Epidemic Models and a Self-Repairing Network with a Simple Lattice

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Abstract

We proposed a self-repairing network where nodes are capable of repairing neighboring nodes by mutually copying. A critical point where faulty nodes can be eliminated has been investigated. This paper further studies dynamics of eradicating faulty nodes by comparing the self-repairing network with mathematical epidemic models such as SIS models. It is shown that the self-repairing network, which is a probabilistic cellular automaton, can be regarded as an epidemic model in some restricted situations.

1. Introduction

Information networks are complex systems with multiple and interacting processes work in parallel on various network structure. Self-recovery of such networks have been studied with a focus on the network structure [1] as well as interacting processes [2]. We have been studying on a self-repairing network with a focus on state propagation and regulation keeping the network model simple enough for analysis [3, 4]. In the model, the two state propagations have been involved: the abnormal state propagation by unsuccessful repair as well as the normal state propagation by successful repair.

Epidemic models have been studied for a long time, and nonlinear properties of the models have investigated in great detail. The models include not only those described by differential equations [5] but by probabilistic cellular automata [6] or even including moving agents [7]. On the other hand, phase transitions have been studied on models extended from an Ising model in the field of statistical physics but involving probabilistic cellular automata [8].

Our model has been already related with a model in statistical physics [3]. This paper specifically focuses on the relation between the self-repairing network and an epidemic model called SIS model.

2. A Self-Repairing Network Model

2.1. A model by a probabilistic CA [3]

The self-repairing network is a network whose nodes can be normal (0) and abnormal (1); and are capable of repairing the neighbor nodes by copying its content. Each node repairs the neighbor nodes with a probability P_r. The repair will be successful with a probability P_{rn} when it is done by a normal node, but with a probability P_{ra} when done by an abnormal node. Further, all the repair must be successful for the target node to be normal when repairing is done by multiple nodes (**Fig. 1**).

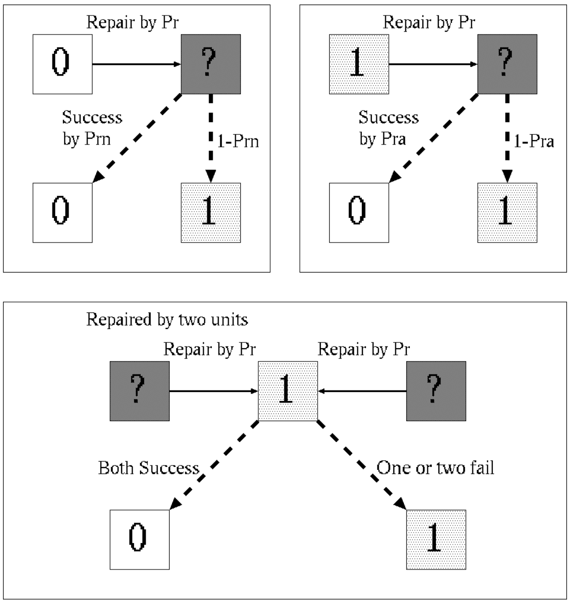

Fig. 1. Repair success rate when repairing is done by normal node (above, left) and by abnormal node (above right); all the repairs by neighbor nodes must be successful for the target node to be normal (below). Solid arcs indicate repairing and dotted arcs indicate state change.

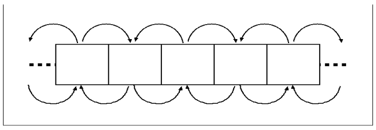

Fig. 2. 1-dimensional structure with two adjacent neighbor nodes

When repairing is done by copying, the marked difference from conventional repairing is that the repairing rather could have contaminated other nodes rather than cleaning: the *double edged sword* in repairing. Thus, it is important to investigate on conditions under which the network may be cleaned. To clarify the conditions, we used a probabilistic cellular automaton [3, 4] which turned out to be Domany-Kinzel model [8] is some particular cases.

In a 1-dimensional structure with two adjacent neighbor nodes (**Fig. 2**), the probabilities for each rule of state change are listed in **Table 1**.

