

# Networks on Earth from the climate data

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## Abstract

We extract scale free networks from air temperature records covering whole earth of NCEP-NCAR reanalysis data. To do this, we use the phase synchronization which analyzes nonlinear nonstationary phase of the time series of the temperature at some locations on the earth and calculate the phase differences with time delay between every pair of them. Networks shows scale free nature which is thought to contribute to the stability of the climate. These studies will help environmental conservation of our earth.

## 1 Introduction

Nowadays a lot of attention is paid to various networks in the world, especially which show scale free nature. But only a few studies have been done about climate networks on the earth. [4],[5] The earth is investigated in various methods like whole earth simulator. Empirical Orthogonal Function (EOF) decomposition is popular method to extract El Niño, Southern Oscillation etc. But networks may give us other kinds of understandings about climate on the earth. To extract meaningful networks from climate data of the earth, we use phase synchronization which describes synchronization of time dependent phase of nonlinear oscillations instead of linear method like cross-correlation. We use temperature of the atmosphere covering the whole surface of the earth. We draw networks by connecting two locations which synchronizes each other. We show some seasonal networks and study their scale free nature.

### 1.1 Data

The global National Centers for Environmental Prediction - National Center for Atmospheric Research (NCEP-NCAR) provides reanalysis dataset of various climate data in every 100-hPa atmospheric pressure levels and every grid with a resolution of  $2^\circ$  latitude  $\times$   $2^\circ$  longitude. We choose 47 locations from the grids

so that density of the locations is almost equal on the earth and use 400-hPa, 500-hPa and 600-hPa and study the daily temperature data from 1979 to 2005.

### 1.2 Phase synchronization

Recently phase synchronization is applied to a lot of complex dynamical systems which shows nonlinear nature and traditional cross-correlation technique may not work well, for example chaotic system, brain activity [1], the binocular fixation eye movements [3] and temperature and precipitation in different regions etc [2]. In this method, we use nonlinear phase which is defined by a Hilbert transform and the generalized phase-difference is  $\varphi_{n,m}(t) = n\phi_1(t) - m\phi_2(t)$  between two oscillators. After taking mod ( $\pi$ ), if the generalized phase-difference is constant, we can say two oscillators are synchronized and shifted by the constant in phase. But generally has time dependence, so we have to see the histogram of  $\varphi_{n,m}$  to judge whether two oscillators are systematically shifted or not. Then we use the Shannon entropy. If the entropy equals 0, they are shifted constantly and if the entropy is large, each cycle of two oscillators are independent so they are not synchronized. Here we have time series  $x_j(t)$  of the temperature at the  $j$ th location on the earth.

- We construct the complex signals

$$z_j(t) = x_j(t) + iy_j(t) = A_j e^{i\phi_j(t)}$$

by Hilbert transform of  $x_j(t)$ . Now we have the nonlinear phase  $\phi_j(t)$  at the  $j$ th location.

- We define the generalized phase-difference  $\varphi_{n,m,i,j}(t)$  for various  $m$  and  $n$  values and every pair of the locations  $i$  and  $j$ .

$$\varphi_{n,m,i,j}(t) = (n\phi_i(t) - m\phi_j(t)) \bmod 2\pi$$

- We create a histogram of  $\varphi_{n,m,i,j}$  with  $M$  bins of size  $2\pi/M$  and we get the frequency distribution  $p_k$  of each bin  $k$ , that show how the phase-difference of the pair of two locations occurs.

- To quantify the systematic occurrence of the phase-difference, we use the Shannon entropy  $S$  and define an index  $\rho_{n,m,i,j}$ .

$$\rho_{n,m,i,j} = \frac{S_{\max} - S}{S_{\max}}, \quad S = - \sum_{k=1}^M p_k \ln p_k$$

By definition, the maximum entropy and the range of the index are  $S_{\max} = \ln M$  and  $0 \leq \rho_{n,m,i,j} \leq 1$ .  $\rho_{n,m,i,j} = 1$  means complete phase synchronization, in this case the distribution of the frequencies  $p_k$  shows sharp peak at a value of  $k$ .

- Next we shift the time  $t \rightarrow t + \tau$  in one of the two time series  $x_i(t)$ ,  $x_j(t)$  and calculate the phase-difference  $\rho_{n,m,i,j}(\tau)$  between one time series with shift  $\tau$  and the other time series without shift. Finally we have max value of the index  $\rho_{n,m,i,j}(\tau)$  among deferent values of  $\tau$ .

The phase synchronization method is beyond the cross-correlation method in the two points. It analyzes the nonlinear relationship and considers the time delay directly. But for now we do not distinguish the negative and the positive correlation in the phase synchronization.

