# Digital Control of Space Robot Manipulators with Velocity Type Joint Controller Using Transpose of Generalized Jacobian Matrix 

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#### Abstract

For free floating space robots having manipulators, we have proposed a discrete-time tracking control method using the transpose of Generalized Jacobian Matrix (GJM). Control inputs of the control method are joint torques of the manipulator. In this paper, the control method is augmented for angular velocity inputs of the joints. Computer simulations have shown the effectiveness of the augmented method.


## 1 Introduction

For space robots having manipulators many control methods of space robot manipulators have been proposed [1]. Most of them, however, use the inverse of Generalized Jacobian Matrix (GJM) which is a coefficient matrix between the end-effector's velocity and the joint velocity of the manipulator. Therefore, if the robot system becomes in a singular configuration, the manipulator is out of control because the inverse of GJM does not exist.

We have proposed discrete time control methods using the transpose of GJM $[2,3]$. The control methods using the transpose of GJM use position and orientation errors between the desired and actual values of the end-tip of the manipulator. Namely, the control methods belong to a class of constant value control such as PID control. Therefore, the value of errors depends on the desired linear and angular velocity of the end-tip based on the desired trajectory.

To obtain higher control performance we have proposed a digital trajectory tracking control method that has variable feedback gains depending on the desired linear and angular velocity [5]. Furthermore, the control method can be applied for cooperative manipulations of a floating object by some space robots [6].

The tracking control method described above can be utilized for manipulators with joint torque controller. It is considered that joint velocity controllers are also used for space robot manipulators. So, we have been proposed a control method using the transpose of GJM for joint velocity controller without trajectory tracking. In this paper, we propose a tracking control method for joint velocity controller. Simulation results show the effectiveness of the control method.

## 2 Tracking Control (Torque input)

Our proposed control method [5] has been designed for a free-floating space robot manipulator as shown in Fig. 1 [4]. It has an uncontrolled base and $n$-DOF manipulator with revolute joints. The target of the end-effector of the manipulator is stationary in an in-


Fig. 1 Space robot model
ertial coordinate frame. Symbols used in this paper are defined as follows:
$\Sigma_{I}$ : inertial coordinate frame
$\Sigma_{B}$ : base coordinate frame
$\Sigma_{E}$ : end-effector coordinate frame
$\Sigma_{T}$ : target coordinate frame
$\boldsymbol{r}_{E}$ : position vector of $\Sigma_{E}$
$\boldsymbol{r}_{T}$ : position vector of $\Sigma_{T}$
$\boldsymbol{v}_{E}$ : linear velocity vector of $\Sigma_{E}$
$\boldsymbol{\omega}_{E}$ : angular velocity vector of $\Sigma_{E}$
$\boldsymbol{q}$ : joint angle vector
$\phi_{*}$ : angle vector representing the orientation of $\Sigma_{*}$
${ }^{I} \boldsymbol{A}_{*}$ : rotation matrix from $\Sigma_{*}$ to $\Sigma_{I}$
$\boldsymbol{E}$ : identity matrix
The tilde operator stands for a cross product such that $\tilde{\boldsymbol{r}} \boldsymbol{a}=\boldsymbol{r} \times \boldsymbol{a}$. All position and velocity vectors are defined with respect to the inertial reference frame.

A discrete-time differential kinematic model of Fig. 1 is given by the following equation $[2,3]$ :

$$
\left[\begin{array}{l}
\boldsymbol{v}_{E}(k)  \tag{1}\\
\boldsymbol{\omega}_{E}(k)
\end{array}\right]=\left[\begin{array}{l}
\boldsymbol{J}_{L}\left(\boldsymbol{\phi}_{B}(k), \boldsymbol{q}(k)\right) \\
\boldsymbol{J}_{A}\left(\boldsymbol{\phi}_{B}(k), \boldsymbol{q}(k)\right)
\end{array}\right] \dot{\boldsymbol{q}}(k)
$$

where $\boldsymbol{J}_{L}$ and $\boldsymbol{J}_{A}$ are called the GJMs of the linear and angular velocities, respectively.

