A Mathematical Model of Planning in the Prefrontal Cortex

Makito Oku[*] Kazuyuki Aihara[†,‡,*]

[*]Department of Mathematical Informatics, Graduate School of Information Science and Technology, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

[†]Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan

[‡]Aihara Complexity Modelling Project, ERATO, Japan Science and Technology Agency (JST), 3-23-5-201 Uehara, Shibuya-ku, Tokyo 151-0064, Japan

Abstract

The prefrontal cortex is involved in a lot of complex cognitive behaviours, such as problem solving, planning, reasoning, and decision making. However, the biological mechanisms of these computations are not clear. To understand the mechanisms, we theoretically consider the experimental result of path-planning task by Mushiake et al., using a mathematical model which we name the potential network model. The result of simulations shows that our model is able to take a correct path in most trials regardless of goal positions and block patterns. Our model also reproduces the characteristics of neurons' activities both in the prefrontal cortex and the primary motor cortex. This study indicates that although the potential network model is abstract, it can be useful for modelling higher brain functions.

1 Introduction

Planning is one of the most complex cognitive functions of human brain. It includes a lot of aspects, such as selection of future actions, anticipation of future events that will occur as a result of those actions, temporal maintenance of sequence of those events, evaluation of the sequence, generation of new strategy if needed, and memorisation of finally decided plan so that the planner will take actions according to the plan. Planning is also related to some major problems about brain, such as working memory,[1] cognitive control,[2] mental imagery,[3] and reward systems.[4]

A lot of studies from neuropsychology and brain imaging show that planning is related to the prefrontal cortex (PFC).[5,6] The PFC has thought to be involved in the executive control of behaviour, and planning is an important aspect of the executive control. Here the question arises: what role does the PFC take during planning of multistep behaviours?

To answer this question, we made a mathematical model of a path-planning task. The path-planning task was a task that required multiple stepwise movements of a cursor within a maze to reach a goal.[7–9] Fig. 1 shows the maze used in the task. Players of this task started from the centre of the maze and tried to reach the goal avoiding obstacles. If the player of the task was monkey, it moved its arm to move around in the maze. If the player was human, the player pushed buttons. The rule which assigned muscular-movements to cursor-movements was replaced for every several trials. By recording of neuron spikes in monkeys' brains, it was shown that many PFC neurons selectively fired when specific cursor-movement was on specific step during both the preparatory period and movement execution.[7]

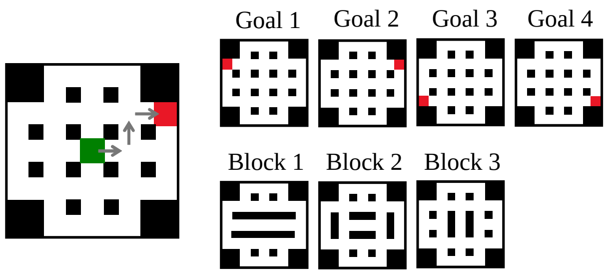

Fig. 1. Path-planning task

On the other hand, Bachmann et al.[10] proposed a mathematically abstract model of neural network. They considered a firing pattern of Hopfield network as a point in high dimensional state space, and attractors as points in the space. This model had no spurious attractors. Furthermore, basins of attractors were well-defined. In this paper, we connected simplified neural assemblies similar to Bachmann's model, considered interactions among them, and named it the potential network model.

2 Potential network model

The potential network model is a network model where each node changes its state continuously according to its potential. The potential is influenced by connected nodes. Suppose a network consists of N nodes. Each node has a state $\boldsymbol{x}_i(t) \in \mathbf{R}^m (i = 1, \cdots, N)$ and a potential $U_i (i = 1, \cdots, N)$. The state of each node is changing on the potential.

$$\frac{\mathrm{d}\boldsymbol{x}_i(t)}{\mathrm{d}t} = -\alpha \nabla U_i(t) \quad (i = 1, \cdots, N).$$

