Control of Three-Link Underactuated Manipulators Using a Switching Method of Fuzzy Energy Regions

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Abstract

In general, manipulators used for industry and academic laboratory have actuators to drive each joint. On the other hand, underactuated manipulators handled by our research have some passive or free joints without actuators and brakes. We recently developed a switching method of fuzzy energy regions to control such manipulators. In such a method, it needs to design parameters related to energy regions and gains of some partly stable controllers based on the computed torque method. The switching method is here applied for a three-link underactuated manipulator. We optimize such design parameters related to fuzzy energy regions by a genetic algorithm. The effectiveness of the present method is illustrated with some simulations.

1 Introduction

Underactuated manipulators have some passive or free joints in general, where the number of inputs is less than the degree of freedom. These passive or free joints cannot generate dynamic torques at all. To control underactuated manipulators, a number of researches have been studied[1]. These systems have complex structural properties, and they have to control a number of generalized coordinates by few inputs. However, reducing the number of actuators brings some advantages such as lightweighting, compactification and cost reduction. The present authors have already proposed a switching control mehtod, in which some partly stable controllers were designed by computed torque method and the related switching lows in fuzzy reasoning or genetic algorithm (GA)[2] were obtained to select one controller among them. However, such a method does not necessarily give a robust result against the change of parameters, such as initial configurations of the manipulator. We discussed the application of a logic based switching method, which has been proposed by Hespanha et al.[3], to systems like underactuated manipulators with drift term. We also recently developed a fuzzy energy region based switching method[4]. Note however that in such a method, it needs to design parameters related to energy regions and gains of some partly stable controllers based on the computed torque method. The present paper is concerned with a three-link underactuated manipulator by applying the similar switching method. We here optimize design parameters related to fuzzy energy regions by a GA. The effectiveness of the present method is illustrated with some simulations.

Table 1: Definition of input torques, generalized coordinates and physical parameters

Symbols	Physical meaning
τ_1, τ_2	Input torque [Nm]
$\theta_1, \theta_2, \theta_3$	Link angle [rad]
m_1, m_2, m_3	Mass of link [kg]
l_1, l_2, l_3	Link length [m]
l_{g1}, l_{g2}, l_{g3}	Distance between joint and center of gravity [m]
I_1, I_2, I_3	Moment of inertia [kgm^2]
μ_1, μ_2, μ_3	Viscous coefficient [Ns/m^2]

2 Underactuated Manipulator

Figure 1 shows a three-link underactuated manipulator where the first and second joints are active and the third joint is passive. In Table 1, τ_1 and τ_2 denote the torques applied to each joint; θ_1, θ_2 and θ_3 denote the angle of each link; and other physical parameters are shown in it. The dynamical model of the underactuated manipulator is given as follows:

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}) = \boldsymbol{\tau} \tag{1}$$