For free-floating space robot manipulators we have proposed the following control law using the transpose of the GJM [5]:

$$
\begin{align*}
\boldsymbol{\tau}_{d}(k) & =\boldsymbol{J}_{L}^{T}(k)\left[\hat{k}_{p}(k) \boldsymbol{e}_{P I}(k)-\hat{\boldsymbol{K}}_{L V}(k) \boldsymbol{v}_{E}(k)\right] \\
& +\boldsymbol{J}_{A}^{T}(k)\left[\hat{k}_{o}(k) \boldsymbol{e}_{O I}(k)-\hat{\boldsymbol{K}}_{A V}(k) \boldsymbol{\omega}_{E}(k)\right] \tag{2}
\end{align*}
$$

where $\boldsymbol{\tau}_{d}(k)$ is the joint torque input vector and

$$
\begin{gathered}
\boldsymbol{e}_{P I}(k)=\boldsymbol{p}_{T}(k)-\boldsymbol{p}_{E}(k), \\
\boldsymbol{e}_{O I}(k)=-\frac{1}{2} \boldsymbol{E}_{X}^{T}(k) \boldsymbol{E}_{O I}(k), \\
\boldsymbol{E}_{O I}(k)=\left[\begin{array}{c}
\boldsymbol{n}_{T}(k)-\boldsymbol{n}_{E}(k) \\
\boldsymbol{s}_{T}(k)-\boldsymbol{s}_{E}(k) \\
\boldsymbol{a}_{T}(k)-\boldsymbol{a}_{E}(k)
\end{array}\right], \quad \boldsymbol{E}_{X}(k)=\left[\begin{array}{c}
\tilde{\boldsymbol{n}}_{E}(k) \\
\tilde{\boldsymbol{s}}_{E}(k) \\
\tilde{\boldsymbol{a}}_{E}(k)
\end{array}\right], \\
\hat{k}_{p}(k)=k_{p}\left\{1+\alpha_{L} \nu_{L}(k)\right\}, \quad \hat{k}_{o}(k)=k_{o}\left\{1+\alpha_{A} \nu_{A}(k)\right\}, \\
\hat{\boldsymbol{K}}_{L V}(k)=\boldsymbol{K}_{L V}\left\{1-\beta_{L} \nu_{L}(k)\right\}, \\
\hat{\boldsymbol{K}}_{A V}(k)=\boldsymbol{K}_{A V}\left\{1-\beta_{A} \nu_{A}(k)\right\}, \\
\nu_{L}(k)=\frac{\left\|\boldsymbol{v}_{E_{d}}(k)\right\|}{v_{d_{\max }}}, \quad \nu_{A}(k)=\frac{\left\|\boldsymbol{\omega}_{E_{d}}(k)\right\|}{\omega_{d_{\max }}} .
\end{gathered}
$$

The vectors $\boldsymbol{n}_{*}, \boldsymbol{s}_{*}$ and $\boldsymbol{a}_{*}(*=T, E)$ are unit vectors along the axes of $\Sigma_{*}$ with respect to $\Sigma_{I}$, i. e.,


Fig. 2 3-link space robot

Table 1 Physical parameters

|  | Length <br> m | Mass <br> kg | Moment of inertia <br> $\mathrm{kg} \cdot \mathrm{m}^{2}$ |
| :---: | :---: | ---: | :---: |
| Base | 3.5 | 2000 | 3587.9 |
| Link 1 | 2.5 | 50 | 26.2 |
| Link 2 | 2.5 | 50 | 26.2 |
| Link 3 | 0.5 | 5 | 0.23 |
| Object | 4.0 | 100 | 200.0 |

${ }^{I} \boldsymbol{A}_{*}=\left[\boldsymbol{n}_{*}(k) \boldsymbol{s}_{*}(k) \boldsymbol{a}_{*}(k)\right] . \boldsymbol{v}_{E_{d}}(k)$ and $\boldsymbol{\omega}_{E_{d}}(k)$ are the desired velocities of $\boldsymbol{v}_{E}(k)$ and $\boldsymbol{\omega}_{E}(k), v_{d_{\max }}$ and $\omega_{d_{\max }}$ are the maximum values of the norm of $\boldsymbol{v}_{E_{d}}(k)$ and $\boldsymbol{\omega}_{E_{d}}(k), \alpha_{\dagger}\left(\alpha_{\dagger} \geq 0\right)$ and $\beta_{\dagger}\left(0 \leq \beta_{\dagger} \leq 1\right)(\dagger=L, A)$ are setting parameters. Furthermore, $k_{p}$ and $k_{o}$ are positive scalar gains for position and orientation, and $\boldsymbol{K}_{L V}$ and $\boldsymbol{K}_{A V}$ are symmetric and positive definite gain matrices for linear and angular velocities of the end-tip of the manipulator.

