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An Underactuated Control for VTOL Aerial Robots with Four Rotors via a Chained Form Transformation

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Abstract

We present a new control strategy for a VTOL aerial robot. A kinematics control law is derived using Astolfi's discontinuous control, after introducing a chained form transformation with one generator and three chains to the original model. This was motivated by the fact that the discontinuous kinematic-model without using a chained form transformation assures only a local stability of the kinematic based control system, instead of guaranteeing a global stability of the control system. Finally, a computer simulation is shown to demonstrate the effectiveness of our approach.

1 Introduction

Unmanned vehicles are important when it comes to performing a desired task in a dangerous and/or inaccessible environment. Unmanned indoor and outdoor mobile robots have been successfully used for some decades. More recently, a growing interest in unmanned aerial vehicles (UAVs) has been shown among the research community. Being able to design a vertical takeoff and landing (VTOL)-UAV, which is highly maneuverable and extremely stable, is an important contribution to the field of aerial robotics, because potential applications are tremendous as seen in high buildings and monuments investigation, rescue missions, film making, etc.

Recently, the study on VTOL type aerial robot attracts the attention of researchers, in which the robot is called "Draganflyer," "Quattrocopter," "X-4 Flyer," or "Quadrotor" and has four rotors in general [1, 2, 3]. The control system for this VTOL type aerial robot can be regarded as an underactuated system [4] that deals with controlling six generalized coordinates with four inputs, and its control becomes complicated, compared to a nonholonomic control where any four states are controlled out of six generalized coordinates by using four inputs.

In this paper, we present a new control strategy for a VTOL aerial robot that is called X4-flyer. A kinematics control law is derived using Astolfi's discontinuous control [5], after introducing a chained form transformation [6] with one generator and three chains to the original model. This was motivated by the fact [7] that the discontinuous kinematic-model without using a chained form transformation assures only a local stability (or controllability) of the kinematic based control system, instead of guaranteeing a global stability of the control system. Finally, a computer simulation is given to demonstrate the effectiveness of our approach.

2 A Chained Form Transformation for a Symmetric Affine System with n State-Four Inputs

Let the controlled objective be a symmetric affine system with n state-four inputs described by

$$\dot{\boldsymbol{q}} = \boldsymbol{g}_1 u_1 + \boldsymbol{g}_2 u_2 + \boldsymbol{g}_3 u_3 + \boldsymbol{g}_4 u_4, \qquad \boldsymbol{q} \in \Re^n \tag{1}$$

Applying the transformations of the state and input such as

$$\boldsymbol{z} = \Phi(\boldsymbol{q}), \qquad \boldsymbol{v} = \Xi(\boldsymbol{q})\boldsymbol{u} \tag{2}$$

to the above equation, the objective is to obtain the following chained form having one-generator and three chains

$$\begin{array}{llll}
\dot{\xi}_0 = v_1 & \dot{\zeta}_0 = v_2 & \dot{\eta}_0 = v_3 & \dot{\gamma}_0 = v_4 \\
 & \dot{\zeta}_1 = \zeta_0 v_1 & \dot{\eta}_1 = \eta_0 v_1 & \dot{\gamma}_1 = \gamma_0 v_1 \\
 & \vdots & \vdots & \vdots \\
 & \dot{\zeta}_{n_2} = \zeta_{n_2-1} v_1 & \dot{\eta}_{n_3} = \eta_{n_3-1} v_1 & \dot{\gamma}_{n_4} = \gamma_{n_4-1} v_1
\end{array}$$

where $n_2 + n_3 + n_4 + 4 = n$ $(n_2 \geq n_3 \geq n_4 \geq 0)$.

It is known [4] that the reachability distribution of a chained form system has rank n for all $\boldsymbol{z} \in \Re^n$, which implies that the transformed system is also globally controllable because it is a symmetric affine system.

As a sufficient condition for implementing a chained form transformation, the input vector fields $\boldsymbol{g}_1 \dots \boldsymbol{g}_4$ satisfy the following forms:

$$\boldsymbol{g}_1 = \frac{\partial}{\partial \boldsymbol{q}_1} + \sum_{i=2}^{n} \boldsymbol{g}_1^i \frac{\partial}{\partial x_i}, \qquad \boldsymbol{g}_2 = \sum_{i=2}^{n} \boldsymbol{g}_2^i \frac{\partial}{\partial x_i}$$
$$\boldsymbol{g}_3 = \sum_{i=2}^{n} \boldsymbol{g}_3^i \frac{\partial}{\partial x_i}, \qquad \boldsymbol{g}_4 = \sum_{i=2}^{n} \boldsymbol{g}_4^i \frac{\partial}{\partial x_i} \tag{3}$$