where

$$
\begin{aligned}
M(\boldsymbol{\theta}) &= \begin{bmatrix} M_{11}(\boldsymbol{\theta}) & M_{12}(\boldsymbol{\theta}) & M_{13}(\boldsymbol{\theta}) \\ M_{12}(\boldsymbol{\theta}) & M_{22}(\boldsymbol{\theta}) & M_{23}(\boldsymbol{\theta}) \\ M_{13}(\boldsymbol{\theta}) & M_{23}(\boldsymbol{\theta}) & M_{33}(\boldsymbol{\theta}) \end{bmatrix} \\
M_{11}(\boldsymbol{\theta}) &= (m_1 l_{g1}^2 + I_1 + m_2 l_1^2 + m_3 l_1^2) \\
&\quad + (m_2 l_{g2}^2 + I_2 + m_3 l_2^2) + (m_3 l_{g3}^2 + I_3) \\
&\quad + 2 l_1 (m_2 l_{g2} + m_3 l_2) \cos\theta_2 \\
&\quad + 2 m_3 l_1 l_{g3} \cos(\theta_2 + \theta_3) \\
&\quad + 2 m_3 l_2 l_{g3} \cos\theta_3 \\
M_{12}(\boldsymbol{\theta}) &= (m_2 l_{g2}^2 + I_2 + m_3 l_2^2) + (m_3 l_{g3}^2 + I_3) \\
&\quad + l_1 (m_2 l_{g2} + m_3 l_2) cos\theta_2 \\
&\quad + m_3 l_1 l_{g3} \cos(\theta_2 + \theta_3) \\
&\quad + 2 m_3 l_2 l_{g3} \cos\theta_3 \\
M_{13}(\boldsymbol{\theta}) &= (m_3 l_{g3}^2 + I_3) + m_3 l_1 l_{g3} \cos(\theta_2 + \theta_3) \\
&\quad + m_3 l_2 l_{g3} \cos\theta_3 \\
M_{22}(\boldsymbol{\theta}) &= (m_2 l_{g2}^2 + I_2 + m_3 l_2^2) + (m_3 l_{g3}^2 + I_3) \\
&\quad + 2 m_3 l_2 l_{g3} \cos\theta_3 \\
M_{23}(\boldsymbol{\theta}) &= (m_3 l_{g3}^2 + I_3) + m_3 l_2 l_{g3} \cos\theta_3 \\
M_{33}(\boldsymbol{\theta}) &= m_3 l_{g3}^2 + I_3 \\
\boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= [h_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \; h_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \; h_3(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})]^T \\
h_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= -l_1 (m_2 l_{g2} + m_3 l_2)(2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \sin\theta_2 \\
&\quad - m_3 l_1 l_{g3} (2\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)(\dot{\theta}_2 + \dot{\theta}_3) \\
&\quad \sin(\theta_2 + \theta_3) - m_3 l_2 l_{g3} (2\dot{\theta}_1 \dot{\theta}_3 + 2\dot{\theta}_2 \dot{\theta}_3 \\
&\quad + \dot{\theta_3}^2) \sin\theta_3 + \mu_1 \dot{\theta}_1 \\
h_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= l_1 (m_2 l_{g2} + m_3 l_2) \dot{\theta}_1^2 \sin\theta_2 + m_3 l_1 l_{g3} \dot{\theta}_1^2 \\
&\quad \sin(\theta_2 + \theta_3) - m_3 l_2 l_{g3} (2\dot{\theta}_1 \dot{\theta}_3 + 2\dot{\theta}_2 \dot{\theta}_3 \\
&\quad + \dot{\theta_3}^2) \sin\theta_3 + \mu_2 \dot{\theta}_2 \\
h_3(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= m_3 l_1 l_{g3} \dot{\theta}_1^2 \sin(\theta_2 + \theta_3) + m_3 l_2 l_{g3} (\dot{\theta}_1 \\
&\quad + \dot{\theta}_2)^2 \sin\theta_3 + \mu_3 \dot{\theta}_3
\end{aligned}
$$

3 Definition of Energy

Energy is defined by using generalized coordinates. The desired joint angle of each link is θ_{di} and the error of joint angle is denoted by

$$e_i \overset{\triangle}{=} \theta_{di} - \theta_i. \tag{2}$$

Then, the energy of each link is defined by

$$E_i \overset{\triangle}{=} e_i^2 + \dot{e}_i^2. \tag{3}$$

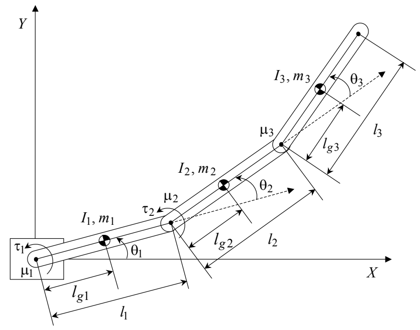

Figure 1: Model of three-link underactuated manipulator

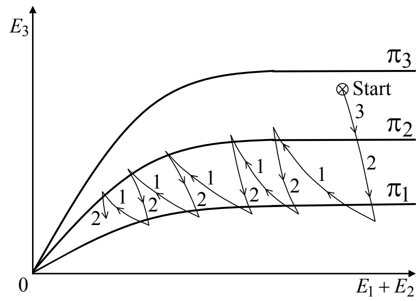

Figure 2: Ideal energy response

4 Fuzzy Energy Region Based Switching

We design controller 1 ($\boldsymbol{C}_1$), controller 2 ($\boldsymbol{C}_2$) and controller 3 ($\boldsymbol{C}_3$) for a three link manipulator. They are used as partial stabilizing controllers to stabilize each link. We can define an energy region related to each controller. Assuming that we use the same fuzzy energy region method as used for two-link manipulator[4], we can express the energy plain in this paper as ideal responses of energy illustrated in Figure 2.

