The Thirteenth International Symposium on Artificial Life and Robotics 2008(AROB 13th '08),
B-Con Plaza, Beppu, Oita, Japan, January 31-February 2, 2008

Trial-to-trial Variability and Its Influence on Higher Order Statistics

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Abstract

Firing patterns of neurons are highly variable from trial to trial, even we record a well-specified neuron exposed to the identical stimulus under same experimental condition. Trial-to-trial variability of spike trains may represent some sort of information and give suggestions about neuronal properties.

We propose a new method for quantifying trial-to-trial variability of spike trains and investigate how characteristics of noisy neural network models affect our proposed measure.

1 Introduction

Firing patterns of neurons are highly variable from trial to trial, even when we record a well-specified neuron exposed to the identical stimulus under same experimental condition ([1][2]). What does the variability between trials mean?

There is a fundamental question about the variability between trials: is it consistent between the trial firing rate of a certain neuron and the population firing rate of a certain trial? This is the question that is pointed out and went into by Masuda and Aihara[3]. They defined the ergodicity of the spike trains as follows: the equivalence between the trial firing rate and the population firing rate[3].

Needless to say, firing rate is a first order statistics. It is important for information coding problem to extend this concept to the higher order statistics[4][5]. In this section, we extend the first order statistics discussed in [3] to the higher order statistics. We quantify the ergodicity, and examine the relationship between the ergodicity and the properties of neuron and the inputs.

2 Statistics of the variability along time and between trial

We characterize the neuronal firing patterns by employing the measure of irregularity C_V[6]. C_V is computed as the standard deviation divided by the mean. C_V take 1 for the purely Poisson process, and 0 for perfectly periodic sequences. C_V indicates the global spiking irregularity.

Generally speaking, C_V is the measure of firing irregularity *along time*. In this study, we use this C_V not only for the conventional meaning, but also for the irregularity *between trials*. Irregularity between trials means how the spike pattern obtained from a certain neuron fluctuates with each trial. For instance, if the spike patterns are the same for each trial, *i.e.* synchronizing among trials, the quantity of the statistics would be small. In general, synchrony indicates the synchrony between neurons, however, trial synchrony is also widely studied[7].

Even if it is not strictly synchronizing between trials, similar spike patterns make the irregularity measure small. Small irregularity between trials means high reproducibility in other words.

Irregularity between trials is measured by the following procedure. First, set all the trials to time, and divide them by the time bin. Second, connect them over trials and produce a new set of ISIs. In this method, all the bins including the ones that have no ISIs would be measured. Finally, measure the statistics for a new set of ISIs.

By using the statistics of irregularity between trials and conventional statistics along time, we will discuss their behaviors in the following sections. For discriminating them, we will denote the irregularity along time as $C_{V\,time}$, irregularity between trials as $C_{V\,trial}$.

In addition, the value of statistics in this study is very small, but this is because we are considering the case of periodic inputs and small noise. It becomes regular along time since the inputs are periodic, and also becomes regular between trials since the spike patterns are almost nearly synchrony.

3 Statistics along time and across trials in LIF model

We use the simple model, leaky integrate-and-fire(LIF) model[8]. This model is the model that treats firing events as a point process[9], which is shown below in case.

$$\mathrm{d}V(t) = (\quad V(t) + I_{ext}(t) + I_{syn}(t))\mathrm{d}t + D\mathrm{d}B_t, \quad (1)$$

where $V(t)$ denotes the membrane potential, denotes the time constant of membrane, $I_{ext}(t)$ denotes the external inputs, $I_{syn}(t)$ denotes the synaptic input from other neurons, D is the intensity of the noise, B_t represents Brownian motion[6]. Noise term $D\mathrm{d}B_t$ contains the background activity which is independent of firing events. If membrane potential $V(t)$ reaches the threshold, membrane potential would be reset to the resting potential.

