Interaction of agents in financial markets and informational method to quantify it

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Abstract

In this article the informational method to quantify behavioral similarities of the market participants is proposed regarding the financial market as a many-body system. An agent-based model of a financial market in which N market participants deal with M financial commodities is considered. In order to measure the agents' interaction the spectral distance defined by the Kullback-Leibler divergence between two normalized spectra of behavioral frequencies is introduced. The validity of the method is evaluated by using the behavioral frequencies obtained from the agent-based model. It is concluded that the perception and decision parameters of agents who treat two commodities tend to be similar when those behavioral frequencies are similar.

1 introduction

A concept of complex systems [1] is expected to provide new insights for the intelligence since the knowledge and wisdom which are composed of the intelligence are widely distributed into the whole human-beings. Recently economically motivated problems seem to become considerable issues in the literature of the artificial intelligence. When one regards economical processes as phenomena which occur in the many-body systems one realize that enormous amount of agents who interact with one another on the field form a complicated dynamical network and that they emerge through such agents' dynamical network [2]. Therefore one recognizes that the agent-based modeling is a promising methodology in order to deepen an understanding of such economical processes at a microscopic level.

In recent years processing performance of computers and storage capacity almost allow us to gather together large amount of data of economical processes [3] and to directedly simulate economical processes for the order of population of the human-beings. Specifically there exist some studies on a method to simulate large size systems and an application of simulation techniques to social/economical phenomena. Moreover an large amount of data about economical processes are available due to spreading of information and communication technology. When we treat with a phenomenon having large degree of freedom or large amount of data it is important to find a rule to reduce the information. When one numerically simulates such agent-based models one faces to the problem for inferring the whole parameters of agents from observable quantities of the objective. And one immediately senses that the number of parameters which one has to infer in order to make the agent-based simulation are explosive. Generally speaking it is hard to infer the whole parameters of a multi-dimensional nonlinear dynamical system from the less observable quantities than the number of parameters. Two kinds of possibilities to cope with this matter are addressed:

(1) to employ the GA algorithm for the agent model, and to infer the whole parameters asymptotically.

(2) to compare the behaviors of agents, to quantify relative differences among agents, and to infer distributions of parameters.

In this article according to the second possibility the method to quantify behavioral similarities among interacting agents is considered [4]. An agent-based model of the financial market is formalized [5] and usefulness of the proposed method is evaluated through the agent-based simulation.

The remainder of this article is organized as follows. In Section 2 the agent-based model of the financial market where N market participants exchange M financial commodities is formalized. In Section 3 the informational method to quantify the behavioral similarity of market participants is considered. As the result of applying the proposal method to the behavioral frequency it is confirmed that the behavioral frequencies have a tendency to be similar when both perception and action parameters of the market participants are similar. Section 4 is devoted into concluding remarks.

2 Model

A model of the financial market where N market participants deal with M financial commodities is considered (see Figure 1). Each market participant perceives information from environment and makes a decision of his/her investment attitude based on the information and make an action. The actions for the market participants to be permitted are buying, selling and waiting.

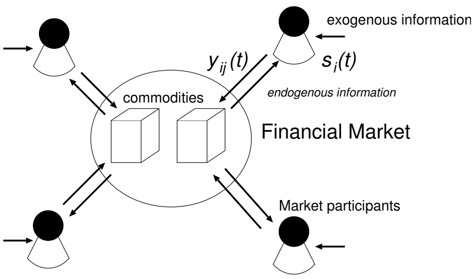

Figure 1: A conceptual illustration of the agent-based model of the financial market where N market participants exchange M financial commodities.

For simplicity the i-th market participant perceives information $x_i(t)$, which is evaluated as a scalar value. This information builds a momentum in which each market participant decides his/her investment attitude. The market participant interprets the information and determines his/her attitude on the basis of the interpretation. Since the possibility of interpretation is very high and dependent on time and market participants uncertainty for the i-th agent to interpret the information $x_i(t)$ at time t is assumed to be modeled by random variables $\xi_i(t)$. And the interpretation $x_i(t) + \xi_i(t)$ drives feeling to determine his/her investment attitude. Furthermore the feeling about the feeling valid the feeling and drives his/her actions. In order to model the feeling about the feeling we introduce a multiplicative factor $a_{ij}(t)$ which represents the feeling about the feeling of the i-th market participant for the j-th financial commodity. If $a_{ij}(t)$ is positive then the feeling about the feeling supports the feeling. If $a_{ij}(t)$ is negative then the feeling about the feeling refutes the feeling. The absolute value of $a_{ij}(t)$ represents intensity of the feeling about the feeling. Since the determination depends on both the feeling and the feeling about the feeling the investment attitude is assumed to be determined from the value $\Phi_{ij}(t) = a_{ij}(t)(x_i(t) + \xi_i(t))$. If it is large then the market participant tends to make a buying decision. Contrarily if it is small then he/she tends to make a selling decision. For simplicity it is assumed that a trading volume can be ignored.

