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A Time-Scaling Method for Near-Time-Optimal Control of an Omni-Directional Robot along Specified Paths

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Abstract: This paper proposes a time-scaling method to determine the near-time-optimal movement of an omni-directional mobile robot along a given reference path. With this strategy, the positions of the trajectory after scaling are the same as the original ones such that the geometric path constraints are not violated. However, the velocities and the accelerations are adjusted to meet the dynamical constraints and to minimize the traveling time. When determining the time-scaling function, a cubic spline interpolation technique is used, in which control points for interpolation are determined simultaneously by a particle swarm optimization (PSO) method based on the integration of a time-scaling function. To show the feasibility of the proposed method, the results of a simulation example is illustrated.

Keywords: Omni-directional robot, Time-scaling, Particle swarm optimization.

I. INTRODUCTION

When applying time-scaling techniques to determine the time-optimal movement of a plant along a specified path, the corresponding scenario is usually as follows. In the beginning, a reference trajectory is synthesized to meet geometric path constraints. Then a new trajectory with a time-scaling relation to the reference trajectory is constructed to satisfy dynamical constraints. The key strategy of the time-scaling procedure is to adjust the velocities and the accelerations of the new trajectory to meet dynamical constraints and to minimize the traveling time while not violating the geometric path constraints.

In the past few years, the time-optimal control problem of omni-directional mobile robots has attracted the attention of several researchers. Chung and Wu [1], Chevallereau [2], Balkcom et al [3], and Kavathekar et al [4], researched control strategies to move a robot from a given initial state to a desired final state while minimizing the maneuver time and/or control effort. Cho et al [5] adopted a robotic manipulator to move a fragile object from an initial point to a specific location in the minimum time without damage. However, in these problems, it should be noted that only the initial and the final states are specified and there are no geometric and/or dynamical constraints imposed on the path. This in turn motivates the research of this paper.

In this paper, a time-scaling technique will be applied to determine the control signals to drive an omni-directional mobile robot to track a specified path in a near-time-optimal manner. When constructing the time-scaling function, the whole time interval of the reference trajectory is divided into many segments, in which the time-scaling function is interpolated as a cubic spline curve according to the chosen control points. However, it should be noted that there are no ways to determine these control points simultaneously in the previous research. Therefore, a PSO method is adopted to overcome this difficulty.

II. DYNAMICS OF AN MNI-DIRECTIONAL MOBILE ROBOT

In this section, it is assumed that the omni-directional mobile robot consists of the orthogonal-wheel assembly mechanism proposed by Pin and Killough [6]. A schematic diagram to illustrate the motion of the omni-directional robot is given in Fig. 1, and a world- frame $[x_w, y_w]^T$ and a moving-frame $[x_m, y_m]^T$ are defined as shown in Fig. 2. The former denotes a frame that everything discussed can be referenced and the latter is a frame attached to the center of the gravity of the robot. From the illustration in Fig. 2, one can find that the transformation between these two frames is described by

$$\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} \quad (1)$$

where ϕ is the angle between these two frames.

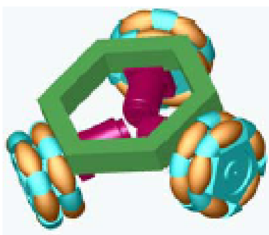

Fig. 1. A schematic diagram of the omni-directional robot.

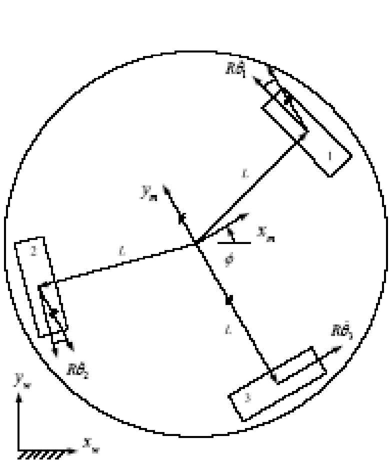

Fig. 2. Definitions of the world-frame $[x_w, y_w]^T$ and the moving- frame $[x_m, y_m]^T$ on an omni-directional robot.

