Digital Spiking Silicon Neuron: Concept and Behaviors in GJ-coupled Network

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Abstract

Silicon neuron is electrical circuit that is analogous to biological neurons. Most spiking silicon neurons comprise analog circuit technology. We propose a new concept of spiking silicon neuron that is composed of only digital circuit technology. The system equations were designed by a mathematical-model-based design method that we proposed for analog silicon neurons in previous works. This allowed us to design a simple digital spiking silicon neuron (DSSN) model that produces rich dynamical behaviors comparable to biological neurons. We analyzed an elemental DSSN model to validate if it possesses fundamental characteristics of neurons, and its behaviors in gap junction (GJ)-coupled networks were studied in order to demonstrate its ability to exhibit rich dynamical behaviors.

1 Introduction

Silicon neuron study is an attempt to produce neuron analog utilizing electrical circuit technology. One of its objectives is to construct artificial silicon neural networks that process information similarly to neural systems in creatures. Although knowledge on the information processing principles in neural systems is inadequate, silicon neuron and neural networks are being studied actively. This is not only because of some realistic potential applications such as associative memory and brain-machine interfacing, but also because analysis by synthesis is one of the most effective tools for brain science. Many silicon neurons have been constructed using analog electrical circuit technology, because neural phenomena are produced by real-valued dynamics. Conventionally, in the analog silicon neuron design trade-off between the circuit size and richness of neuronal properties has been a disadvantage. In the previous works [1][2], we proposed a mathematical-model-based design policy that allows us to implement a compact silicon neuron that possesses rich neuronal dynamics.

For silicon neurons, digital circuit technology has not been applied as extensively as analog technology. However, it has some appealing features such as insensitivity to the fluctuations in the environment and continuous improvement in the fabrication technology. Additionally, field programmable gate arrays (FPGAs) allow users to construct their own ICs. Currently, most digital silicon neurons and neural networks are dedicated processors for classical non-spiking neuron models and network models or simulators for spiking neuron models [3][4]. In this paper, we propose a new concept for digital silicon neuron design. By applying a mathematical-model-based design policy to digital circuits, we can design a digital spiking silicon neuron (DSSN) that possesses properties supported by theoretical models for biological neurons with compact circuitry. It is intended to be a constituent element for digital spiking neural networks that operate in real time, and has the potential to be an alternative to analog silicon neurons.

In the next section, we introduce the concept and design a model for an elemental DSSN model. In the third section, we report complex behaviors observed in a GJ-coupled network of the model to demonstrate that it has potential to exhibit rich dynamical behaviors comparable to analog silicon neurons. Finally, we will briefly refer to the implementation of our model.

2 Concept and Model of DSSN

The basic concept of DSSN is a dedicated system for solving differential equations of spiking neuron models. The hardware is an ordinary arithmetic circuit used for numerical integration. The keypoint lies in the designation of system equations. To implement a solver for a biological neuron model, massive hardware resources are required because most of the models are described using complex differential equations. We can avoid this problem by designing the system equations by a mathematical-model-based design method. Here, equations that have topological structures in their phase portrait similar to some theoretical models are designed first, and then, their parameters are tuned on the basis of bifurcation analysis.

The most fundamental property of biological neurons is the generation of action potentials. Neural excitability is another property, which classifies neurons according to the firing frequency at the onset of repetitive firing induced by a sustained stimulus that is increased gradually. Neurons with Class I excitability begin to fire repetitively with arbitrarily zero frequency, whereas those with Class II excitability begin with a non-zero frequency. Theoretical studies have elucidated the mathematical structure behind these properties by utilizing the phase portrait and bifurcation analyses [5][6]. These studies not only succeeded in explaining the mechanism of various properties of action potentials but also showed that saddle-node on invariant circle and Hopf bifurcations of a stable equilibrium corresponding to a resting state produce Class I and II excitabilities, respectively.

