Mathematical Modeling of Frogs' Calling Behaviors and its Possible Application to Artificial Life and Robotics

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Abstract

This paper theoretically describes calling behaviors of Japanese tree frogs *Hyla japonica* with a simple model of phase oscillators. Experimental analysis showed that while isolated single frogs called nearly periodically, a pair of interacting frogs called alternately. We model these phenomena as a system of coupled phase oscillators, where each isolated oscillator behaves periodically as a model of the calling of a single frog and two coupled oscillators shows anti-phase synchronization, reflecting the alternately calling behavior of two interacting frogs. Then, we extend the model to a system of three oscillators corresponding to three interacting frogs and analyse the dynamics. We also discuss a biological meaning of the calling behaviors and its possible application to Artificial Life and Robotics.

1 Introduction

Nonlinear dynamics like synchronization has been both experimentally and theoretically analyzed in many biological systems [1–7] with respect to possible functions. In this paper, we consider calling behaviors of frogs from the viewpoint of nonlinear dynamics. There have been some experimental studies on synchronization of calls of frogs. Loftus-Hills studied the synchronization in calling behaviors of frogs *Pseudacris streckeri* [8], where tape-recorded calls were used to evoke response of frogs. Lemon and Struger studied acoustic entrainment to randomly generated calls in frogs *Hyla crucifer* [9]. Here, we theoretically study spontaneous calling behaviors [10, 11] of Japanese tree frogs *Hyla japonica* shown in Fig. 1 and discuss a possible application to artificial life and robotics.

Figure 1: Japanese tree frog *Hyla japonica*.

2 Experimental Results

Male Japanese tree frogs *Hyla japonica* which were collected from breeding assemblages in paddy fields in Kyoto, Japan were used for the experiment. Spontaneous mating calls were recorded and analyzed.

Figure 2 shows an example of the waveforms of the calls recorded from (a) a single frog calling alone and (b) two interacting frogs calling together. While a single frog called nearly periodically as shown in Fig. 2(a), two frogs called alternately as shown in Fig. 2(b). The detail of the experiment was reported elsewhere [10, 11].

3 Mathematical Modeling of Frogs' calling behaviors

3.1 Phase oscillator model

We model the calling behaviors of frogs as phase oscillators. The calling of a single frog is regarded

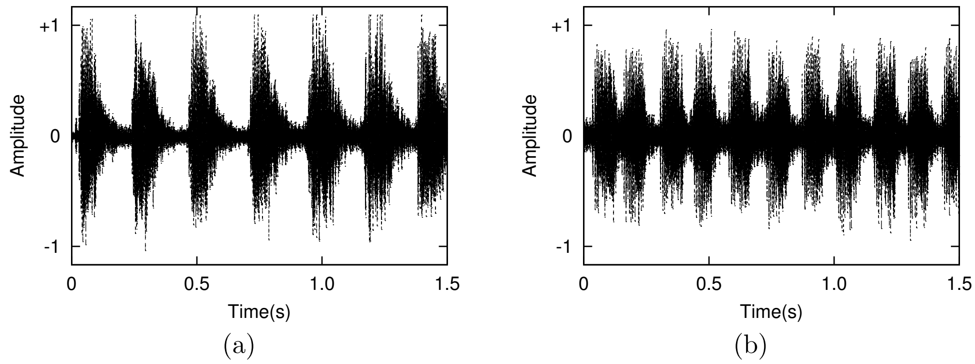

Figure 2: The waveforms of the calls of (a) a single frog and (b) two interacting frogs. A single frog called nearly periodically, and a pair of frogs called alternately.

as a periodic phase oscillator, and the calling of two interacting frogs as two coupled phase oscillators.

First, we consider the situation that each frog calls alone. The calling behavior of respective frogs is described as a phase oscillator with the phase variable θ with $\theta \in \mathbb{S}^1 = (\mathbb{R} \mod 2\pi) = [-\pi, \pi]/\{-\pi \equiv \pi\}$ [6] as follows:

$$\frac{d\theta}{dt} = \omega, \tag{1}$$

where ω is an intrinsic natural frequency. It is assumed that $\theta = 0$ correspond to each call. This model represents the property that single frog calls periodically.