For a horizontal planar 3-DOF robot shown in Fig. 2 and an object, computer simulation using Eq. (2) has been done [5]. The simulation condition is follows. Physical parameters of the robot and object are shown in Table 1. A point of interest of the object moves along a straight path from the initial position to the target position and the object angle is set up to the initial value. The sampling period is $T=0.01 \mathrm{~s}$ and setting parameters of the control law are $k_{p}=k_{o}=50000, \boldsymbol{K}_{L V}=\operatorname{diag}\{5000,5000\}$, $\boldsymbol{K}_{A V}=5000, \alpha_{A}=\alpha_{L}=0.8$ and $\beta_{A}=\beta_{L}=0.3$.

Fig. 3 and 4 show the simulation result. And Fig. 5 shows the relation between the actual joint input torque and joint angular velocity. From Fig. 5 we can see that the value of angular velocity is varying with the constant torque input during the sampling interval $T$. In other words, if the control inputs vary roughly for manipulators with joint velocity controllers the joint controllers give large torques to the robot.


Fig. 3 Motion of the robot (Torque input)


Fig. 4 Simulation result (Torque input)


Fig. 5: Relation between torque and velocity (joint 2)

## 3 Tracking Control (Velocity input)

For manipulators with joint velocity controllers the control law (2) cannot be applied directly. To obtain similar control performance to the case of the joint torque controllers, we use the dynamic equation of the robot.

Equation of motion of the space robot shown in Fig. 1 can be described as follows [1]:

$$
\begin{equation*}
\boldsymbol{H}(\boldsymbol{q}) \ddot{\boldsymbol{q}}(t)+\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})=\boldsymbol{\tau}(t) \tag{3}
\end{equation*}
$$

where $\boldsymbol{H}$ is the symmetric and positive definite inertia matrix, and $\boldsymbol{C}$ is the vector of Coliolis and centrifugal forces.

Discretizing Eq. (3) by the sampling period $T_{1}(T=$ $n T_{1}, n$ is positive integer) and applying the forward Euler approximation to $\ddot{\boldsymbol{q}}\left(k_{1}\right)$, we have

$$
\begin{equation*}
\dot{\boldsymbol{q}}\left(k_{1}\right)=\dot{\boldsymbol{q}}\left(k_{1}-1\right)-T_{1} \boldsymbol{H}^{-1}\left(k_{1}\right)\left\{\boldsymbol{C}\left(k_{1}\right)-\boldsymbol{\tau}\left(k_{1}\right)\right\} . \tag{4}
\end{equation*}
$$

Here, we assume that $\boldsymbol{q}$ is constant during the sampling interval $T$. Then for Eq. (4) the actual joint velocity control input $\dot{\boldsymbol{q}}_{d}\left(k_{1}\right)$ is determined as

$$
\begin{equation*}
\dot{\boldsymbol{q}}_{d}\left(k_{1}\right)=\dot{\boldsymbol{q}}\left(k_{1}-1\right)-T_{1} \boldsymbol{H}^{-1}(k)\left\{\boldsymbol{C}\left(k_{1}\right)-\boldsymbol{\tau}_{d}(k)\right\} . \tag{5}
\end{equation*}
$$

To verify the validity of the proposed control law (2), with Eq. (5) simulation is performed. The condition is same to the torque input case and the sampling period for Eq. (5) is $T_{1}=0.001 \mathrm{~s}(n=10)$.

The simulation result is shown in Fig. 6 and 7. Furthermore, Fig. 8 shows the difference of the joint angular velocity between the case of torque input control and velocity input control. From these figures, the both control performances are similar and good control performance can be achieved using the control law (2) with Eq. (5).


Fig. 6 Motion of the robot (Velocity input)


Fig. 7 Simulation result (Velocity input)


Fig. 8 Velocity difference between Fig. 4 and 7

## 4 Conclusion

In this paper, a digital tracking control method for space robot manipulators with joint velocity controller was proposed. The simulation result showed the effectiveness of the proposed method.

## References

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