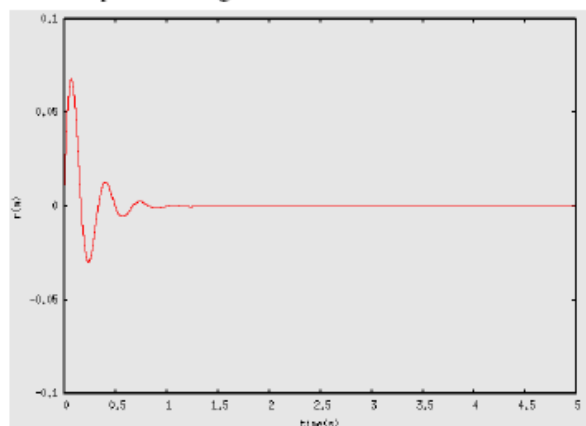


Figure.8. Reference signal.

The simulation is run in same way section III. From Figure 9, the suspension deflection is shown preventing limit of  $\pm 0.08$

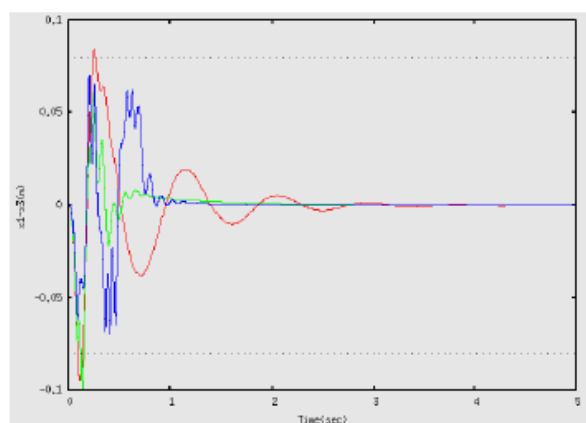


Figure.9. Suspension deflection,  
Passive(red), LPV(green), Reference shaping(blue)

## V. CONCLUSION

In this paper, the authors proposed a method to improve the control performance of the active vehicle suspension system. The “reference shaping” method is applied in order to satisfy the constraint about suspension deflection. However, the response becomes oscillative. This is also due to the fact that the two controllers are gathered piecewise linearly. This defect may be improved by designing smooth interpolation properly.

## VI. REFERENCES

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