3.1 Variability of refractory period

In general case, neuron does not fire right after the spike event, where the term is called absolute refractory period. Here, we examine how the length of the refractory period affects the firing statistics. External input $I_{ext}(t)$ is added as below.

$$\mathrm{d}V(t) = (\quad V(t) + I_{ext}(t) + I_{syn}(t))\mathrm{d}t + D\mathrm{d}B_t, \quad (2)$$

$$I_{ext}(t) = (1 + \frac{_{ref}}{80})(4.9 \quad 10^{-2} + 3.0 \quad 10^{-3}\sin(\frac{2\pi t}{T})). \quad (3)$$

$_{ref}$ denotes the length of the refractory period, T is the period of the external input. Neuron cannot fire in the term of $_{ref}$. To get rid of the effect of firing rate difference, the term of $_{ref}$ is added to $I_{ext}(t)$ to keep the firing rate constant.

Figure1 represents the firing statistics of neuron that receives the sinusoidal input affected by the length of the absolute refractory period.

As in figure 1, $C_{V\,time}$, which denotes the firing irregularity along time decreases as the refractory period increases, which means the firing pattern along time is becoming more regularly. While, $C_{V\,trial}$, which is the firing statistics across the trials is represented as in figure 2.

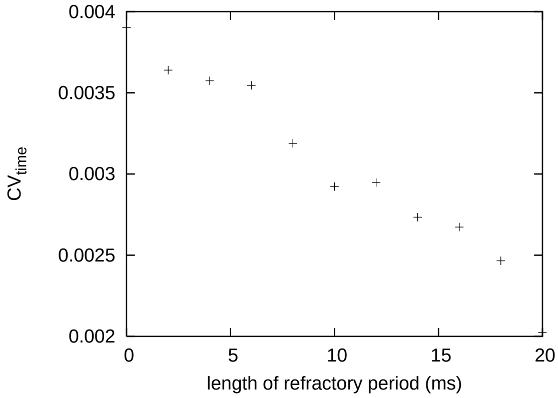

Figure 1: Irregularity of spike trains along time in the case of variable refractory period

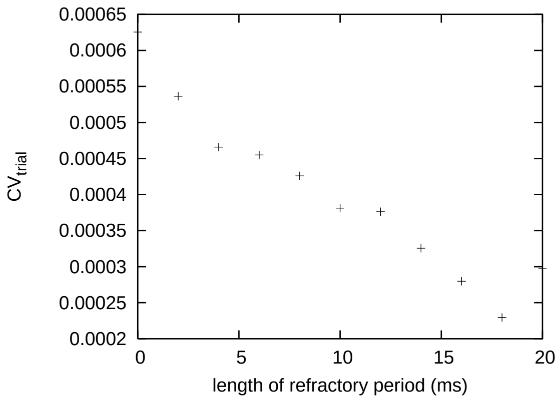

Figure 2: Irregularity of spike trains across trials in the case of variable refractory period

As in figure 2, $C_{V\,trial}$, which denotes the firing irregularity along time also decreases as the refractory period increases, which means the firing pattern across the trials is becoming more regularly. Regular firing pattern across trials indicates the firing is more reproducible as the length of refractory period increases. These results correspond to the former theoretical results[10].

3.2 Variability of input intensity

Next, we consider the effect of input to each statistics. We add cosine wave to LIF model, and consider the relationship between input intensity and the statistics. Model is represented as below.

$$\mathrm{d}V(t) = (\quad V(t) + I_{ext}(t) + I_{syn}(t))\mathrm{d}t + D\mathrm{d}B_t, \quad (4)$$

$$I_{ext}(t) = \overline{I_{ext}(t)} + amp \quad \cos(\frac{2\pi t}{T}), \qquad (5)$$

$$I_{syn}(t) = 0, \qquad (6)$$

where $\overline{I_{ext}(t)}$ denotes the temporal average of $I_{ext}(t)$, amp is the intensity of the input. We neglect the synaptic input by setting $I_{syn}(t) = 0$, since our aim is to consider the effect of external inputs to the firing statistics.

Figure 3 and 4 represent the behavior of statistics along time and across trials, in the case of variable input intensity amp.