Table 1. Transition probabilities of the one dimensional probabilistic cellular automaton [4]

State change	Transition Probability
000→1	$P_r(1-P_{rn})(P_rP_{rn}-P_r+2)$
001→1	$P_r^2(1-P_{rn}P_{ra})+P_r(1-P_r)((1-P_{rn})+(1-P_{ra}))$
101→1	$P_r(1-P_{ra})(2-P_r(1-P_{ra}))$
010→1	$1-P_rP_{rn}(2(1-P_r)+P_rP_{rn})$
011→1	$1-P_r((P_{rn}+P_{ra})(1-P_r)+P_rP_{ra}P_{rn})$
111→1	$1-P_rP_{ra}(P_rP_{ra}+2(1-P_r))$

2.2 Steady state analysis with mean field approximation [4]

Let *y* denote a fraction of abnormal nodes, and let the probability of being abnormal (1) be approximated by *y* at any nodes. Then the dynamics of *y* can be described by the equation (1) where *a, b,* and *c* are constants determined by the three parameters of the self-repairing network.

$$\frac{dy}{dt}=ay^2+by+c, \qquad (1)$$

$$a=-P_r^{\,2}(P_{rn}-P_{ra})^2,$$
$$b=-2P_r(1-P_{rn})(P_r(P_{rn}-P_{ra})+1)+P_r(P_r-2P_{ra}),$$
$$c=P_r(1-P_{rn})(2-P_r(1-P_{rn}))$$

When $P_{rn}>P_{ra}$ hence $a<0$, the steady state y_∞ can be calculated as follows.

$$y_\infty=\frac{1}{2a}\left(-b-\sqrt{b^2-4ac}\right)$$

In order for abnormal nodes eradicated, c must be 0 i.e. $P_{rn}=1$, for suppose otherwise normal nodes could have spread abnormal states. When $c=0$, the following condition (2) must be satisfied for abnormal states eradication, since the time derivative $\frac{dy}{dt}$ must be negative in the equation (1).

$$\frac{P_{ra}}{P_r}\geq\frac{1}{2} \qquad (2)$$

2.3 Simulation Results

Although the mean field approximation above suggested parameter conditions for abnormal node extinction, it must be verified by computer simulation for the self-repairing model by a probabilistic cellular automaton. To investigate the dynamics of normal node population when P_{rn} varies, a computer simulation is conducted with parameters listed in **Table 2**. **Fig. 4** plots the time evolution of normal nodes when P_{rn} varies as well as numerical calculation of the mean field approximation.

Table 2. Simulation Parameters

Parameters	Value
Time steps for each trial	1500
Number of trials	10
Number of nodes	400
Initial number of normal nodes	200
Pr	1.0
Prn	0.6, 0.9, 1.0
Pra	0～1(0.02)

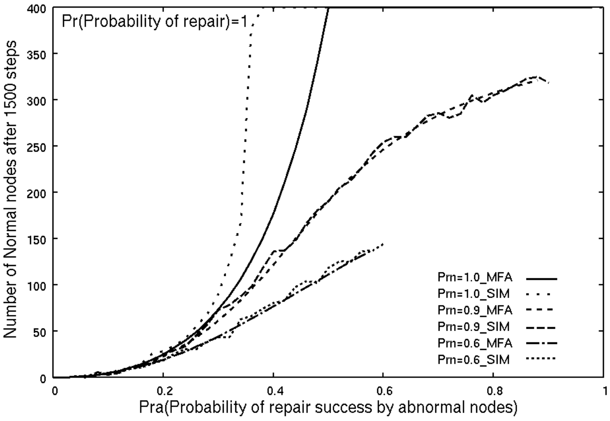

Fig. 4. Time evolution of the number of normal nodes when P_{rn} varies (legends)

3. Self-Repairing Network and SIS Model

3.1 An SIS model

Epidemic models assume the states such as S (susceptible), I (infected), and R (recovered). Combining such states allowed, epidemic models varies from SI, SIS, and SIR. Among them, in an SIS model assumes that susceptible nodes will fall into the infected state with an infection rate β when the neighbor nodes are infected. The infected nodes will be recovered with a recovery rate γ and become susceptible state again. The SIS model can describe, for example, a sexually transmitted diseases; venereal disease gonorrhea [5].

