A Relationship between the Accepting Powers of Alternating Finite Automata and Nondeterministic On-Line Tessellation Acceptors on Four-Dimensional Input Tapes

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Abstract

Recently, due to the advances in dynamic image processing, computer animation, and so forth, it has become increasingly apparent that the study of four-dimensional pattern processing should be very important. Thus, we think that the research of four-dimensional automata as the computational model of four-dimensional information processing has been significant. During the past about five years, automata on a four-dimensional tape have been proposed and several properties of such automata have been obtained. One model is the four-dimensional alternating finite automaton (4-AFA) which is an alternating version of a four-dimensional finite automaton, and another is the four-dimensional nondeterministic on-line tessellation acceptor (4-NOTA) which is a natural extension of the three-dimensional nondeterministic on-line tessellation acceptor to four dimensions. In this paper, we mainly investigate a relationship between the accepting powers of 4-AFA's and 4-NOTA's.

Key Words : alternation, finite automaton, four-dimensional input tape, nondeterminism, on-line tessellation acceptor

1 Introduction and Preliminaries

The question of whether processing four-dimensional digital patterns is much difficult than two- or three-dimensional ones is of great interest from the theoretical and practical standpoints. In recent years, due to the advances in many application areas such as dynamic image processing, computer animation, and so on, the study of four-dimensional pattern processing has been of crucial importance. Thus, we think that the research of four-dimensional automata as the computational model of four-dimensional pattern processing has been meaningful. This paper mainly deals with *four-dimensional alternating finite automaton* (4-AFA) and *four-dimensional nondeterministic on-line tessellation acceptor* (4-NOTA), and investigate some results concerning a relationship between the accepting powers of 4-AFA's and 4-NOTA's [1,4].

Let $\sum$ be a finite set of symbols. A *four-dimensional tape* over $\sum$ is a four-dimensional rectangular array of elements of $\sum$. The set of all four-dimensional tape over $\sum$ is denoted by $\sum^{(4)}$. Given a tape $x \in \sum^{(4)}$, for each $j(1 \leq j \leq 4)$, we let $l_j(x)$ be the length of x along the jth axis. When $1 \leq i_j \leq l_j(x)$ for $j(1 \leq j \leq 4)$, let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4). Furthermore, we define $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 4)$, as the four-dimensional tape y satisfying the following:

(i) for each $j(1 \leq j \leq 4)$, $l_j(y) = i'_j - i_j + 1$;

(ii) for each r_1, r_2, r_3, r_4 $(1 \leq r_1 \leq l_1(y), 1 \leq r_2 \leq l_2(y), 1 \leq r_3 \leq l_3(y), 1 \leq r_4 \leq l_4(y))$, $y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 - 1,$

r_2+i_2-1, r_3+i_3-1, r_4+i_4-1). We let the input tapes, through this paper, be restricted to ones which each sidelength is eguivalent in order to increase the theoretical interest,as shown in Fig.1.

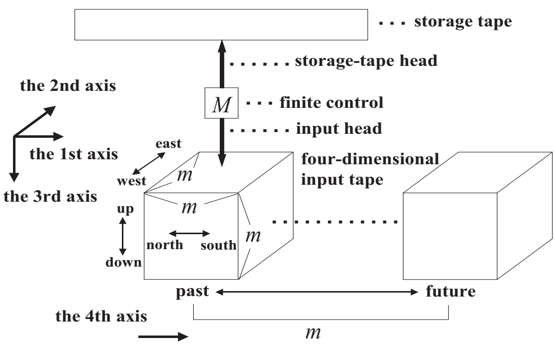

Fig. 1: Four-dimensional input tape.

A 4-AFA M is a four-dimensional finite automaton whose state set is partitioned into *universal* and *existential* states. The machine M has a read-only four-dimensional tape. A *step* of M consists of reading one symbol from the input tape, moving the input head in specified direction $d\in${east, west, south, north, up, down, future, past, no more}, and entering a new state, in accordance with the next-move relation. A *seven-way four-dimensional alternating finite automaton* (SV4-AFA) is a 4-AFA whose input head can move in only seven directions — east, west, south, north, up, down, or future.