With the transformation in (1) and the Newton's Second Law of Motion, one can derive the dynamical equations of the omni-directional robot as follows:

$$\frac{d}{dt}\begin{bmatrix}\dot{x}_w\\ \dot{y}_w\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix}a_1 & -a_4\dot{\phi} & 0\\ a_4\dot{\phi} & a_1 & 0\\ 0 & 0 & a_3\end{bmatrix}\begin{bmatrix}\dot{x}_w\\ \dot{y}_w\\ \dot{\phi}\end{bmatrix} + T\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} \quad (2)$$

where $a_1 = -3c/(3I_\omega + 2MR^2)$,

$a_3 = -3cL^2/(3I_\omega L^2 + I_v R^2)$, $a_4 = 3I_\omega/(3I_\omega + 2MR^2)$,

$$T = \begin{bmatrix}-b_1(\sqrt{3}\sin\phi + \cos\phi) & b_1(\sqrt{3}\sin\phi - \cos\phi) & 2b_1\cos\phi\\ b_1(\sqrt{3}\cos\phi - \sin\phi) & -b_1(\sqrt{3}\cos\phi + \sin\phi) & 2b_1\sin\phi\\ b_2 & b_2 & b_2\end{bmatrix},$$

$b_1 = k_a R/(3I_\omega + 2MR^2)$, $b_2 = k_a RL/(3I_\omega L^2 + I_v R^2)$.

M is the mass of the robot, L is the distance between any wheel and the center of gravity of the robot, R is the radius of the wheel, c is the viscous friction factor of the wheel, I_ω is the moment of inertia of the wheel around the driving shaft, I_v is the moment of inertia of the robot, k_a is the driving gain factor, and u_i is the driving inputtorque.

Suppose that the trajectory of an omni-directional mobile robot is given as

$$\beta_r(t) = [x_w(t) \quad y_w(t) \quad \phi(t)]^T, \ t \in [0, t_f] \quad (3)$$

which specifies a pre-defined geometric path and will be called a reference trajectory hereafter. For this reference trajectory, define a new trajectory as

$$\beta_s(\tau) = \beta_r(t) \quad (4)$$

where the new time variable τ is defined as

$$\tau = \int_0^t s(t)dt \quad (5)$$

and $s(t)$ is a time-scaling function to be determined for $t \in [0, t_f]$.

In order to physically implement the new trajectory $\beta_s(\tau)$, it is obvious that the value of $s(t)$ must be positive. With the condition of $s(t) > 0$, τ becomes a monotonically increasing function of t. Therefore, there is one-to- one correspondence between τ and t such that $\beta_s(\tau)$ is a time-scaled function of $\beta_r(t)$.

Taking time derivatives with respect to τ on both sides of (4), one obtains

$$\dot{\beta}_s(\tau) = \frac{d\beta_s(\tau)}{d\tau} = \frac{d\beta_r(t)}{d\tau} = \frac{\dot{\beta}_r(t)}{s(t)} \quad (6)$$

Similarly, taking time derivatives with respect to τ on both sides of (6) yields

$$\ddot{\beta}_s(\tau) = \frac{d}{d\tau}\dot{\beta}_s(\tau) = \frac{1}{s^2(t)}\left[\ddot{\beta}_r(t) - \frac{\dot{s}(t)}{s(t)}\dot{\beta}_r(t)\right] \quad (7)$$

From (4), (6), and (7), one can find that $\beta_s(\tau)$ traces the same geometric path as $\beta_r(t)$, but with different velocity and acceleration. Then from (2), (6), and (7), the dynamics of the new trajectory $\beta_s(\tau)$ is given as

$$\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} = T^{-1}\left(\frac{1}{s^2(t)}\begin{bmatrix}\ddot{x}_w\\ \ddot{y}_w\\ \ddot{\phi}\end{bmatrix} - \frac{\dot{s}(t)}{s^3(t)}\begin{bmatrix}\dot{x}_w\\ \dot{y}_w\\ \dot{\phi}\end{bmatrix} - \frac{1}{s(t)}\begin{bmatrix}a_1 & -a_4\dot{\phi} & 0\\ a_4\dot{\phi} & a_1 & 0\\ 0 & 0 & a_3\end{bmatrix}\begin{bmatrix}\dot{x}_w\\ \dot{y}_w\\ \dot{\phi}\end{bmatrix}\right) \quad (8)$$

Letting t_f and τ_f denote the traveling times of the reference and the new trajectories, respectively, the near-time-optimal control problem of an omni-directional mobile robot along a specified path can be formulated as follows:

***Problem A*:**
Given the reference trajectory $\beta_r(t)$ and the new trajectory $\beta_s(\tau)$ described in (3) through (5), find a time-scaling function $s(t)$ to minimize

$$\tau_f = \int_0^{t_f} s(t)dt \quad (9)$$

subject to

$$s(t) > 0 \ \text{for} \ t \in [0, t_f] \quad (10)$$

and

$$[u_{1,min} \quad u_{2,min} \quad u_{3,min}]^T \le \mathbf{u}(\tau) \le [u_{1,max} \quad u_{2,max} \quad u_{3,max}]^T \quad \text{for } \tau \in [0, \tau_f] \quad (11)$$

where $\mathbf{u}(\tau)$ is defined in (8).