Susceptible (S) and infected (I) state respectively correspond to normal (0) and abnormal (1) state in the self-repairing network. **Fig. 5** shows the state transition between S(0) and I(1).

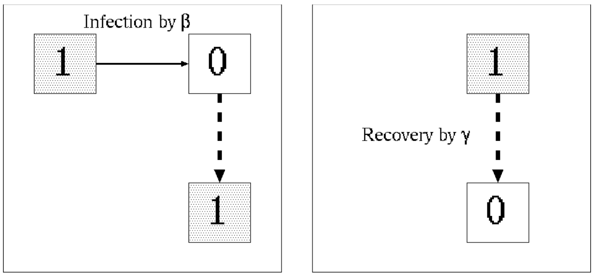

Fig. 5. State Transition in an SIS Model. Solid arcs indicate repairing and dotted arcs indicate state change**.** Solid arcs indicate repairing and dotted arcs indicate state change.

3.2 Parameter correspondence between an SIS model and a self-repairing network

In a random graph with a mean degree $\langle k \rangle$, the dynamics of the fraction of infected (abnormal) nodes y can be described as follows.

$$\frac{dy}{dt} = \beta \langle k \rangle (1-y) y - \gamma y$$

$$= -\beta \langle k \rangle y^2 + (\beta \langle k \rangle - \gamma) y$$

Thus, when P_{rn}=1 and hence c=0 in the equation (1), parameters a, b of a self-repairing network will be related to the parameters of the above SIS model as follows.

$$a = -\beta \langle k \rangle$$

$$b = \beta \langle k \rangle - \gamma$$

Since we consider one-dimensional cellular automaton for the self-repairing network, $\langle k \rangle$ could be evaluated as 2, and hence;

$$\beta = \frac{P_r^{\,2} (1-P_{ra})^2}{2} \tag{3}$$

$$\gamma = P_r P_{ra} \left(P_r P_{ra} + 2(1-P_r) \right) \tag{4}$$

3.3 Simulation Results

To examine the correspondence between the SIS Model and the self-repairing network, simulations are conducted with parameters listed in **Table 3**. **Fig. 6** plots the number of normal nodes varying the repair success rate P_{ra} when repaired by abnormal nodes. The SIS model and the self-repairing network mostly matches, however, both models do not match with the numerical solution obtained from the mean filed approximation (1), particularly when P_{ra} is greater than 0.3.

Table 3. Simulation Parameters

Parameters	Value
Time steps for each trial	500
Number of trials	10
Number of nodes	400
Initial number of normal nodes	200
Pr	1.0
Prn	1.0
Pra	0～1(0.02)
β	$\frac{P_r^2(1-P_{ra})^2}{2}$
γ	$P_r P_{ra}(P_r P_{ra}+2(1-P_r)$

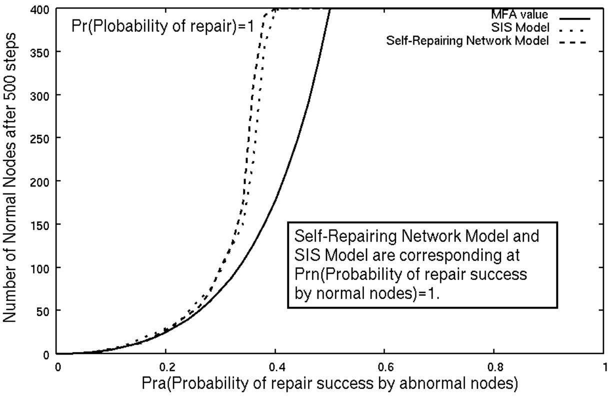

Fig. 6. Relation between SIS Model and Self-Repairing Model.

4. Conclusion

The self-repairing network involves repairing by mutual copying in a network. Hence, it models not only abnormal state propagation (unsuccessful repair) but normal state propagation (successful repair). Thus, it differs from the epidemic models which models only abnormal state propagation (infection), and recovery is modeled as an independent event (not as propagation). This paper shows, in spite of the difference, self-repairing network can be reduced to an SIS model under certain conditions.

Acknowledgements

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