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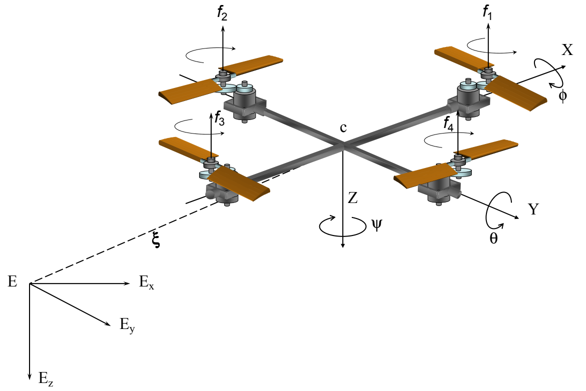

Fig. 1: Coordinate definition of X4-flyer

where the vector fields $\boldsymbol{g}_1 \ldots \boldsymbol{g}_4$ are to be smooth, linearly independent each other.

Define the distributions:

$$
\begin{aligned}
G_0 &= \mathrm{span}\{g_2, g_3, g_4\} \\
G_1 &= \mathrm{span}\{g_2, g_3, g_4, \mathrm{ad}_{g_1} g_2, \mathrm{ad}_{g_1} g_3, \mathrm{ad}_{g_1} g_4\} \\
&\vdots \\
G_m &= \mathrm{span}\{ad^i_{g_1} g_2, \mathrm{ad}^i_{g_1} g_3, \mathrm{ad}^i_{g_1} g_4\} \qquad (0 \le i \le m)
\end{aligned} \tag{4}
$$

where $G_0 \ldots G_m$ are all involutive and G_m has rank $n-1$.

In addition, we have to find four functions, $h_1 \ldots h_4$ to be not unique, such that

$$
\begin{array}{lll}
dh_1 \perp G_j & 0 \le j \le m & \\
dL^k_{g_1} h_2 \perp G_j & 0 \le j \le n_2 - 1 & 0 \le k \le n_2 - 1 - j \\
dL^k_{g_1} h_3 \perp G_j & 0 \le j \le n_3 - 1 & 0 \le k \le n_3 - 1 - j \\
dL^k_{g_1} h_4 \perp G_j & 0 \le j \le n_4 - 1 & 0 \le k \le n_4 - 1 - j
\end{array}
$$

where the Lie derivative of a scalar function $\phi(x)$ along a vector field $f(x)$ is the following scalar function defined by

$$
L_f \phi(x) = \frac{\partial \phi}{\partial x} f(x) \tag{5}
$$

When all of the above conditions are satisfied, $\Phi(\boldsymbol{q})$ and $\Xi(\boldsymbol{q})$ can be reduced to

$$
\Phi(\boldsymbol{q}) = \left[h_1 \;\; L^{n_2}_{g_1} h_2 \; \cdots \; h_2 \;\; L^{n_3}_{g_1} h_3 \; \cdots \; h_3 \;\; L^{n_4}_{g_1} h_4 \; \cdots \; h_4 \right]^T
$$

$$
\Xi(\boldsymbol{q}) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
L^{n_2+1}_{g_1} h_2 & L_{g_2} L^{n_2}_{g_1} h_2 & L_{g_3} L^{n_2}_{g_1} h_2 & L_{g_4} L^{n_2}_{g_1} h_2 \\
L^{n_3+1}_{g_1} h_3 & L_{g_2} L^{n_3}_{g_1} h_3 & L_{g_3} L^{n_3}_{g_1} h_3 & L_{g_4} L^{n_3}_{g_1} h_3 \\
L^{n_4+1}_{g_1} h_4 & L_{g_2} L^{n_4}_{g_1} h_4 & L_{g_3} L^{n_4}_{g_1} h_4 & L_{g_4} L^{n_4}_{g_1} h_4
\end{bmatrix}
$$

3 Kinematics of X4-Flyer

Let $E = \{E_x \; E_y \; E_z\}$ denote a right-hand inertial frame such that E_z denotes the vertical direction downwards into the earth (see Fig. 1). Let the vector $\boldsymbol{\xi} = [x\, y\, z]^T$ denote the position of the center of mass of the airframe in the frame E relative to a fixed origin $O \in E$. Let c be a (right-hand) body fixed frame for the airframe. When defining the rotational angles $\boldsymbol{\eta} = [\phi \;\; \theta \;\; \psi]^T$ around X-, Y-, and Z-axis in the frame c, the orientation of the rigid body is given by a rotation $R : c \to E$, where $R \in \Re^{3\times3}$ is an orthogonal rotation matrix.