Based on the above mentioned information, we designed a model for elemental DSSN, whose equations are:

$$\frac{dv}{dt} = \frac{\phi}{\tau}(f(v) - n + I_0 + I_{stim}) \quad \text{and} \tag{1}$$

$$\frac{dn}{dt} = \frac{1}{\tau}(g(v) - n), \tag{2}$$

where

$$f(v) \equiv \begin{cases} a_n(v + b_n)^2 - c_n & \text{when } v < 0, \\ -a_p(v - b_p)^2 + c_p & \text{when } v \geq 0, \end{cases} \tag{3}$$

$$g(v) \equiv \begin{cases} k_n(v - p_n)^2 + q_n & \text{when } v < r, \quad \text{and} \\ k_p(v - p_p)^2 + q_p & \text{when } v \geq r. \end{cases} \tag{4}$$

Parameters a_x, b_x, c_x, k_x, p_x, q_x, and r determine the form of the nullclines, and ϕ and τ are time constant parameters (for $x = n$ and p). These equations were designed so that they could reproduce topological structures in the phase plane of the Morris-Lecar model [7]. It is one of the simplest models that show both Class I and II excitabilities depending on parameter sets. In our equations, multiplication operations between variables are significantly reduced because they consume large hardware resources (cubic curve is constructed by two quadratic curves). Note that multiplication between a parameter and a variable can be implemented by the shift operation if we select the parameter from $\{2^n | n \in \mathbf{Z}\}$.

We selected the parameter values so that our model reproduced the phase plane structure in the Morris-Lecar model in the Class I and II modes (see Appendix for values). In Fig. 1 (a), (b), and (c), the phase planes for Class I and II modes of our DSSN model are shown. These topological structures in the phase plane were proved to exhibit fundamental properties of action potentials by theoretical studies [5]. By selecting the descending limb of the n-nullcline, we can select the neural excitability classes. Fig. 2 (a), (b), and (d) show the results of the bifurcation analysis. They confirmed that our parameter sets produced the expected classes of excitability. Additionally, in the Class II mode, our DSSN model produced chaotic re-

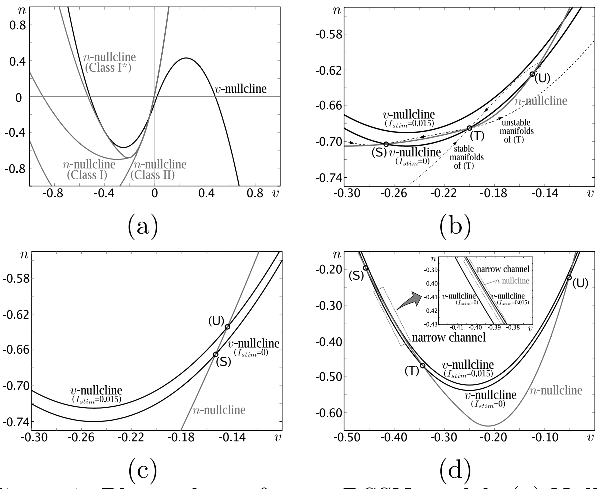


Figure 1: Phase planes for our DSSN model. (a) Nullclines for Class I, II, and I* modes. (b), (c), and (d) Closeup around critical structures for Class I, II, and I* modes, respectively. (S) is a stable equilibrium (resting state); (T), a saddle; and (U), an unstable equilibrium. Stimulus current (I_{stim}) shifts the v-nullcline up, resulting in repetitive oscillation. In the Class I* mode, v- and n-nullclines are very close to each other (narrow channel) around $v = -0.4$.

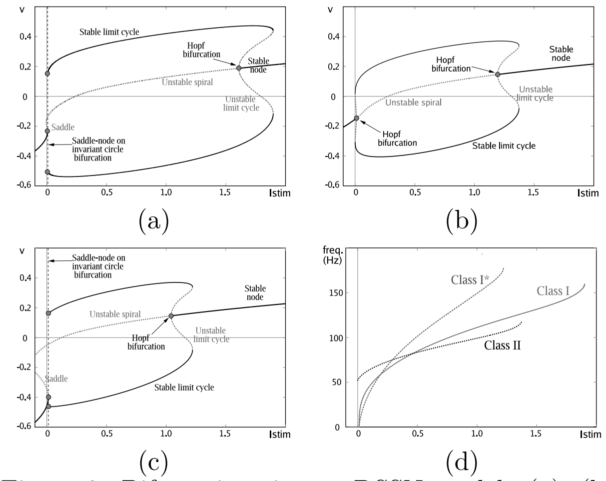


Figure 2: Bifurcations in our DSSN model. (a), (b), and (c) Bifurcation of variable v. Limit cycles are represented by the maximum and minimum values. (d) Bifurcation of frequency for limit cycles.

sponses against repetitive pulse stimuli similar to those in biological neurons [8] and an analog silicon neuron [1]. This indicates the potential of our DSSN model to exhibit rich dynamical behavior.