### 1.3 Climate Networks

We extract climate networks as follows. Here we consider only  $n = 1$  and  $m = 1$  phase-difference  $\rho_{1,1,i,j}$ . We standardize the max value of the index  $\rho_{1,1,i,j}$  by dividing the average value of them among deferent values of time delays  $\tau$ .

$$\rho_{\max,i,j} = \max(\rho_{1,1,i,j}(\tau)) / \text{mean}(\rho_{1,1,i,j}(\tau))$$

So every  $i$ th- $j$ th location pare has the set values of  $(\rho_{\max,i,j}, \tau_{\max,i,j})$ . To decide the links of the networks, we use the thresholds  $(\tilde{\rho}, \tilde{\tau})$  in both values of  $(\rho_{\max,i,j}, \tau_{\max,i,j})$  and define the network matrix  $N_{i,j}$

$$N_{i,j} = \begin{cases} 1 & \text{if } (\rho_{\max,i,j} > \tilde{\rho}) \wedge (\tau_{\max,i,j} < \tilde{\tau}) \\ 0 & \text{other wise} \end{cases}$$

To test the consistency of the networks, we check the common links of data in the consecutive pressure levels data and in the consecutive 6 years terms in 25 years data. Changing the thresholds  $(\tilde{\rho}, \tilde{\tau})$ , we calculate the correlation of two network matrixes in the two consecutive data Fig.1 shows the correlation of the two networks and the fraction of existing common links. We see the correlation of the two networks and

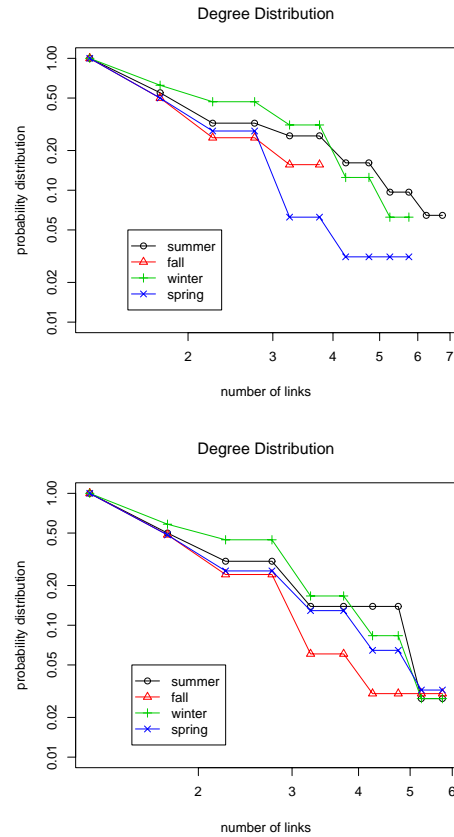


Figure 3: Degree distribution about outgoing links ( left ) and incoming links ( right )

the fraction of existing common links are trade-off. When we need high consistency of the network and a lot of existing common links, this is multi-objective optimization problem. So we pay attention to Pareto-optimal frontier. We see  $\tilde{\tau} \sim 10$  days calculations stay near the Pareto-optimal frontier and when we changes another threshold  $\tilde{\rho}$ , it almost reaches peak at  $\tilde{\rho} \sim 4$ . So we choose the two threshold  $(\tilde{\tau}, \tilde{\rho}) = (10, 4)$ . Now we can draw directed networks with the direction of time delays. Fig.2 show the networks in each seasons. We study the distribution of number of links and it shows scale free like Fig.3.

### References

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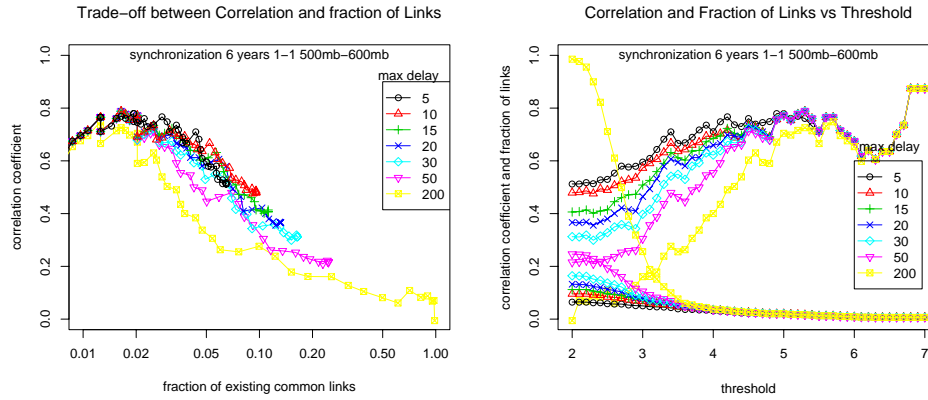


Figure 1: To test the consistency of the networks, we investigate the common links of data in the consecutive pressure levels data and in the consecutive 6 years terms in 25 years data. Changing the thresholds ( $\tilde{\rho}, \tilde{\tau}$ ), we calculate the correlation of two network matrixes in the two consecutive data Fig.1 shows the correlation of the two networks and the fraction of existing common links.