A 4-NOTA M is an infinite mesh-connected four-dimensional array of cells. Each cell of the four-dimensional array consists of a nondeterministic finite-state machine. The nondeterministic finite-state machines are all identical. M decides whether a four-dimensional tape is accepted or not by on-line fashions. For more details of the definitions of 4-AFA and 4-NOTA, see [4] and [1], respectively.

Let T(M) be the set of four-dimensional tapes accepted by a machine M, and let $\mathcal{L}$[4-AFA]={T|T=T(M) for some 4-AFA}. $\mathcal{L}$[SV4-AFA] and $\mathcal{L}$[4-NOTA] are defined in the same way as $\mathcal{L}$[4-AFA]. Further, for a set T(M) of four-dimensional tapes accepted by a machine M, the complementation of T(M) is denoted by $\bar{\mathrm{T}}$(M).

2 Results

We first investigate a relationship between the accepting powers of 4-AFA's and 4-NOTA's.

Lemma 2.1. $\mathcal{L}$[4-AFA]$\not\subseteq\mathcal{L}$[4-NOTA].

Proof : We consider the four-dimensional tape embedding of directed bipartite graphs with equal number of vertices on both sides [2,3]. Let $\sum$={v_i, e, w, s, n, u, d, f, p, +, x, 0} be a finite set of symbols used for the embedding. We use the following embedding rule. The symbol v_i represents the ith vertex for each i ($1\leq i\leq 2n$), symbol + means an intersection of two edges (i.e., where they join or split), symbol x is for a cross-over of two-edges, symbol 0 represents a blank space, and symbol e, w, s, n, u, d, f, and p are the symbols needed to form eastward, westward, southward, northward, upward, downward, future, and past edges, respectively. Let $\underline{P}$ be a four-dimensional tape embedding of a directed bipartite graph with k=2m(m+1)-m vertices on both sides. The size of $\underline{P}$ will be (4m+3)$\times$(4m+3)$\times$(4m+3)$\times$(4m+3) (including the boundary symbols). The (2m+2)th plane of some two cubes of $\underline{P}$ defines 2k vertices of a bipartite graph, where the westmost k vertices on the (2m+2)th plane of one cube form one group and eastmost k vertices on the (2m+2)th plane of the other cube form the other group. The 2k v_i's are placed such that there are blanks separating the first m vertices of one cube from the second m vertices of the other cube on each odd-numbered row, and there is a blank between consecutive v_i's in both the eastmost and weatmost groups. An example of such embedding is given in Fig.2. Consider language L_1={P|P$\in\sum^{(4)}$ and $\underline{P}$ is a four-dimensional tape embedding of some acyclic directed bipartite graph with equal number of vertices on both sides}. We can show that L_1 can be accepted by a 4-AFA but not by any 4-NOTA. □

Lemma 2.2. *For every* 4-AFA A, $\bar{\mathrm{L}}$(A) *is accepted by a* 4-NOTA.