Each node has several fixed points $\boldsymbol{s}_i^1, \cdots, \mathbf{s}_i^{L_i} \in \mathbf{R}^m (i = 1, \cdots, N)$. These points are called attractors because they could be local minimum potentials. Fig. 2 shows an example. When node A's state is near enough to one of its attractors, then the node affects next node B's potential so that one of attractors in node B gets stable. For simplicity, each node's potential is formed by linear summation of all attractors of connected nodes.

$$U_j(t) = \sum_{i \in P_j} \sum_{k=1}^{L_i} f(\boldsymbol{x}_i, \boldsymbol{s}_i^k) U_{ij}^k \quad (j = 1, \cdots, N).$$

Here U_{ij}^k is the potential from k-th attractor of i-th node to j-th node and P_j is a set of nodes connected to j-th node.

f is a closeness function. It is a function of distance between the state of the node and each attractor. In this paper, we use Gaussian function as the closeness function.

$$f(\boldsymbol{x}_i, \boldsymbol{s}_i^k) = \beta \exp\Big(-\frac{\|\boldsymbol{x}_i - \boldsymbol{s}_i^k\|^2}{\sigma^2}\Big).$$

3 Simulations of path-planning task

3.1 Setup of simulations

We constructed a potential network model for the path-planning task (Fig. 3). We assumed that cursor-movements for three steps were represented separately in the PFC. 1st, 2nd, and 3rd nodes were corresponding to the three steps and each of them had four attractors corresponding to four directions (up, down, left, and right). Goal and block information were represented in goal node and block node respectively. These two nodes biased the three nodes to select appropriate path. Step node made strong potential in the cursor node to inhibit action execution during preparatory period. When cue signals were displayed, this node also reactivated corresponding node to trigger action execution.

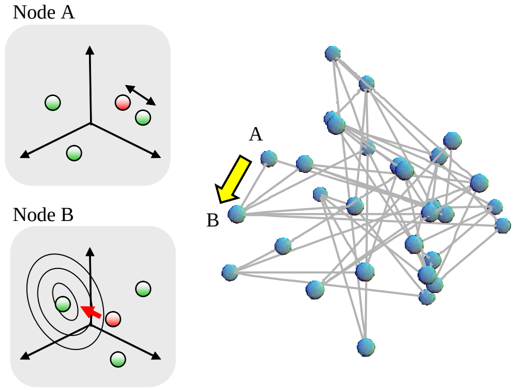

Fig. 2. Schematic diagram of the potential network model

The forms of potentials were mixture of Gaussian functions. They biased an attractor or several attractors. There was a potential with its centre at the origin of the state space, which prevented the state from approaching any attractors.

In the original experiment, assignment rule between cursor-movements and arm-movements was changed for every several trials. Rule, arm, and motion nodes were expected to perform the translation from cursor-movements to arm-movements. In this paper, however, we did not add these three nodes.

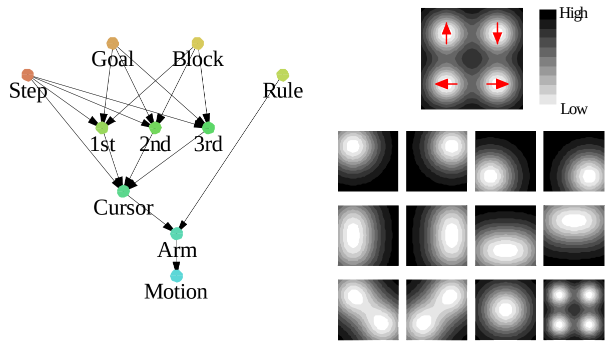

Fig. 3. Potential network model for path-planning and examples of potential patterns

3.2 Result of simulations

Fig. 4 shows an example of state transition of 1st, 2nd, 3rd, and cursor nodes in block 3 condition. Temporal sequence of events during the task was the same to the original experiment. Two shadowed regions correspond to goal displaying period and block displaying period. Three black bars indicate movement executions for each step. During preparatory period, states of 1st, 2nd, and 3rd nodes were biased by goal node and block node to approach attractors corresponding to proper cursor movements. In contrast, state of cursor node was under strong inhibitive potential made by step node so that it stayed far from any attractors. After preparatory period, states of the three nodes fluctuated in the state space because of noise. So they left from attractors. When step node reactivated the three nodes, their states approached attractors again, which changed the state of cursor node.