If an exponential function is used, the design parameters of boundary curve to divide energy region are denoted by the amplitude and the time constant. Control responses of manipulators depend on these parameters. Such parameters need to be set in an ideal way. But it is difficult to set at once, so that the boundary curve is denoted by fuzzy expression in this research. If a boundary curve has any fuzziness denoted by the present fuzzy reasoning, then there appears an advantage of present method in setting the design parameters roughly. We first consider a straight line approximation shown in Figure 3. After obtain-

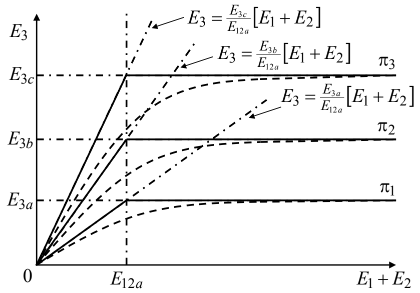

Figure 3: Region approximations for a logical switching

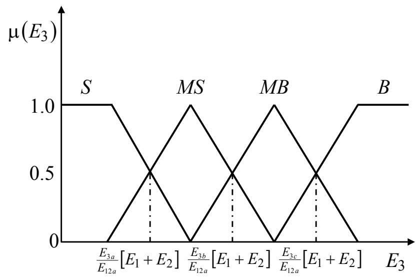

Figure 4: Membership functions for $E_1 + E_2 \leq E_{12a}$

ing such an approximation, fuzzy sets for E_3 can be defined for $E_1 + E_2 \leq E_{12a}$ and $E_1 + E_2 > E_{12a}$ cases respectively, as shown in Figure 4 and Figure 5, where E_{12a}, E_{3a}, E_{3b}, and E_{3c} are the design parameters of fuzzy sets. In order to realize ideal energy responses, fuzzy rules are given as follows:

Rule 1: If $E_3 = S$ then $s_1 = 1$
Rule 2: If $E_3 = MS$ and $I_{t-1} = 1$ then $s_2 = 1$
Rule 3: If $E_3 = MS$ and $I_{t-1} = 2$ then $s_3 = 2$
Rule 4: If $E_3 = MS$ and $I_{t-1} = 3$ then $s_4 = 2$
Rule 5: If $E_3 = MB$ and $I_{t-1} = 1$ then $s_5 = 2$
Rule 6: If $E_3 = MB$ and $I_{t-1} = 2$ then $s_6 = 2$
Rule 7: If $E_3 = MB$ and $I_{t-1} = 3$ then $s_7 = 3$
Rule 8: If $E_3 = B$ then $s_8 = 3$

Here, a parameter I_{t-1} which means the index of controller for one-step delay, is introduced, because one-step delayed controller must be retained in the overlapped energy region according to the ideal energy response. s_i is the index of controller that must be used in the fuzzy rule i. The corresponding control system is shown in Figure 6.

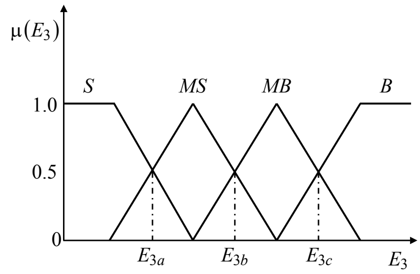

Figure 5: Membership functions for $E_1 + E_2 > E_{12a}$

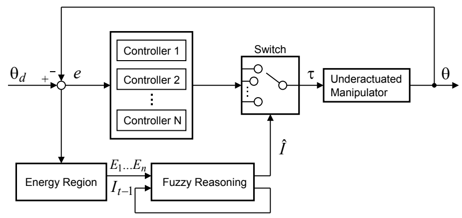

Figure 6: Block diagram of the proposed control system

5 Optimization by GA

We discuss about the design parameters of fuzzy rules using GA. These parameters related to the energy regions are E_{12a}, E_{3a}, E_{3b}, and E_{3c}. Each parameter is assumed to be encoded by 32 [bit]. A cost function is defined by using the error energy of time responses for different two initial state vectors:

$$
\begin{aligned}
{}^1\boldsymbol{x}(0) &= [{}^1\boldsymbol{\theta}^T(0)\ {}^1\dot{\boldsymbol{\theta}}^T(0)]^T \\
&= [\ \pi/4\ \ \pi/4\ \ \pi/4\ \ 0\ \ 0\ \ 0\ ]^T \\
{}^2\boldsymbol{x}(0) &= [{}^2\boldsymbol{\theta}^T(0)\ {}^2\dot{\boldsymbol{\theta}}^T(0)]^T \\
&= [\ \pi/6\ \ \pi/6\ \ \pi/6\ \ 0\ \ 0\ \ 0\ ]^T
\end{aligned}
$$