3 GJ-coupled DSSN models

A gap junction (GJ) is a type of physical connection between neuronal cells, which is electrically equivalent to linear resistance. Various regions in the brain are known to contain numerous GJs, whose functions have attracted considerable interest from many researchers. In 2004, Fujii and Tsuda [9] found that some of Class I neuron models exhibited characteristic chaotic behaviors when interconnected via GJs. They classified these neuron models as Class I* and indicated that the following two conditions in phase plane structures supported this class. The first condition is the existence of a phase plane structure called narrow channel, which means that the nullclines for the membrane and ionic conductance variables remain close to each other in certain regions. The second one requires the unique crosspoint of the above two nullclines to be an unstable spiral equilibrium. These conditions are satisfied when a weak stimulus current is given to a neuron model with nullclines having specific shapes. In Fig. 1 (a) and (d), we show the phase planes for our DSSN model in the Class I* mode (see Appendix for the parameter set). A narrow channel is formed in the region shown by the dashed square, when $I_{stim} = 0.015$. Bifurcation analysis demonstrated that our DSSN model belonged to Class I in this mode (Fig. 2 (c) and (d)).

We calculated the maximum lyapunov exponent for the GJ-coupled network of DSSN models to demonstrate that our model with this parameter setting could operate as a Class I* neuron. This network is composed of a one-dimensional array of 20 DSSN models interconnected with two nearest neighbors via GJs (see Fig. 3). The current through GJ applied to the i-th DSSN model (I_{gj}^i) is given as follows:

$$I_{gj}^i = (v_{i+1} + v_{i-1} - 2v_i)/R_{gj}, \qquad (5)$$

where i is the index number for DSSN model (from 1 to 20); v_i, v for the i-th DSSN model ($v_0 \equiv v_1$ and $v_{21} \equiv v_{20}$); and R_{gj}, the resistance of the GJ. These currents are added to I_{stim} for each neuron. In Fig. 4 (a), the maximum lyapunov exponents for the Class I, II, and I* modes are shown. It is large in the Class I* mode when R_{gj} is approximately between 2.5 and 20, whereas it is approximately zero, independent of R_{gj} in the Class I and II modes. In the Class I* mode,

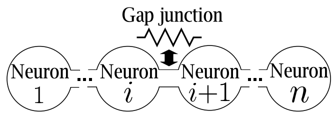


Figure 3: One dimensional GJ-coupled network.

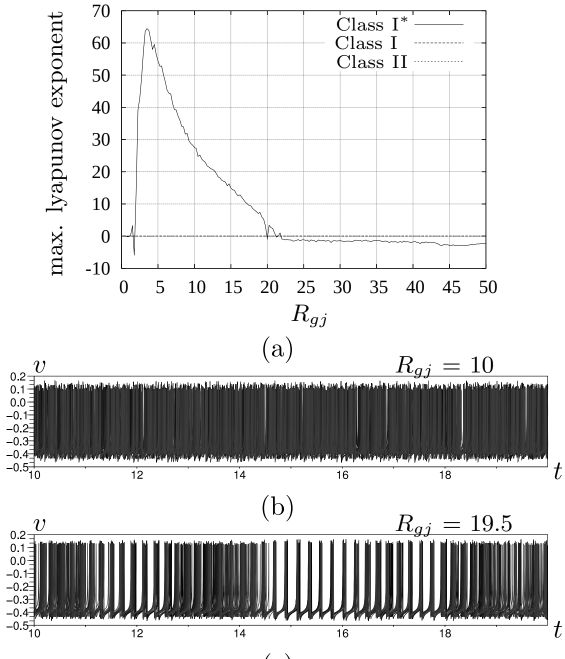
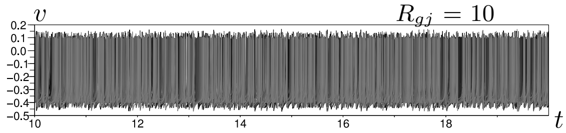