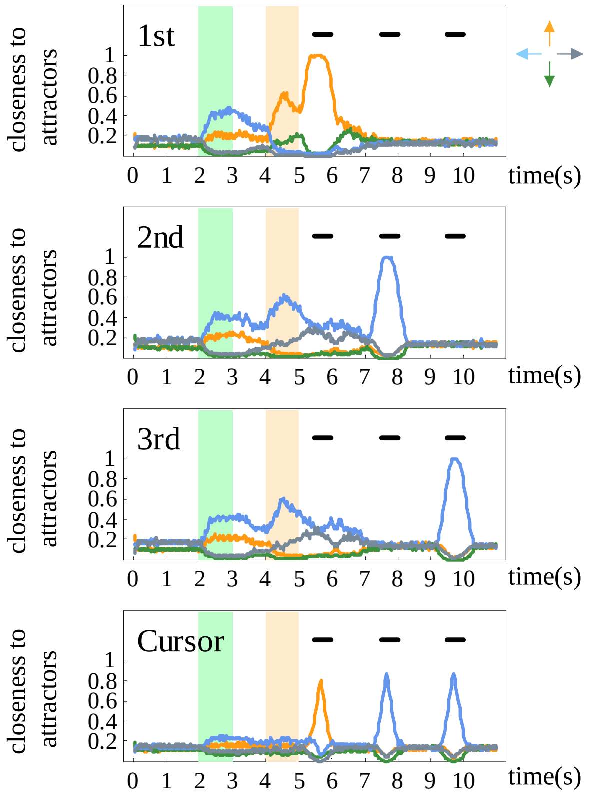

Fig. 4. An example of time series of state transition

Fig. 5 shows the probability of cursor-movement selection of each step. In most cases this model chose a correct path. Some errors were seen in the 3rd step in all conditions. The state of nodes seemed to fluctuate getting away from the basins of attractors. Another errors were seen in the 1st step of block 3 condition, which was failure to overwrite reflective response.

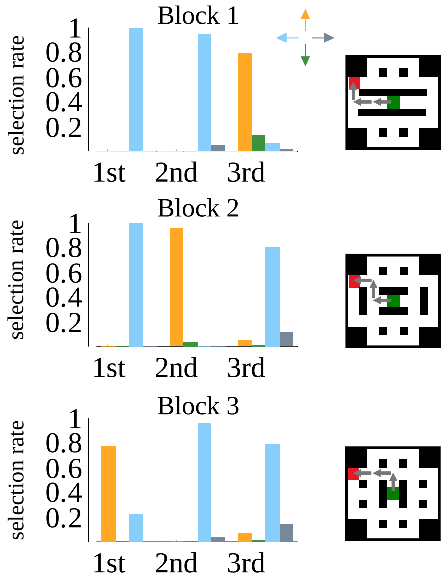

Fig. 5. Cursor-movement selection rate

4 Discussion

Medium of modelling of cognitive functions In general, there are two streams of modelling of human's complex cognitive behaviours: production system[11,12] and connectionist model.[13] In addition, there are some hybrid studies.[14,15] The production system is good at highly complex problems. However, its biological basis is not clear. On the other hand, connectionist model has biological background and it has flexible performance. However, this approach has a risk to be redundant and to have complexity irrelevant to the essence of computation.

The potential network model we proposed here was intrinsically based on connectionist model. At the same time, it was easy to implement if-then rules in this model because each attractor's basin and effect were well-defined. Therefore, this model had advantages of both sides.

Role of prefrontal cortex There are many hypotheses about the role of the PFC.[16–19] Among them, the *cognitive control hypothesis* is broadly accepted. This study was consistent with the hypothesis. In the framework of cognitive control hypothesis, the PFC sends bias signal to posterior cortex to overwrite reflective, innate, or prepotent responses.[2] In our model, goal and block node made potentials to the three nodes corresponding to each step, biasing appropriate action.

