Effect of facility closure in the SEIR epidemic model

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Abstract

Avian influenza H5N1 has caused large outbreaks in birds in Southeast Asia. This virus is highly virulent in humans who have been infected directly from birds. But, the virus has not achieved human-to-human transmission. If a new virus of bird flu capable of human-to-human transmission appears, this change could cause pandemic influenza. We propose the SEIR epidemic model of influenza transmission to assess the influence of facility closure as a containment strategy. Mathematical models are important tools in analyzing the spread and control of infectious diseases. If the fraction of infected individuals exceeds the set threshold, we execute the facility closure for the set period and there are assumed to be no transmission among a population. If the basic reproduction number R_0 was assumed to be 2.0, our model suggested that it was not necessarily the case that long period of facility closure reduced the prevalence. The final size of an epidemic depended on the number of infected individuals when the susceptible fraction was equal to $1/R_0$.

1 Introduction

The threat of pandemic influenza has increased for decades [1]. H5N1 highly pathogenic avian influenza is causing outbreaks among poultry in Southeast Asia. The transmission from birds to human and other mammalian species has been sporadic. The virus has not required the ability to be transmitted from human to human. But, if mutation of the virus occurs, the novel variant could be capable of sustaining human-to-human transmission. Besides, pandemic influenza can cause a public health crisis because many people are immunologically naive to the new virus. Although antiviral drugs offer protection against infection, production delays would limit availability in the first months of pandemic [2].

Influenza prevention and containment strategies can be considered under the broad categories of antiviral, vaccine, and nonpharmaceutical measures. In this study, we focus on nonpharmaceutical measures, especially in the facility closure. For example, schools are known to be the primary context of influenza transmission [3]. However, no data or analyses exist for recommending illness thresholds or rates of change that lead to considering closing or reopening schools [4].

The purpose of this study is to clarify the impact of facility closure by changing the threshold or the duration of closure. Here we construct a simple epidemic model of influenza transmission with deterministic differential equations. Mathematical models and computer simulations are useful tools for building and testing theories, assessing quantitative conjectures, answering specific questions, and determining sensitivities to changes in parameter values [5]. The model formulation process clarifies assumptions, variables, and parameters. We can use mathematical models in comparing, planning, implementing, and evaluating various detection, prevention, and control programs.

2 Model and general theory

We use a mathematical model called the SEIR epidemic model, which is represented as follows:

$$\frac{dS}{dt} = -\beta SI, \tag{1}$$

$$\frac{dE}{dt} = \beta SI - \sigma E, \tag{2}$$

$$\frac{dI}{dt} = \sigma E - \gamma I, \tag{3}$$

$$\frac{dR}{dt} = \gamma I. \tag{4}$$

$S(t)$, $E(t)$, $I(t)$, and $R(t)$ are the number of susceptible, exposed, infective, and recovered individuals, respectively. This model is based on the Kermack-McKendrick model [6].

The susceptible class S contains individuals who have a risk of becoming infected. When there is a contact of a susceptible with an infective so that transmission occurs, the individual enters the exposed class E in the latent period. An exposed individual is infected but non-contagious. After the latent period, the individual enters the infective class I in the infection period. An infective individual is contagious, that is capable of transmitting the infection. After the infection period, the individual enters the recovered class R. A recovered individual is permanently immunity to further infection.

Movements out of the class E and I are governed by σE and γI, respectively. It is shown that these terms correspond to exponentially distributed waiting times. For example, the transfer rate γI corresponds to $P(t) = e^{-\gamma t}$ as the fraction that is still in the infective class t units after entering this class and to $1/\gamma$ as the mean waiting time. We define the duration of latent period and infection period as $1/\sigma$ and $1/\gamma$.

The key value governing the time evolution of these equations is the basic reproduction number R_0, which is defined as the mean number of secondary infections generated by a primary infection in a susceptible population [7]. R_0 for the SEIR model is given by

$$R_0 = \frac{\beta N}{\gamma}, \tag{5}$$

where N is the total number of individuals so that $N = S(t) + E(t) + I(t) + R(t)$. If $R_0 < 1$, one infected individual will infect fewer than one susceptible individual before recovering. The infection will die out certainly. If $R_0 > 1$, one infected individual will infect more than one susceptible individual before recovering. There is some possibility of a major epidemic. Therefore, R_0 is considered as the threshold that determines whether an infection can persist in a population or not.