Then, we model the situation that two frogs call together through interaction. The system is described as two coupled phase oscillators with two phase variables θ_A and θ_B as follows (see also [10, 11]):

$$\frac{d\theta_A}{dt} = \omega + g_{AB}(\theta_B - \theta_A), \tag{2}$$

$$\frac{d\theta_B}{dt} = \omega + g_{BA}(\theta_A - \theta_B), \tag{3}$$

where ω is the intrinsic frequency that is assumed to be the same between two frogs, and g_{AB} and g_{BA} are 2π-periodic functions that represent the mutual interaction. To examine whether two oscillators synchronize, we analyze the dynamics of the phase difference $\phi \equiv \theta_A - \theta_B$ with $\phi \in \mathbb{S}^1$. Subtracting Eq.(3) from Eq. (2), we obtain the following equation on ϕ:

$$\frac{d\phi}{dt} = g_{AB}(-\phi) - g_{BA}(\phi). \tag{4}$$

Here, we assume g_{AB} and g_{BA} to be a sinusoidal function for the sake of simplicity, according to the former studies [2, 3], then, Eq.(4) is calculated as follows:

$$\frac{d\phi}{dt} = 2K \sin\phi, \tag{5}$$

where K is a positive coupling coefficient as schematically shown in Fig. 3(a). The stable equilibrium point ϕ^* which satisfies $\left.\frac{d\phi}{dt}\right|_{\phi=\phi^*} = 0$ and $\left.\left(\frac{d}{d\phi}\left(\frac{d\phi}{dt}\right)\right)\right|_{\phi=\phi^*} < 0$ is given by $\phi^* = \pi$. This stable equilibrium point reproduces the experimental result qualitatively, namely these two oscillators synchronize in anti-phase.

3.2 Extension to a system of three frogs

Next, we extend this model to a system of three coupled oscillators as follows:

$$\frac{d\theta_A}{dt} = \omega - K_1 \sin(\theta_B - \theta_A) - K_3 \sin(\theta_C - \theta_A), \tag{6}$$

$$\frac{d\theta_B}{dt} = \omega - K_1 \sin(\theta_A - \theta_B) - K_2 \sin(\theta_C - \theta_B), \tag{7}$$

$$\frac{d\theta_C}{dt} = \omega - K_3 \sin(\theta_A - \theta_C) - K_2 \sin(\theta_B - \theta_C), \tag{8}$$

where K_i's $(i = 1, 2, 3)$ are symmetrical coupling coefficients between two frogs as schematically shown in Fig. 3(b). Here, for the simplicity, we assume that $\omega = 1.0$ and $K_1 = K_3 = 1.0$. In order to examine dynamical properties in this system, we define the phase differences $\phi_1 \equiv \theta_A - \theta_B$ and $\phi_2 \equiv \theta_B - \theta_C$. Then, we change the value of K_2 from 0 to 1.0 as the bifurcation parameter and numerically examine the stable equilibrium points ϕ_1^* and ϕ_2^*.

The bifurcation diagram is shown in Fig. 4. In the region $0 < K_2 < 0.5$, oscillators A and B synchronize in anti-phase and oscillators B and C synchronize

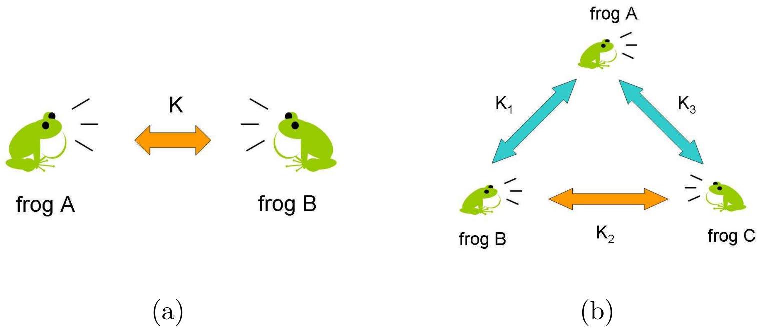

Figure 3: Schematic diagrams in modeling of a system of (a) two frogs and (b) three frogs.