Other issues involved in planning Our model for the path-planning task concerned only limited aspects of planning. For example, the players of the path-planning task were well-trained, so they could accurately find correct path immediately. In fact, neuron spike data from monkeys' brains suggested that 1st, 2nd, and 3rd cursor-movement-selective neurons in the PFC started to fire simultaneously during the preparatory period.[7] Accordingly, stepwise rehearsal of event sequence during the preparatory period was not performed in our model. In Addition, our model did not include evaluation system and learning mechanism, which is our future works.

5 Summary

To understand the mechanism of planning in the prefrontal cortex, We theoretically considered the experimental result of path-planning task by Mushiake et al., using the potential network model. The result of simulations showed that our model was able to take a correct path in most trials regardless of goal positions and block patterns. Our model also reproduced the characteristics of neurons' activities both in the PFC and the primary motor cortex. This study indicated that although our model was abstract and concerned only limited aspects of planning, it could be useful for modelling higher brain functions.

Acknowledgements

This study is partially supported by Grant-in-Aid No. 17022012 for Scientific Research on Priority Areas System study on higher-order brain functions from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

References

[1] Baddeley A (2003), Working memory: looking back and looking forward. *Nat. Rev. Neurosci.* 4(10):829–839

[2] Miller EK (2000), The prefrontal cortex and cognitive control. *Nat. Rev. Neurosci.* 1(1):59–65

[3] Kosslyn SM (2005), Mental images and the brain. *Cogn. Neuropsychol.* 22(3–4):333–347

[4] Schultz W, Dayan P, Montague PR (1997), A neural substrate of prediction and reward. *Science* 275(5306):1593–1599

[5] Unterrainer JM, Owen AM (2006), Planning and problem solving: from neuropsychology to functional neuroimaging. *J. Physiol. Paris* 99(4–6):308–317

[6] Goel V, Grafman J (2000), Role of the right prefrontal cortex in ill-structured planning. *Cogn. Neuropsychol.* 17(5):415–436

[7] Mushiake H, Saito N, Sakamoto K, et al. (2006), Activity in the lateral prefrontal cortex reflects multiple steps of future events in action plans. *Neuron* 50(4):631–641

[8] Mushiake H, Saito N, Sakamoto K, et al. (2001), Visually based path-planning by Japanese monkeys. *Cogn. Brain Res.* 11(1):165–169

[9] Mushiake H, Saito N, Furusawa Y, et al. (2002), Orderly activations of human cortical areas during path-planning task. *Neuroreport* 13(4):423–426

[10] Bachmann CM, Cooper LN, Dembo A, et al. (1987), A relaxation model for memory with high storage density. *Proc. Natl. Acad. Sci. U.S.A.* 84(21):7529–7531

[11] Anderson JR, Bothell D, Byrne MD, et al. (2004), An integrated theory of the mind. *Psychol. Rev.* 111(4):1036–1060

[12] Kieras DE, Meyer DE (1997), An overview of the EPIC architecture for cognition and performance with application to human-computer interaction. *Human-Computer Interaction* 12(4):391–438

[13] Dehaene S, Changeux JP (1997), A hierarchical neuronal network for planning behaviour. *Proc. Natl. Acad. Sci. U.S.A.* 94(24):13293–13298

[14] Polk TA, Simen P, Lewis RL, et al. (2002), A computational approach to control in complex cognition. *Cogn. Brain Res.* 15(1):71–83

[15] Newman SD, Carpenter PA, Varma S, et al. (2003), Frontal and parietal participation in problem solving in the Tower of London: fMRI and computational modeling of planning and high-level perception. *Neuropsychologia* 41(12):1668–1682

[16] Wood JN, Grafman J (2003), Human prefrontal cortex: processing and representational perspectives. *Nat. Rev. Neurosci.* 4(2):139–147

[17] Miller EK, Cohen JD (2001), An integrative theory of prefrontal cortex function. *Annu. Rev. Neurosci.* 24:167–202

[18] Duncan J (2001), An adaptive coding model of neural function in prefrontal cortex. *Nat. Rev. Neurosci.* 2(1):820–829

[19] Fuster JM (2001), The prefrontal cortex–an update: time is of the essence. *Neuron* 30(2):319–33