We propose the new epidemic model with facility closure. If the proportion of infective individuals exceeds the threshold of closure θ, we assume that facilities are closed for d days and that there is no transmission among the people. The dynamics with closure is represented as follows:

$$\frac{dS}{dt} = 0, \tag{6}$$

$$\frac{dE}{dt} = -\sigma E, \tag{7}$$

$$\frac{dI}{dt} = \sigma E - \gamma I, \tag{8}$$

$$\frac{dR}{dt} = \gamma I. \tag{9}$$

If the proportion of infective individual is less than the threshold after the facility closure of d days, facilities reopen and the transmission occurs again. The dynamics with or without closure follows equations (1)-(4). Figure 1 shows the dynamics with or without closure schematically.

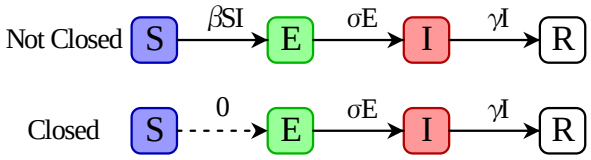

Figure 1: The dynamics of the SEIR epidemic model with or without closure

3 Results

We performed a numerical simulation to investigate the dynamics of our epidemic model. As an initial state, we set $\{S(0), E(0), I(0), R(0)\} = \{99, 1, 0, 0\}$. Since recent estimates of the basic reproduction number of the 1918 pandemic strain were in the range 2-3 [8], we assumed that $R_0 = 2.0$. We also assumed distributions of infectiousness consistent with previous studies [9], giving mean latent and infection periods of 1.9 days and 4.1 days, respectively. These assumed parameters are shown in Table 1. We estimated the value of β, σ, and γ from equation (5) and Table 1.

Table 1: Parameters for transmission

Parameters	Description	Value
R_0	Basic reproduction number	2.0
$1/\sigma$	Mean latent period	1.9
$1/\gamma$	Mean infection period	4.1

Figure 2 shows the transition of the susceptible fraction, exposed fraction, infective fraction, and the sum of exposed and infective fraction without facility closure. The horizontal auxiliary line shows that the susceptible fraction is 0.5 and the vertical auxiliary one shows the day when it is 0.5. Both the susceptible fraction and infective fraction has the maximum on about 30th day. About 80% of a population is infected on the 180th day.

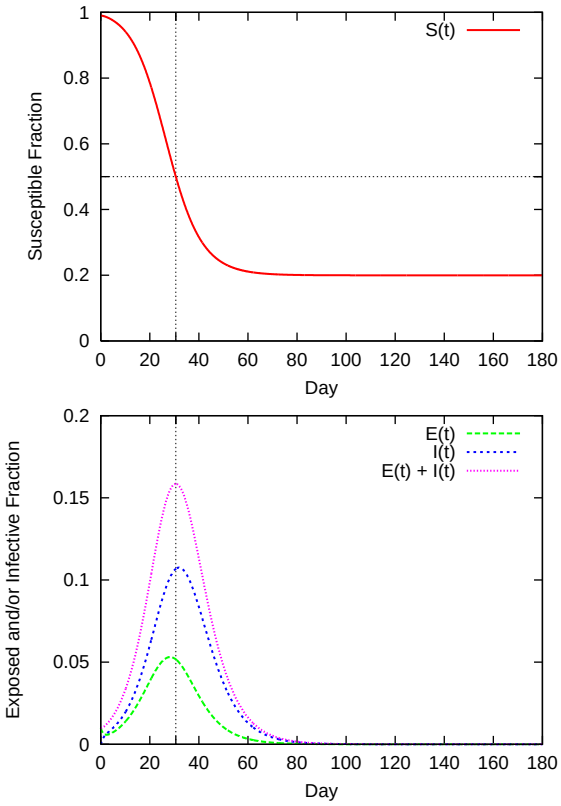

Figure 2: The transition of susceptible fraction, exposed fraction, infective fraction, and the sum of exposed and infective fraction without facility closure

Figure 3 shows the relation between the threshold of closure and the prevalence, which is the proportion of recovered individuals. We examined three parameters for the duration of closure d, i.e. three, five, and seven days. The prevalence doesn't reduce monotonically with reducing the threshold of closure regardless of the closed period. Many of the long term closure reduce the prevalence broadly if the threshold of closure is fixed. But, there is a little possibility that the long term closure produce somewhat high prevalence than short term one.

Figure 4 shows the transition of susceptible fraction and the sum of exposed and infective fraction with facility closure for five days. Thresholds of closure θ were assumed to be 0.06, 0.07, and 0.09. When the threshold is 0.06, the facility closure is implemented twice on the 20th and 40th day.