Figure 4: Behaviors of a GJ-coupled network of 20 DSSN models. (a) Maximum lyapunov exponents in Class I, II, and I* modes. Large values are obtained only in the Class I* mode. (b) and (c) Superimposed waveforms for v in the Class I* mode when R_{gj} is 10 and 19.5, respectively. Intermittently chaotic behavior is observed when R_{gj} in (c), when the maximum lyapunov exponent is relatively small.

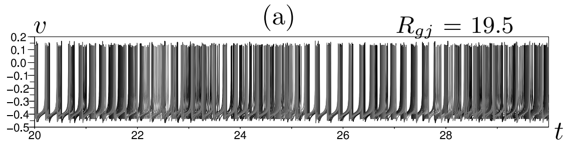
the network is synchronous when R_{gj} is sufficiently small, becomes chaotic (Fig. 4 (b)) as R_{gj} increases, and then returns to the synchronous state when R_{gj} is sufficiently large. We observed intermittently chaotic behaviors for the R_{gj} values during the transition from the chaotic to synchronous states (Fig. 4 (c)). These behaviors are consistent with that of an analog silicon neuron model we designed in the previous work [10].

4 Concluding remark

In the previous sections, we proposed a DSSN model and indicated that it has potential to reproduce the fundamental characteristics of neurons. We also showed that our model could exhibit complex behav-



(a)



(b)

Figure 5: Behaviors of a GJ-coupled network of 20 DSSNs. (a) $R_{gj} = 10$. (b) $R_{gj} = 19.5$. Numerical integration was performed by Euler's method and precision was 28 bit fixed-point. Chaotic and intermittently chaotic behavior that are consistent with model simulation (Fig. 4 (b) and (c)) were observed.

ior with monotonic stimulus in GJ-coupled networks.

We are planning to implement our DSSN model in FPGA devices and are performing numerical simulations. Reasonable results are obtained when we select Euler's method and fixed-point expressions for numerical integration. For example, we observe complex behaviors in the Class II mode of the DSSN model when it is stimulated repetitively (not shown), and chaotic and intermittently chaotic behaviors in a GJ-coupled network of 20 DSSNs (Fig. 5) that is consistent with the accurate simulation of the model. These results are obtained with a time step of 1.0^{-5} for Eular's method and 28 bit fixed-point expression. Numerical precision is an important factor that affects the circuit size and operation speed. However, it is quite a difficult problem to give objective criteria for determining the required precision. This is because interconnected neurons and even a single neuron are complex systems and the assessment of their dynamical behavior is a complicated subject.

For the implementation of a digital spiking silicon neural network, a silicon synapse circuit is required. We will design this circuit in the near future by using a mathematical-model-based method referring to kinetic models for biological synapses.

Acknowledgements

This study is supported by Aihara Complexity Modelling Project, ERATO, JST, Japan.

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Appendix

A Common parameters:

Par.	Value	Par.	Value	Par.	Value
a_n	8.0	b_n	0.25	c_n	0.5
a_p	8.0	b_p	0.25	c_p	0.5
k_p	16.0	p_p	–0.2125	q_p	–0.6875

B Class I parameters:

Par.	Value	Par.	Value	Par.	Value
k_n	2.0	p_n	–0.3	q_n	–0.705
ϕ	1.0	τ	0.003	r	–0.2
I_0	–0.205				

C Class II parameters:

Par.	Value	Par.	Value	Par.	Value
k_n	4.0	p_n	–0.55	q_n	–1.295
ϕ	0.6	τ	0.003	r	–0.1
I_0	–0.24				

D Class I* parameters:

Par.	Value	Par.	Value	Par.	Value
k_n	4.0	p_n	–0.1	q_n	–0.755
ϕ	0.6	τ	0.002	r	–0.25
I_0	–0.25				