Then, the sampling interval is 0.01 [s], the simulation time is 30 [s], and the desired state vector is to be converged to zero.

The size of a population is 100 and the maximum number of generations is 500. Simulation condition used here are shown in Table 2.

The cost function is given by

$$
f_c = \sum_{i=1}^{2} f_i \tag{4}
$$

$$
f_i = \begin{cases} \sum_{j=2501}^{3000} \sum_{k=1}^{3} E_k(j) \\ \quad \text{if } E_1(j) + E_2(j) \leq 40 \text{ and } E_3(j) \leq 40 \\ 120(3000 - j_d) \quad \text{otherwise} \end{cases}
$$

Table 2: Setting parameters of simurations

Conditions	Setting value
Mass of each link	m_1=0.346 [kg]
	m_2=0.236 [kg]
	m_3=0.079 [kg]
Length of each link	l_1=0.20, l_2=0.20 [m]
	l_3=0.22 [m]
Distance between center of gravity and each joint	l_{g1}=0.1, l_{g2}=0.1 [m]
	l_{g3}=0.11 [m]
Coefficient of viscous friction of each joint	μ_1=0.00 [Ns/m^2]
	μ_2=0.00 [Ns/m^2]
	μ_3=0.02 [Ns/m^2]
proportional gain	K_{p1}=25.0, K_{p2}=25.0
	K_{p3}=25.0
derivative gain	K_{v1}=10.0, K_{v2}=10.0
	K_{v3}=10.0

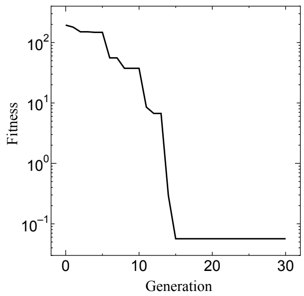

Figure 7: Generation history of GA

where i is the index of simulation trains, j is the index of discrete time, k is the index of energy of each link, and j_d is the index of discrete time when a certain link energy diverges to infinite. As a result, a training history in cost function is shown in Figure 7. At this stage, the parameters related to the energy region are converged. The parameters of fuzzy energy region are then set to E_{12a}=7.463264, E_{3a}=1.329838, E_{3b}=12.482597, and E_{3c}=13.929500.

Now, the other initial state vector is set to

$$\begin{aligned} \boldsymbol{x}(0) &= [\boldsymbol{\theta}^T(0)\ \dot{\boldsymbol{\theta}}^T(0)]^T \\ &= [\ \pi/2\ \ \pi/3\ \ 0\ \ 0\ \ 0\ \ 0\ ]^T \end{aligned}$$

We evaluated the total error energy of each link, where the evaluation function, f_e was applied only for last 15 seconds when the control converged to zero roughly. Then, the evaluation function was f_e=0.113969. The response of each link angle is shown in Figure 8. Thus, we confirmed that the present method converged with a satisfied condition.

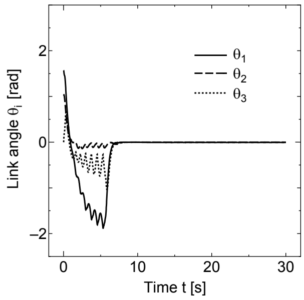

Figure 8: Simulation results with initial state vector [$\pi/2$ $\pi/3$ 0 0 0 0]

6 Conclusions

In this paper, we have applied a switching control method using fuzzy energy regions to a three-link underactuated manipulator. Since it was assumed to use the same fuzzy energy region method as used in two-link manipulator, we naturally expressed the energy plain for three-link. Therefore, several design parameters related to the fuzzy energy region were able to be trained by genetic algorithm, introducing a cost function to be used in the optimization process. In the furure, we want to check the projection form of energies for three-link underactuated manipulators.

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