The action is determined on the basis of his/her feeling about the feeling. Since the decision and action have strong nonlinearity the action is determined with Granovetter type threshold dynamics [6]. In order to separate three actions at least two thresholds are needed. Defining the threshold for the i-th market participant to determine buying the j-th financial commodity as $\theta^B_{ij}(t)$ and selling it as $\theta^S_{ij}(t)$ ($\theta^B_{ij}(t) > \theta^S_{ij}(t)$), three investment attitudes (buying: 1, selling: -1, and waiting: 0) are determined by

$$y_{ij}(t) = \begin{cases} 1 & (\Phi_{ij}(t) \geq \theta^B_{ij}(t)) \\ 0 & (\theta^B_{ij}(t) < \Phi_{ij}(t) < \theta^S_{ij}(t)) \\ -1 & (\Phi_{ij}(t) \leq \theta^S_{ij}(t)) \end{cases} . \tag{1}$$

Furthermore it is assumed that the information is described as the endogenous factor, moving average of log return over T, plus the exogenous factor, $s_i(t)$:

$$x_i(t) = \sum_{k=1}^{M} C_{ik}(|\theta^S_{ik}(t)|, |\theta^B_{ik}(t)|) \times \frac{1}{T}\sum_{\tau=1}^{T} R_j(t - \tau\Delta t) + s_i(t), \tag{2}$$

where $C_{ij}(|\theta^S_{ij}(t)|, |\theta^B_{ij}(t)|)$ represents focal points of the i-th market participant for the j-th financial commodity. It seems reasonable to assume that $C_{ij}(x, y)$ is a monotonically decreasing function of x and y.

The excess demand for the j-th financial commodity, $N^{-1}\sum_{i=1}^{N} y_{ij}(t)$, drives the market price of the j-th financial commodity [7]. To guarantee positive market prices, we choose a log return,

$$R_j(t) = \log S_j(t + \Delta t) - \log S_j(t), \tag{3}$$

and define the log returns as the excess demand,

$$R_j(t) = \gamma N^{-1} \sum_{i=1}^{N} y_{ij}(t), \tag{4}$$

where γ is a positive constant to show a response of the return to the excess demand. Furthermore the total behavioral frequency to submit their quote request of the j-th financial commodity is defined as

$$A_j(t) = \frac{1}{\Delta t} \sum_{i=1}^{N} |y_{ij}(t)|. \tag{5}$$

In order to simulate the agent model we assume $C_{ij}(x,y) = CA_j(k)/(x^2+y^2)$ ($C>0$), and $a_{ij}(t) = a(t) + w_{ij}(t)$, where $a(t)$ and $w_{ij}(t)$ is sampled from the Gaussian distribution. $a(t)$ is assume to be a more slowly varying random variable than $w_{ij}(t)$. As shown in Figure 2 market price changes $R_j(t)$ show fat-tailed fluctuations. The probability density function of price changes is leptokurtic and have fat-tails.

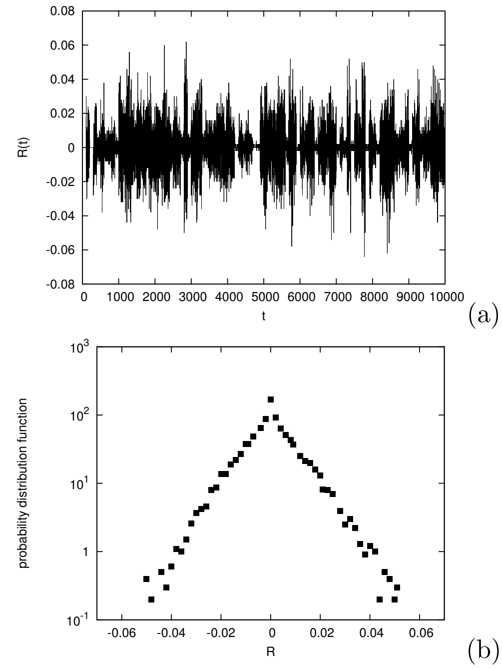

Figure 2: Time series of market price changes (a) and a probability density functions for them (b) for $N=500$, $M=10$, $T=50$, $\Lambda=0.23$, $\sigma=0.1$, $\bar{a}=0.1$, $\sigma_a=2.0$, $C=100$.

3 Method

Market participants operate terminal computers and submit their quotation when they want to buy or sell several amount of financial commodities. Since their intention (internal states of the market participants) drives their behavior, observing their behaviors seems to be equivalent to inferring their internal states. Hence comparing their behavior may lead to comparing their perception and decision parameters.