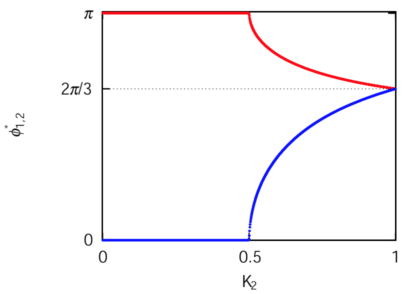

Figure 4: The bifurcation diagram in the system of three coupled oscillators, where the red line represents the phase difference ϕ_1^*, and the blue line ϕ_2^*.

in-phase. With increasing the value of K_2 in the region $0.5 < K_2 < 1.0$, on the other hand, the property of synchronization in this system gradually changes, and finally three oscillators synchronize in tri-phase at $K_2 = 1.0$.

4 A Possible Application to Artificial Life and Robotics

We discuss a biological meaning of the calling behaviors and a possible application to artificial life and robotics. Generally speaking, it is said that the main purpose of calling by male frogs is to attract females and tell their own positions to other males [12].

If one male frog mates with one female in a one-to-one manner, it is important for two males to make females distinguish them each other. In fact, many kinds of frogs are known to mate in such a one-to-one manner [13], including the mating in Japanese tree frogs [14]. Thus, it is probable that two male Japanese tree frogs call alternately to be distinguished by a female. On the other hand, male Japanese tree frogs are known to inhabit with a low density in breeding assemblages [12]. Then, we suppose that the anti-phase synchronization of two male frogs can play a role of telling their own positions to other males, resulting in sparse distribution. In that meaning, anti-phase synchronization would be applicable for multiple artificial agents and robots to prevent collisions each other in some real or abstract spaces.

In the actual system of male frogs, coupling coefficients would depend on the distance between male frogs, because they interact by calling and hearing. It was numerically confirmed by varying the coupling coefficient K_2 that the system of three coupled oscillators shows more complicated properties than that of two frogs does similarly to coupled chemical oscillators [15]. Therefore, the calling behaviors in a system of many frogs should be much more complicated. For the purpose of understanding such a system, it is important to extend the model to a system of many oscillators. A simple extension of our model to a larger system composed of N frogs is given as follows:

$$\frac{d\theta_i}{dt} = \omega_i - \frac{1}{N-1}\sum_{i=1}^{N} K_{ij}\sin(\theta_j - \theta_i), \tag{9}$$

where for the ith frog ($i = 1, 2, \ldots, N$), θ_i is the phase variable, ω_i is an intrinsic natural frequency, and K_{ij} represents interaction with the jth frog. It is an important future problem to analyze such a system of many frogs both experimentally and theoretically.

Moreover, such a study would provide useful mechanisms of controlling distributed systems composed of many artificial agents and robots. For example, in-phase synchronization and anti-phase one may represent cooperation and competition between agents and robots. Moreover, frogs that call in phase together can be interpreted as a cooperative cluster, which may produce emergence of a kind of communication.

5 Conclusion

We have theoretically modeled the calling behaviors of Japanese tree frogs *Hyla japonica* as a system of coupled phase oscillators where two coupled phase oscillators synchronize in anti-phase like the real calling behaviors of two frogs. Biologically speaking, the anti-phase synchronization would be important for a frog to tell his own position to the other frog. In this meaning, anti-phase synchronization would be applicable to prevent collisions of multiple agents and robots. Then, we have extended the model to a system of three coupled oscillators and confirmed that such a system shows more complicated properties than that of two oscillators does. For the purpose of application to a system of many agents and robots, it would be an important future problem to analyze a system of many frogs both experimentally and theoretically. It is also our future problem to modify our models more realistic, for example, by considering phase shift parameters and distribution of intrinsic frequencies of frogs.

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