The above formulations (4), (6), and (7) indicate that the new trajectory traces the same geometric path as the reference trajectory. Meanwhile, under the constraints on the control torques, its velocity and acceleration will be adjusted by the time-scaling function to make the traveling time τ_f as small as possible.

III. FORMULATION OF TIME-SCALING FUNCTIONS

Formulation of the time-scaling function will be described in this section. The first step is to divide the traveling time t_f of the reference trajectory $\beta_r(t)$ into N equal sub-intervals. This means that

$$t_{i+1} - t_i = \frac{t_f}{N} = \Delta t \quad \text{for } i = 0, 1, \cdots, N-1 \tag{12}$$

Then, for each sub-interval, it is assumed that the time-scaling function is interpolated by a cubic spline curve as follows:

$$s(t) = s_i(\lambda_i) = a_i\lambda_i^3 + b_i\lambda_i^2 + c_i\lambda_i + d_i \tag{13}$$

where a_i, b_i, c_i, and d_i are coefficients to be detemined and $\lambda_i = t - t_i / \Delta t$ for $i = 0, 1, \cdots, N-1$.

To determine the time-scaling function in (13) uniquely, one can assume that a cubic polynomial function $s(t)$ is continuously differentiable and has a continuous second derivative for $0 \le t \le t_f$ (Burden and Faires [7]). This means that the following boundary conditions should be satisfied.

$$s_i(1) = s_{i+1}(0) \quad \text{for } i = 0, 1, \cdots, N-2 \tag{14}$$

$$\dot{s}_i(1) = \dot{s}_{i+1}(0) \quad \text{for } i = 0, 1, \cdots, N-2 \tag{15}$$

$$\ddot{s}_i(1) = \ddot{s}_{i+1}(0) \quad \text{for } i = 0, 1, \cdots, N-2 \tag{16}$$

Meanwhile, one also can assume that the initial and final velocities of the new trajectory are the same as those in the reference trajectory. To meet these requirements, from (6), one can find that the following two boundary conditions are also required.

$$s_0(0) = 1,\ s_{N-1}(1) = 1 \tag{17}$$

Similarly, in order to make the initial and final values of $\ddot{\beta}_z(\tau)$ be equal to those of $\ddot{\beta}_r(t)$, from (7) and (17), one can find that two more boundary conditions are needed.

$$\dot{s}_0(0) = 0,\ \dot{s}_{N-1}(1) = 0 \tag{18}$$

From the above illustration, one can find that there are $(3N+1)$ conditions given in (14) through (18). For the uniqueness of the time-scaling function, the positions at time instants t_i, $i = 1, 2, \cdots, N-1$, should be determined. These values will be called the control points of the time-scaling function from hereafter. The values of control points have significant influence on the value of the final traveling time τ_f. Therefore, how to use a systematic approach to determine the values of control points while minimizing the value of τ_f should be studied. A PSO method will be adopted for searching the optimal control points simultaneously.

PSO is a population-based stochastic searching technique developed by Kennedy and Eberhart [8]. When applying a PSO method, possible solutions must be encoded into particle positions and a fitness function must be chosen. Since the goal is to minimize the traveling time of the new trajectory $\beta_z(\tau)$, the fitness function will be defined as

$$fitness = \frac{1}{\tau_f} = \frac{1}{\int_0^{t_f} s(t)dt} \tag{19}$$

Meanwhile, a particle position is represented as

$$\mathbf{P} = [p_1, p_2, \cdots, p_{N-1}] = [s(t_1), s(t_2), \cdots, s(t_{N-1})] \tag{20}$$

At each iteration, the particles update their velocities and positions based on the local best and global best solutions. In PSO method, the inertia weight, the cognitive parameter, and the social parameter are linearly adaptable over the evolutionary procedure (Ratnaweera et al [9]).

IV. SIMULATION RESULTS

In this section, an omni-directional mobile robot is utilized to demonstrate the near-time-optimal control problem along a given referenced path. For convenience, the dynamical equations used in this example are the same as those of Watanabe et al [10]. This means that the parameters of the mobile robot are chosen as $L = 0.178$ m, $M = 9.4$ kg, $I_v = 11.25$ kg·m^2, $I_\omega = 0.02108$ kg·m^2, $R = 0.0245$ m, $k_d = 1$, and $c = 5.983 \times 10^{-6}$ kg·m^2/sec, respectively. Meanwhile, the constraints on the control torques are assumed to be

$$-0.1\ \text{N}\cdot\text{m} \le u_i \le 0.1\ \text{N}\cdot\text{m} \quad \text{for } i = 1, 2, 3 \tag{21}$$