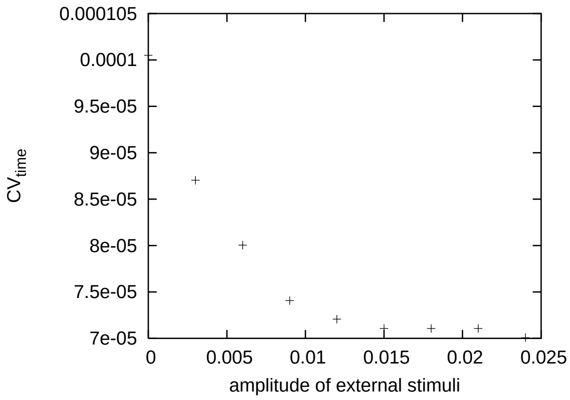

Figure 3: Irregularity of spike trains along time in the case of variable input intensity

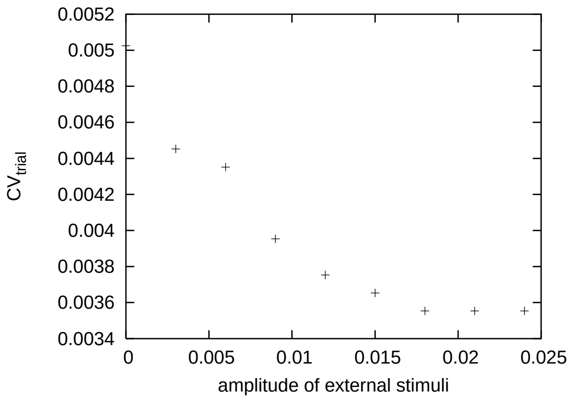

Figure 4: Irregularity of spike trains across trials in the case of input intensity

It is evident from figure 3 and 4 that both of the statistics, along time and across trials, decrease as the input intensity increases. This means that firing pattern is more regular and reproducible as the input intensity increases.

4 Firing statistics in neuronal population

In the former section, we considered the effects of refractory period and the input intensity to the firing statistics. Those discussions are based on the neuronal and input properties of single neuron, and quantified the ergodicty that is discussed in [3] by introducing the statistics across trials.

In this section, we consider the statistics in the case of multiple neurons, multiple trials. Concretely speaking, we consider the behaviors of firing irregularity across trials of single neuron, and firing irregularity between neurons by connecting multiple LIF models.

Firing irregularity across trials is measured as in the former section. The procedure of measuring the firing irregularity between neurons is depicted as follows.

First, set all the neuronal firing patterns of single trial and divide them by the time bin. Second, connect them over different neurons and produce a new set of ISIs. Finally, measure the statistics for a new set of ISIs. To avoid producing nonexistent ISIs by connecting the edges, the ISIs that lie on edges of a bin is excluded. In this method, all the bins including the ones that have no ISIs would be excluded. We denote the irregularity between other neurons as $C_{V\,pop}$. $C_{V\,pop}$ can be interpreted as the statistics which represents the synchrony and the correlations between neurons.

To consider the firing irregularity between neurons, we construct a simple model depicted as follows.

First, connect 100 neurons randomly. External input $I_{ext}(t)$ is added common to all the neurons. If the neuron fires, 30 postsynaptic neurons are randomly selected, and synaptic inputs are provided to them by input intensity ϵ.

Concretely, the model is depicted as follows.

$$\mathrm{d}V(t) = (\quad V(t) + I_{ext}(t) + I_{syn}(t))\mathrm{d}t \qquad (7)$$

$$+D\mathrm{d}B_t, \qquad (8)$$

$$I_{ext}(t) = A(4.9 \quad 10^{-2} + 5.6 \quad 10^{-3} \sin(\frac{2\pi t}{T}), \qquad (9)$$

$$|I_{syn}(t)| = \epsilon. \qquad (10)$$

Here, A represents the intensity of external input, ϵ represents the intensity of synaptic input. Intensity balance of A and ϵ determines the superiority occupied in the inputs.

Figure 5 shows the irregularity between neurons and across trials when the balance of external inputs and synaptic inputs varies.