In order to measure similarity of the agents' parameters from their behavior it is necessary to specify a representative quantity. One of candidates for specifying such a representative quantity is the behavioral frequency. If the behavioral frequencies of agents are similar then one may thinks that perception and decision parameters of these agents have a tendency to be similar. In order to quantify the similarity of two behavioral frequencies the informational method with an asymmetric spectral distance defined by the Kullback-Leibler divergence between two normalized spectra of behavioral frequencies is employed. The reason employing this method is because the spectral distance can measure similarity of the underlying dynamics as well as that of time series.

The idea to measure the similarity of the behavioral frequencies $A_i(t)$ of the i-th financial commodity is to evaluate the power normalized spectra. By using the discrete Fourier transform of $A_i(t)$,

$$\tilde{A}_i(n) = \sum_{t=0}^{N-1} x_i(t) \exp\Big(-2\pi i n t/N\Big) \quad (0 \leq n \leq N-1), \tag{6}$$

the power spectrum is estimated by

$$P_i(n) = \tilde{A}_i(n)\tilde{A}_i(n)^* \tag{7}$$

where * represents the complex conjugate. Because the Kullback-Leibler divergence (KL) is a functional of two normalized functions [8, 9] one needs to normalize the power spectrum in order to apply it to power spectra,

$$p_i(n) = \frac{P_i(n)}{\sum_{n=1}^{N-1} P_i(n)}. \tag{8}$$

The similarity of two spectra is defined by

$$K_{ij} = \sum_{n=1}^{N-1} p_i(n) \log \frac{p_i(n)}{p_j(n)}. \tag{9}$$

which is a non-negative $K_{ij} \geq 0$, and asymmetric $K_{ij} \neq K_{ji}$. $K_{ij}=0$ if and only if $p_i(n) = p_j(n)$. What the direct current component is ignored is equivalent to eliminating the mean value of the behavioral frequency.

In general an asymmetric matrix can be described as $K_{ij} = J_{ij} + I_{ij}$ by using the transformation $J_{ij} = (K_{ij} + K_{ji})/2$ and $I_{ij} = (K_{ij} - K_{ji})/2$. Specifically J_{ij} is called a symmetric Kullback-Leibler divergence (SKL). The SKL possesses a symmetric and non-negative properties: $J_{ij} = J_{ji}$ and $J_{ij} \geq 0$ if and only if $p_i(n) = p_j(n)$. Of course an alternative symmetrical divergence, for example, Jensen-Shannon divergence [9], is also applicable for measuring the difference between power spectra.

Under the assumption that their whole actions are observable one can calculate the spectral distance of the sum of actions for the ith commodity and the

jth commodity. If the spectral distance is small then the agents who treat the ith commodity and the jth commodity seem to have similar focusing points and strategies. Namely we may estimate the rate of the market participants who have the same focusing points and the same strategies through the behaviors of agents.

This method to quantify the behavioral similarities of the market participants is demonstrated by using the agent-based model of financial market. From the numerical simulation of the agent-model of the financial market in which 10 financial commodities are dealt relationships between the similarities of the agents' parameters and the behavioral similarities are calculated. Figure 3 shows the SKL matrix as a fully connected network in which the thin/thick lines between two nodes (financial commodities) exhibit similar/dissimilar behavior between these financial commodities. It is found that the total behavioral frequencies of two financial commodities fluctuate similarly if the agents' parameters are similar.

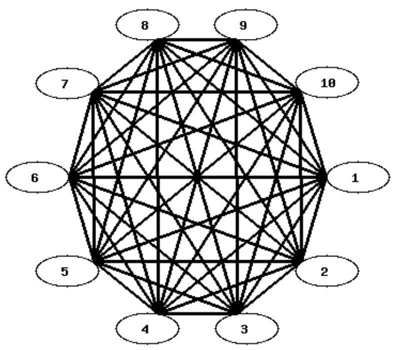

Figure 3: The SKL matrix among 10 financial commodities calculated from the behavioral frequencies obtained from the agent-based model for $N = 500$, $M = 10$, $T = 50$, $\Lambda = 0.23$, $\sigma = 0.1$, $\bar{a} = 0.1$, $\sigma_a = 2.0$, $C = 100$ is shown as a fully connected network in which the thin/thick lines exhibit similar/dissimilar nodes (financial commodities).

4 Summary

The agent-based model where N market participants exchange M financial commodities was formalized. In order to measure the similarity of two total behavioral frequencies the spectral distance matrix defined by the Kullback-Leibler divergence between two normalized power spectra of the behavioral frequency was calculated. It is conclude that the parameters of the agents who treat two commodities tend to be similar when the behavioral frequencies for two commodities are similar.

Acknowledgements

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