Proof : Let A be a 4-AFA. Define the complement 4-AFA of A to be $\bar{\mathrm{A}}$. That is, $\bar{\mathrm{A}}$ is obtained by swapping the universal and existential states, and the accepting and nonaccepting states of A. Note that in general, $\bar{L}(A) \neq L(\bar{A})$, since A may reject an input by entering an infinite loop. We construct a 3-NOTA M to accept $\bar{L}(A)$. Let x be an input pattern. Given x, M tries to guess and verify the existence of a (possibly infinite) computation tree of $\bar{A}$ on x whose leaves are all labeled with accepting configurations. Let π denote the computation tree of $\bar{A}$ on x that M will guess. Let $R(i_1, i_2, i_3, i_4)$ denote the set of all states of $\bar{A}$ when its input head is at the (i_1, i_2, i_3, i_4) cell, $1 \leq i_1 \leq l_1(x)$, $1 \leq i_2 \leq l_2(x)$, $1 \leq i_3 \leq l_3(x)$, $1 \leq i_4 \leq l_4(x)$, in the guessed computation tree π. For each $q \in R(i_1, i_2, i_3, i_4)$, call$(x, (i_1, i_2, i_3, i_4), q)$ a configuration (of $\bar{A}$) represented by q. Generally, the (i_1, i_2, i_3, i_4) cell of M operates as follows. It receives the sets $R(i_1-1, i_2, i_3, i_4)$ and $R(i_1, i_2, i_3, i_4)$ from the (i_1-1, i_2, i_3, i_4) cell, the sets $R(i_1, i_2-1, i_3, i_4)$ and $R(i_1, i_2, i_3, i_4)$ from the (i_1, i_2-1, i_3, i_4) cell, the sets $R(i_1, i_2, i_3-1, i_4)$ and $R(i_1, i_2, i_3, i_4)$ from the (i_1, i_2, i_3-1, i_4) cell, and the sets $R(i_1, i_2, i_3, i_4-1)$ and $R(i_1, i_2, i_3, i_4)$ from the (i_1, i_2, i_3, i_4-1) cell, and verifies that $R(i_1, i_2, i_3, i_4)$ is consistent with the neighboring sets $R(i_1-1, i_2, i_3, i_4)$, $R(i_1, i_2-1, i_3, i_4)$, $R(i_1, i_2, i_3-1, i_4)$, $R(i_1, i_2, i_3, i_4-1)$, $R(i_1+1, i_2, i_3, i_4)$, $R(i_1, i_2+1, i_3, i_4)$, $R(i_1, i_2, i_3+1, i_4)$, $R(i_1, i_2, i_3, i_4+1)$. That is, the following conditions must hold : (a) none of the members of $R(i_1, i_2, i_3, i_4)$ represents a terminating nonaccepting configuration; (b) if $q \in R(i_1, i_2, i_3, i_4)$ and q is universal, then all immediate successors of the configuration $(x, (i_1, i_2, i_3, i_4), q)$ are represented by the states contained in $R(i_1-1, i_2, i_3, i_4) \bigcup R(i_1, i_2-1, i_3, i_4) \bigcup R(i_1, i_2, i_3-1, i_4) \bigcup R(i_1, i_2, i_3, i_4-1) \bigcup R(i_1+1, i_2, i_3, i_4) \bigcup R(i_1, i_2+1, i_3, i_4) \bigcup R(i_1, i_2, i_3+1, i_4) \bigcup R(i_1, i_2, i_3, i_4+1) \bigcup R(i_1, i_2, i_3, i_4)$; and (c) if $q \in R(i_1, i_2, i_3, i_4)$ and q is existential, then at least one of the immediate successors of the configuration $(x, (i_1, i_2, i_3, i_4), q)$ is represented by the states contained in $R(i_1-1, i_2, i_3, i_4) \bigcup R(i_1, i_2-1, i_3, i_4) \bigcup R(i_1, i_2, i_3-1, i_4) \bigcup R(i_1, i_2, i_3, i_4-1) \bigcup R(i_1+1, i_2, i_3, i_4) \bigcup R(i_1, i_2+1, i_3, i_4) \bigcup R(i_1, i_2, i_3+1, i_4) \bigcup R(i_1, i_2, i_3, i_4+1) \bigcup R(i_1, i_2, i_3, i_4)$. Also, the (i_1, i_2, i_3, i_4) cell passes the sets $R(i_1, i_2, i_3, i_4)$ and $R(i_1+1, i_2, i_3, i_4)$ to the (i_1+1, i_2, i_3, i_4) cell, the sets $R(i_1, i_2, i_3, i_4)$ and $R(i_1, i_2+1, i_3, i_4)$ to the (i_1, i_2+1, i_3, i_4) cell, and so on. It addition, the(1,1,1,1) cell makes sure that R(1,1,1,1) contains q_0.