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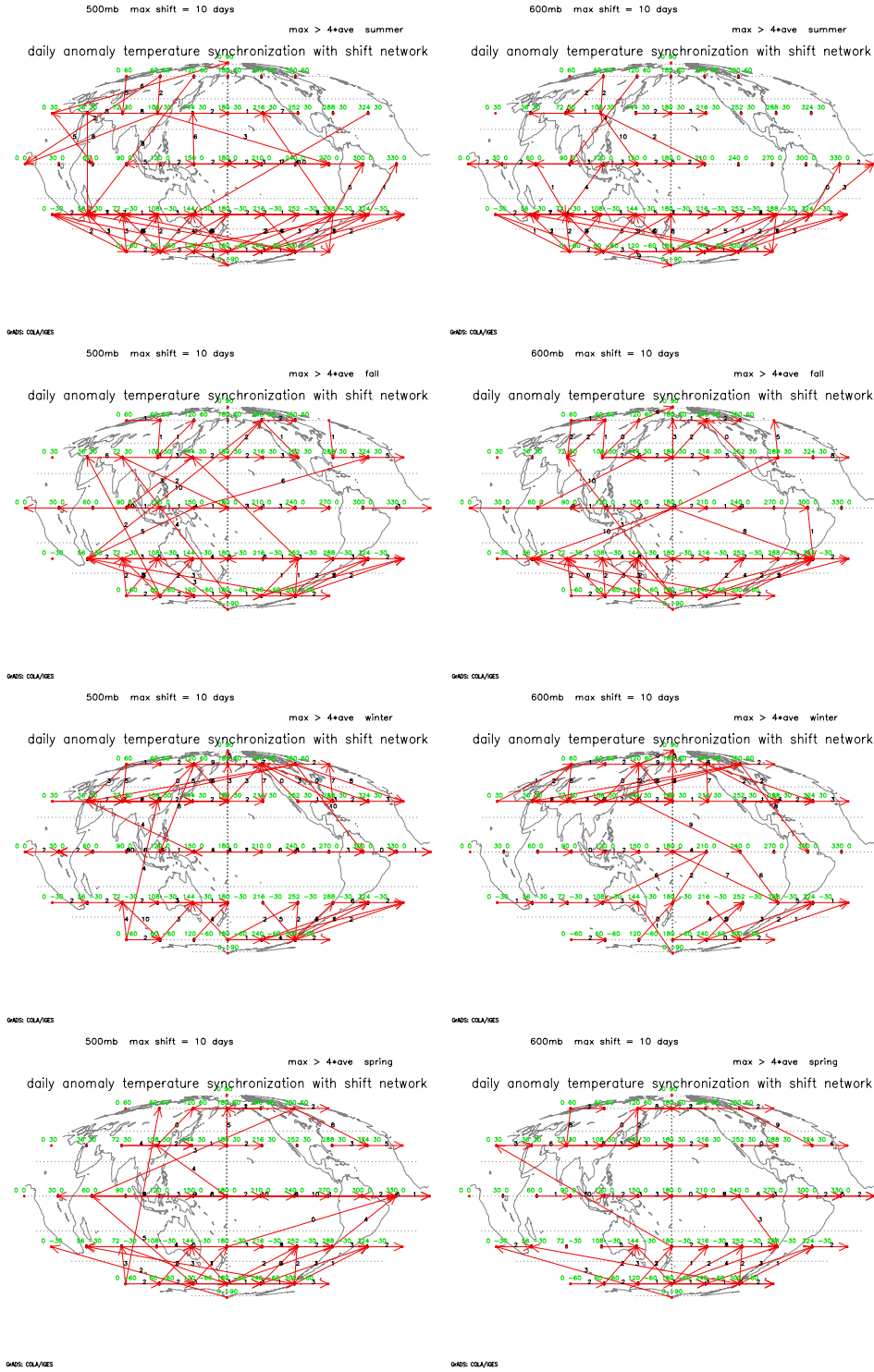


Figure 2: Seasonal networks by temperature of pressure level 500-hPa ( left ) and 600-hPa (right) with threshold  $(\bar{\tau}, \bar{\rho}) = (10, 4)$