In the simulation, the reference trajectory of the robot, $\beta_r(t) = [x_w(t)\ \ y_w(t)\ \ \phi(t)]^T$, $0 \text{ sec} \le t \le 50\pi$ sec, is given as follows:

$$x_w = 5\cos\left(\frac{\pi}{2}\sin(0.04t + \frac{\pi}{2})\right) \tag{22}$$

$$y_w = 5\sin\left(\frac{\pi}{2}\sin(0.04t + \frac{\pi}{2})\right) \tag{23}$$

$$\phi = \pi\cos\left(\frac{\pi}{2}\sin(0.04t + \frac{\pi}{2})\right) \tag{24}$$

The initial and final states of the omni-directional mobile robot are $\beta_r(0) = [0 \text{ m}\ \ 5 \text{ m}\ \ 0 \text{ rad}]^T$ and $\beta_r(50\pi) = [0 \text{ m}\ \ 5 \text{ m}\ \ 0 \text{ rad}]^T$, respectively. Considering the computation burden, the number of sub-intervals is chosen as $N = 10$ and 9 control points are to be determined by the PSO method.

When performing the PSO approach, referring to the works of Ratnaweera et al [9], the maximal number of iterations and the population size are chosen to be 1000 and 20, respectively. Applying the proposed PSO method with

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the fitness function defined in (19), the time-scaling function $s(t)$ is determined as shown in Table 1 and the traveling time is reduced significantly from 50π sec to 73.04 sec.

Table 1. The time-scaling functions determined by the proposed PSO method.

$s_i(t)$	a_i	b_i	c_i	d_i
$s_1(t)$	0.7101	-1.3098	0	1
$s_2(t)$	-0.3363	0.8205	-0.4894	0.4003
$s_3(t)$	0.0967	-0.1883	0.1428	0.3951
$s_4(t)$	-0.0853	0.1019	0.0564	0.4464
$s_5(t)$	0.0705	-0.1540	0.0044	0.5194
$s_6(t)$	-0.0306	0.0576	-0.0920	0.4404
$s_7(t)$	0.0408	-0.0341	-0.0684	0.3754
$s_8(t)$	-0.0652	0.0882	-0.0142	0.3138
$s_9(t)$	0.2548	-0.1073	-0.0333	0.3226
$s_{10}(t)$	-0.6101	0.6570	0.5164	0.4368

The plots of $s(t)$ and the time integration of $s(t)$ for $0 \le t \le t_f$ are shown in Fig. 3. Since $s(t)$ is greater than zero within the time interval $[0, t_f]$, the time integration of $s(t)$ is monotonically increasing as shown in Fig. 3(b).

V. CONCLUSION

In this paper, a time-scaling procedure is applied to determine the near-time-optimal movement of an omni-directional mobile robot along a specific path. The strategy is to find a reference trajectory that satisfies the geometric path constraints first. Then the positions of the trajectory are kept the same while the velocities and the accelerations are adjusted by a time-scaling function to meet the dynamical constraints and to minimize the traveling time simultaneously.

The time-scaling function in this paper is interpolated by cubic spline functions, in which control points are determined simultaneously by a PSO method. The feasibility of the proposed method is verified by the simulation results. Moreover, it should also be noted that there is no evidence to show that the solution obtained is globally optimal. This explains why the term "near-time-optimal" is used in this paper instead of the term "time-optimal". How to determine the globally optimal control points is suggested as a topic for further research if one is interested in this issue.

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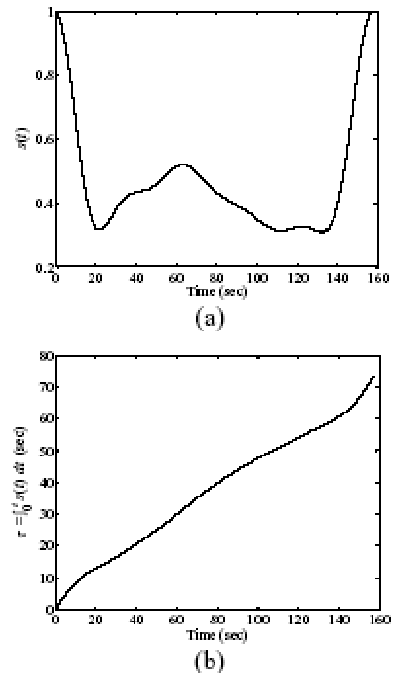

Fig. 3. The plots of (a) the time-scaling function $s(t)$ (b) the time integration of $s(t)$.