Using such a rotational matrix and exchanging $\dot{x}$ and $\dot{z}$ in the kinematic model of X4-flyer [7], gives the following model:

$$
\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{x} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}
=
\begin{bmatrix}
\cos\phi\cos\theta & 0 & 0 & 0 \\
\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & 0 & 0 & 0 \\
\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \dot{z}_b \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{6}
$$

where $\dot{z}_b$ denotes the Z-directional translational velocity. When defining $\boldsymbol{q} = [z\; y\; x\; \phi\; \theta\; \psi]^T$ and $\boldsymbol{u} = [\dot{z}_b\; \dot{\phi}\; \dot{\theta}\; \dot{\psi}]^T$, it can be rewritten in the symmetric affine form of six states-four inputs:

$$
\dot{\boldsymbol{q}} = \boldsymbol{f}_1 u_1 + \boldsymbol{f}_2 u_2 + \boldsymbol{f}_3 u_3 + \boldsymbol{f}_4 u_4, \qquad \boldsymbol{q} \in \Re^6 \tag{7}
$$

4 A Chained Form Transformation for X4-Flyer

In order to satisfy the conditions of (3), the input vector fields $\boldsymbol{f}_1, \cdots, \boldsymbol{f}_4$ are changed to

$$
\boldsymbol{g}_1 = \frac{\boldsymbol{f}_1}{\cos\phi\cos\theta}, \quad \boldsymbol{g}_2 = \boldsymbol{f}_2 \quad \boldsymbol{g}_3 = \boldsymbol{f}_3, \quad \boldsymbol{g}_4 = \boldsymbol{f}_4 \tag{8}
$$

which can be reduced to

$$
\boldsymbol{g}_1 = \begin{bmatrix} 1 \\ \tan\theta\sin\psi - \tan\phi\frac{\cos\psi}{\cos\theta} \\ \tan\theta\cos\psi + \tan\phi\frac{\sin\psi}{\cos\theta} \\ 0 \\ 0 \\ 0 \end{bmatrix}
\quad
\boldsymbol{g}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}
\quad
\boldsymbol{g}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
\quad
\boldsymbol{g}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

The corresponding distributions are given by

$$
\begin{aligned}
G_0 &= \mathrm{span}\{g_2, g_3, g_4\} \\
G_1 &= \mathrm{span}\{g_2, g_3, g_4, \mathrm{ad}_{g_1} g_2, \mathrm{ad}_{g_1} g_3\}
\end{aligned}
$$

where G_0 and G_1 are involutive, and G_1 has rank 5.

Since $n_2 + n_3 + n_4 + 4 = 6$, if $n_2 = n_3 = 1$ and $n_4 = 0$, then the conditions for determining $h_1 \ldots h_4$ are given by

$$
dh_1 \perp G_0, \quad dL^0_{g_1} h_2 \perp G_0, \quad dL^0_{g_1} h_3 \perp G_0, \quad dh_1 \perp G_1
$$

where note that the scalar h_4 can be selected arbitrarily because of $n_4 = 0$.

The concrete distributions G_0 and G_1 are as follows:

$$G_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos\psi}{\cos^2\phi\cos\theta} & -\frac{\sin\psi}{\cos^2\theta} + \tan\phi\frac{\sin\theta}{\cos^2\theta}\cos\psi \\ 0 & 0 & 0 & \frac{\cos\psi}{\cos^2\phi\cos\theta} & -\frac{\sin\psi}{\cos^2\theta} + \tan\phi\frac{\sin\theta}{\cos^2\theta}\cos\psi \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

In order to satisfy the conditions:

$$dh_1 \perp G_0, \qquad dh_1 \perp G_1 \tag{9}$$

h_1 should be

$$dh_1 = [* \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0] \tag{10}$$

where $*$ denotes any free element that the designer can select it arbitrarily. From this fact, we select h_1 such as

$$h_1 = z \tag{11}$$

The scalars h_2 and h_3 have to satisfy the conditions:

$$dL^0_{g_1}h_2 \perp G_0, \qquad dL^0_{g_1}h_3 \perp G_0 \tag{12}$$

which yields the following candidates in a derivative form:

$$dh_2 = [* \;\; * \;\; * \;\; 0 \;\; 0 \;\; 0], \quad dh_3 = [* \;\; * \;\; * \;\; 0 \;\; 0 \;\; 0] \tag{13}$$