The 4-NOTA M constructed above verifies that for every configuration in the guessed tree π, either it is a terminating accepting configuration or it is nonterminating and all of its immediate successor configurations exist. It is easy to see that, if x is rejected by A, then there exists a (possibly infinite) computation tree of $\bar{A}$ on x whose leaves are all labeled with accepting configurations, and vice versa. Hence, M accepts $\bar{L}(A)$. □

Lemma 2.3. $\mathcal{L}$[4-NOTA]$\not\subseteq\mathcal{L}$[3-AFA].

Proof : Suppose that $\mathcal{L}$[4-NOTA]$\subseteq\mathcal{L}$[4-AFA]. Let T_1 be the same language that we considered in the proof of Lemma 2.1. From Lemma 2.2 and hypothesis, $\bar{L}_1$ is accepted by a 4-AFA. By Lemma 2.2, L_1 is accepted by a 4-NOTA. But L_1 is not accepted by any 4-NOTA, as shown in the proof of Lemma 2.1. This is a contradiction. Hence, $\mathcal{L}$[4-NOTA]$\not\subseteq\mathcal{L}$[3-AFA]. □

From Lemmas 2.1 and 2.3, we have the following result.

Theorem 2.1. $\mathcal{L}$[4-AFA] and $\mathcal{L}$[4-NOTA] are incomparable.

Next, we investigate a relationship between the accepting powers of SV4-AFA's and 4-NOTA's.

Lemma 2.4. $\mathcal{L}$[SV4-AFA] *is closed under complementation.*

Proof : Let A be an SV4-AFA and $\bar{A}$ be the complement of A as in the proof of Lemma 2.2. By using the same idea as in the proof of Theorem 4.4 in [3], we can construct an SV4-AFA A' from $\bar{A}$ such that $L(A') = \bar{L}(A)$. □

Theorem 2.2. $\mathcal{L}$[FV4-AFA]$\subsetneq\mathcal{L}$[4-NOTA].

Proof : From Lemma 2.4, $\mathcal{L}$[SV4-AFA] is closed under complementation. The inclusion follows from Lemma 2.2. That it is proper follows since $\mathcal{L}$[SV4-AFA]$\subseteq\mathcal{L}$[4-AFA] and $\mathcal{L}$[4-AFA] is incomparable with $\mathcal{L}$[4-NOTA]. □

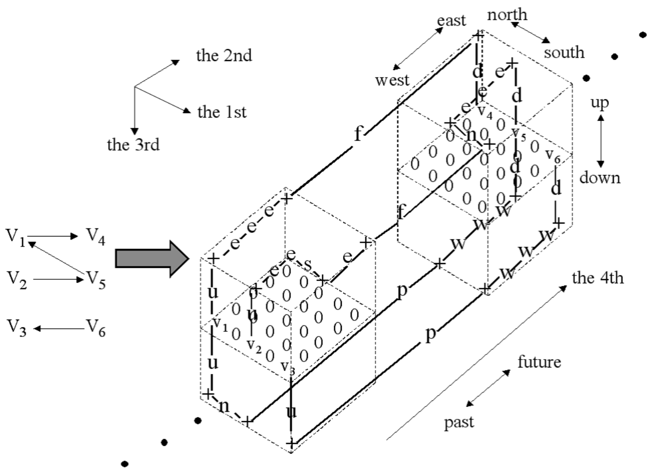

Fig. 2: An example of embedding (m=1).

3 Conclusion

This paper mainly investigated a relationship between the accepting powers of alternating finite automata and nondeterministic on-line tessellation acceptors on four- dimensional input tapes. It is interesting to investigate closure properties about their four-dimensional automata. We will treat this problem in further papers.

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