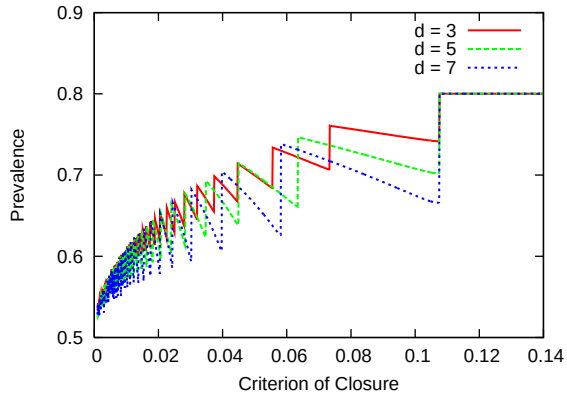

Figure 3: The relation between the threshold of closure and the prevalence

4 Discussion

Figure 2 represents that the number of exposed and infective individuals reaches a peak on the 30th day. After this time, the following formula is formed:

$$\frac{dE}{dt} + \frac{dI}{dt} < 0. \tag{10}$$

From equations (2), (3), and (5), we can obtain the following formula:

$$\frac{S}{N} < \frac{1}{R_0}. \tag{11}$$

This formula indicates that the number of exposed and infective individuals start to reduce when the susceptible fraction is less than $1/R_0$. This threshold of reduction is 0.5 because we assumed that $R_0 = 2.0$. This analytical solution gives good agreement with experimental results in Figure 2.

From equations (10) and (11), at least a half of a population is infected in this model. Therefore, the number of exposed and infected individuals when the susceptible fraction is equal to $1/R_0$ is important value to reduce the prevalence.

Figure 4 is an example of why zigzag lines are drawn in Figure 3. First, the transition of the sum of exposed and infective fraction achieves a peak twice when the threshold of closure θ is 0.09 or 0.07. If the threshold changes from 0.09 into 0.07, the sum of exposed and infective fraction in the first peak is reduced but in the second peak is increased. In the second peak, both susceptible fraction is the same by 0.5, and more prevalence is produced by more exposed and infective individuals with $\theta = 0.07$ than with $\theta = 0.09$. Second, if we reduce θ from 0.07 much further more, the

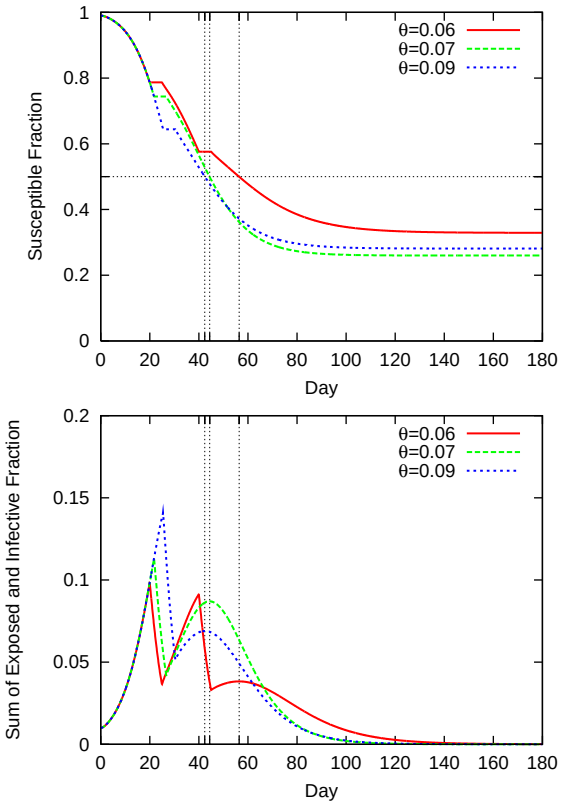

Figure 4: The transition of susceptible fraction and the sum of exposed and infective fraction with facility closure for five days

transition of the sum of exposed and infective fraction achieves a peak three times as $\theta = 0.06$ in Figure 4. The sum of exposed and infective fraction with $\theta = 0.06$ is less in the third peak when the susceptible fraction is equal to 0.5 than with $\theta = 0.07$. These two mechanism generate the zigzag line in Figure 3.

In this study, we reveal that the low prevalence is generated by the small number of exposed and infective individuals when the susceptible fraction is equal to $1/R_0$. However, since R_0 of a future newly emergent influenza strain is unknown, we can not change the number of exposed and infective individuals purposely. It seems to be the desirable measure to close facilities at the low threshold and for a long term if at all possible.

5 Conclusions

We propose the simple epidemic model based on the SEIR epidemic model to explore the effect of facility closure. Long period closure has the high possibility to reduce the prevalence than short period closure. However, there are some situations it is better to close facilities for long period.

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