Taking account of the fact that the variable z has been already selected for h_1, it follows that

$$h_2 = y, \qquad h_3 = x \tag{14}$$

Since h_4 can be selected freely as pointed out above, we decide

$$h_4 = \psi \tag{15}$$

From these discussions, it is found that

$$\Phi(\boldsymbol{q}) = \begin{bmatrix} h_1 \\ L^1_{g_1}h_2 \\ h_2 \\ L^1_{g_1}h_3 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} z \\ \tan\theta\sin\psi - \tan\phi\frac{\cos\psi}{\cos\theta} \\ y \\ \tan\theta\cos\psi + \tan\phi\frac{\sin\psi}{\cos\theta} \\ x \\ \psi \end{bmatrix}$$

$$\Xi(\boldsymbol{q}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ L^2_{g_1}h_2 & L_{g_2}L^1_{g_1}h_2 & L_{g_3}L^1_{g_1}h_2 & L_{g_4}L^1_{g_1}h_2 \\ L^2_{g_1}h_3 & L_{g_2}L^1_{g_1}h_3 & L_{g_3}L^1_{g_1}h_3 & L_{g_4}L^1_{g_1}h_3 \\ L^1_{g_1}h_4 & L_{g_2}L^0_{g_1}h_4 & L_{g_3}L^0_{g_1}h_4 & L_{g_4}L^0_{g_1}h_4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{\cos\psi}{\cos^2\phi\cos\theta} & \frac{\sin\psi}{\cos^2\theta} - t\phi\frac{\sin\theta}{\cos^2\theta}\cos\psi & t\phi\frac{\sin\psi}{\cos\theta} + t\theta\cos\psi \\ 0 & \frac{\sin\psi}{\cos^2\phi\cos\theta} & \frac{\cos\psi}{\cos^2\theta} + t\phi\frac{\sin\theta}{\cos^2\theta}\sin\psi & t\phi\frac{\cos\psi}{\cos\theta} - t\theta\sin\psi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $t\alpha$ denotes $\tan\alpha$.

5 Discontinuous Control

The kinematic model of X4-flyer with a chained form transformation is stabilized by using the Astolfi's discontinuous feedback control [5].

The resultant system with a chained form transformation is described by

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \dot\xi_0 \\ \dot\zeta_0 \\ \dot\zeta_1 \\ \dot\eta_0 \\ \dot\eta_1 \\ \dot\gamma_0 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \zeta_0 v_1 \\ v_3 \\ \eta_0 v_1 \\ v_4 \end{bmatrix} \tag{16}$$

In order to make the above system discontinuous, applying a coordinate transformation as a σ process yields

$$y_1 = \xi_0, \qquad y_2 = \zeta_0, \qquad y_3 = \frac{\zeta_1}{\xi_0}$$
$$y_4 = \frac{\eta_0}{\xi_0}, \qquad y_5 = \frac{\eta_1}{\xi_0^2}, \qquad y_6 = \frac{\gamma_0}{\xi_0} \tag{17}$$

When defining $Z_1 = \xi_0$ and $\boldsymbol{Z}_2 = [\zeta_0 \;\; \zeta_1 \;\; \eta_0 \;\; \eta_1 \;\; \gamma_0]^T$ as the coordinates with no transformation, this is equivalent to select that $\sigma = \xi_0^2$ and $\Phi = [\zeta_0\xi_0^2 \;\; \zeta_1\xi_0 \;\; \eta_0\xi_0 \;\; \eta_1 \;\; \gamma_0\xi_0]^T$ in the transformed coordinates $Y_1 = \xi_0$, $\boldsymbol{Y}_2 = \Phi(Z_1, \boldsymbol{Z}_2)/\sigma(Z_1)$. Here, it is satisfied that $\sigma(0) = 0$ and $\Phi(0, \boldsymbol{Z}_2) = [0 \;\; 0 \;\; 0 \;\; \eta_1 \;\; 0]^T \neq 0$.

Differentiating the above new state variables, defining $v_3 = \xi_0\hat{v}_3$ and$v_4 = \xi_0\hat{v}_4$, and rearranging it gives

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ (y_2 - y_3)\frac{1}{y_1} & 0 & 0 & 0 \\ -\frac{y_4}{y_1} & 0 & 1 & 0 \\ (y_4 - 2y_5)\frac{1}{y_1} & 0 & 0 & 0 \\ -\frac{y_6}{y_1} & 0 & 0 & 1 \end{array}\right]\begin{bmatrix} v_1 \\ v_2 \\ \hat{v}_3 \\ \hat{v}_4 \end{bmatrix}$$
$$= \left[\begin{array}{c|c} \boldsymbol{g}_{11} & \boldsymbol{g}_{12} \\ \hline \boldsymbol{g}_{21} & \boldsymbol{g}_{22} \end{array}\right]\boldsymbol{v} \tag{18}$$