When the external input is superior, the value of $C_{V\,pop}$ is high, while the value of $C_{V\,trial}$ is high when

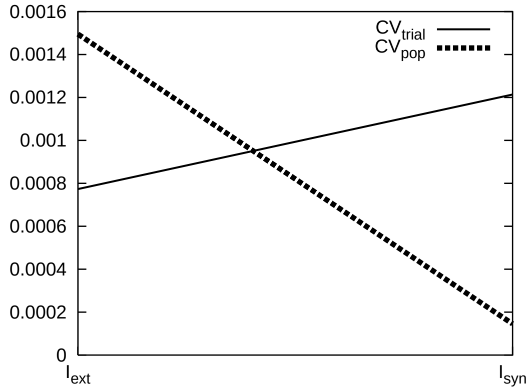

Figure 5: The irregularity between neurons and across trials when the balance of external inputs and synaptic inputs varies.

the synaptic input is superior. This means that irregularity between neuron increases when external input is superior, which indicates that the firing patterns gets more different between neurons. Conversely, when synaptic input is superior, irregularity across trials increases, which indicates the less reproducible firing. These results show that synaptic inputs induce the regular firing of neuronal population, while external inputs induce the regular firing across trials.

5 Discussion

We introduced the new statistics of irregularity along time, across trials, and between neurons, to quantify the ergodicty of spike trains that is discussed in [3]. For the spike trains obtained from single neuron, the following results are confirmed. 1. More regular and reproducible spike trains are obtained as the refractory period lengthens. 2. More regular and reproducible spike trains are obtained as the input intensity increases. The statistics along time and across trials both behave similarly for the modulation of refractory period and input intensity.

In the case of spike trains observed from multiple neurons, regular firing in neuronal population is obtained when synaptic inputs are superior, while reproducible firing is obtained when external inputs are superior. The statistics across trials and between neurons exhibited contrary behaviors for the input properties. This result might be useful to estimate the property of inputs.

Generally, variability of firing patterns for each trial is neglected, and statistics of ISIs are often averaged over trials. In the statistical analysis, statistics along time has only been treated. By introducing the statistics across trials and between neurons as in this study, it may be possible to extract the information that has not been considered. Therefore, it is also important from the viewpoint of information coding [4][5].

It is a future issue to apply these statistics for physiological data and decode from the data and estimate the properties of neuron and inputs.

References

[1] W. R. Softky and C. Koch. The highly irregular firing of cortical cells is inconsistent with temporal integration of random epsps. *J. Neurosci.*, 13:334–350, 1993.

[2] M. N. Shadlen and W. T. Newsome. The variable discharge of cortical neurons: Implications for connectivity, computation, and information coding. *J. Neurosci.*, 18:3870–3896, 1998.

[3] N. Masuda and K. Aihara. Ergodicity of spike trains: When does trial averaging make sense? *Neural Computation*, 15:1341–1372, 2003.

[4] E. Salinas and T. J. Sejnowski. Correlated neuronal activity and the flow of neural information. *Nature Reviews Neuroscience*, 2:539–551, 2001.

[5] A. Benucci, P. F. Verschure, and P. König. Existence of high-order correlations in cortical activity. *Phys. Rev. E*, 68(4):041905, Oct 2003.

[6] H. C. Tuckwell. *Introduction to Theoretical Neurobiology*, volume 2. Cambridge University Press, 1988.

[7] J. D. Hunter, J. G. Milton, P. J. Thomas, and J. D. Cowan. resonance effect for neural spike time reliability. *J. Neurophysiol.*, 80:1427–1438, 1998.

[8] K. Fujiwara, H. Fujiwara, M. Tsukada, and K. Aihara. Reproducing bursting interspike interval statistics of the gustatory cortex. *Biosystems*, 90:442–448, 2007.

[9] W. Gerstner and W. M. Kistler. *Spiking Neuron Models*. Cambridge University Press, 2002.

[10] D. Hansel, G. Mato, and C. Meunier. synchrony in excitatory neural networks. *Neural Computation*, 7:307–337, 1995.