Furthermore, defining $Y_1 = y_1$ and $\boldsymbol{Y}_2 = [y_2 \;\; y_3 \;\; y_4 \;\; y_5 \;\; y_6]^T$, and setting $v_1 = -ky_1$ gives

$$\boldsymbol{g}_{21} \times v_1 = -k\begin{bmatrix} 0 \\ y_2 - y_3 \\ -y_4 \\ y_4 - 2y_5 \\ -y_6 \end{bmatrix} \triangleq f(\boldsymbol{Y}_2) \tag{19}$$

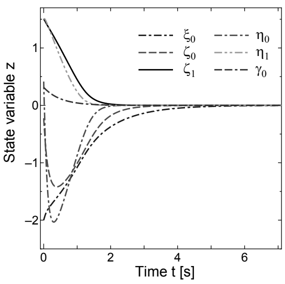

Fig. 2: Controlled state variables in a chained form

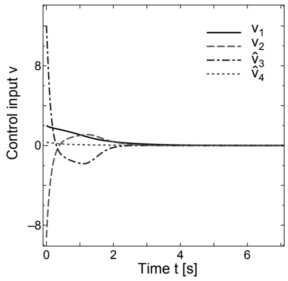

Fig. 3: Discontinuous control inputs

so that

$$\dot{\boldsymbol{Y}}_2 = f(\boldsymbol{Y}_2) + \boldsymbol{g}_{22} \begin{bmatrix} v_2 \\ \hat{v}_3 \\ \hat{v}_4 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -k & k & 0 & 0 & 0 \\ 0 & 0 & k & 0 & 0 \\ 0 & 0 & -k & 2k & 0 \\ 0 & 0 & 0 & 0 & k \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ \hat{v}_3 \\ \hat{v}_4 \end{bmatrix} \tag{20}$$

which is shown to be controllable. Therefore, it is easy to find a continuous function as a linear state feedback that can asymptotically stabilize this system.

6 Simulation

Setting $k = 1$ and assigning the closed-loop poles as $[-1\ \ -2\ \ -3\ \ -4\ \ -5]$ for the σ transformed system, the resultant feedback gain matrix is obtained, so that the input $[v_2\ \ \hat{v}_3\ \ \hat{v}_4]^T$ can be described by

$$\begin{bmatrix} v_2 \\ \hat{v}_3 \\ \hat{v}_4 \end{bmatrix} = -\begin{bmatrix} 8 & -18 & 1 & -6 & 0 \\ 1 & -3 & 10 & -32 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} \tag{21}$$

The simulation results are shown in Fig. 2 to Fig. 5, where the initial state vector was set to $\boldsymbol{q}_0 = [1.5\ \ 1.5\ \ -2.0\ \ \pi/10\ \ \pi/10\ \ \pi/10]^T$ and the desired value was $\boldsymbol{q}_r = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$.

Fig. 2 shows the time-responses of the state variables for the system with the chained form transformation, whereas Fig. 3 denotes the control inputs for such a chained form system. It is found from Figs. 4 and 5 that the latitude, horizontal position, and all attitude angles converged to the desired values in a shot time by applying the proposed discontinuous control method.

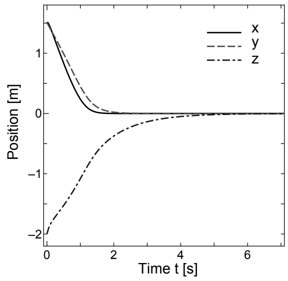

Fig. 4: Position control

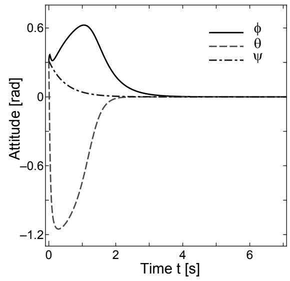

Fig. 5: Attitude control

7 Conclusion

An underactuated control method has been considered for a VTOL aerial robot with four rotors, where a kinematic model was used to rely on a discontinuous control approach. To assure a globally asymptotic stability for the kinematic model based control system (or a global controllability for the controlled objective), the canonical form of a chained form, consisting of a kinematic model with one generator-three chains, was obtained. Then, the Astolfi's discontinuous control approach was applied for such a canonical form to realize an underactuated control method that controls six states by four rotor inputs. The effectiveness of the method was proved through a simulation.

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