A Study on Quantum-Inspired Evolutionary Algorithm based on Pair Swap

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Abstract

Quantum-Inspired Evolutionary Algorithm (QEA) is proposed as one of approximate algorithms to solve combinatorial optimization. QEA is evolutionary computation that uses quantum bits and suerposition states in quantum computing. Although conventional QEA is a coarse-grained parallel algorithm, it involves many parameters that must be adjusted manually. This paper proposes a new method of Pair Swap which exchanges each best solution information between two individuals. Experimental result shows that the proposed method is a simpler algorithm and can find high quality solution in binary Knapsack Problem.

Keywords

evolutionary computation, quantum computing, quantum bit, pair swap, knapsack problem

1 Introduction

Quantum computer[1, 2] is a computation model using quantum mechanical principles such as superposition state, interference effect, and entanglement state. Recently, stochastic combinatorial search algorithms combined with evolutionary algorithm have been recently proposed by incorporating quantum mechanical principles or quantum bits[3, 4, 5, 6].

Han et al.[5, 6] have proposed Quantum-inspired Evolutionary Algorithm (QEA) in which each gene is represented by a quantum bit. QEA can do single-point search and automatically shift from global search to local search like Simulated Annealing (SA)[7]. QEA can also perform multi-point search like Classical Genetic Algorithm (CGA) in order to solve large-scale optimization problems.

In QEA, there are more than one subpopulations (groups) like Island GA (IGA)[8, 9], and inter- and intra-group migration procedures are performed. Evolution in each group enables coarse-grained parallelization and prevents premature convergence, and the migration procedures can control search diversification and intensification. However, the adjustment of a number of parameters is required for the number of group and migration intervals for each problem.

Therefore, we propose Quantum-inspired Evolutionary Algorithm based on Pair-Swap method (QEAPS). We have showed that the performance of QEAPS is better than that of QEA in 0-1 Knapsack Problem(KP)[10]. In this paper, we keen on evaluating the performance of QEAPS against that of QEA in 0-1 KP, and we show that the effectiveness of QEAPS. To be concrete, the search performance and the robustness of QEAPS are verified by performance comparison of constraint handling methods (when the sum total of the item exceeds the capacity of the knapsack in 0-1KP).

2 Quantum-Inspired Evolutionary Algorithm based On Pair Swap

2.1 Quantum Bit Representation of Gene

QEA and QEAPS uses a quantum bit (qubit) as a gene, while, in conventional genetic algorithm (CGA), a gene is usually a definite value of binary, integer, real number, or character. The individual i in the generation t is composed of the chromosome represented as a tensor product of the qubits, $q_i = q_{i1} \otimes q_{i2} \otimes \cdots \otimes q_{im}$ and the best solution information that is binary string discovered in search process (Personal Best) $b_i = [b_{i1}, b_{i2}, \ldots, b_{im}]$. Here, m is the number of genes or qubits included in an chromosome. The qubit $q_{ij} (j = 1, ..., m)$ has stochastic superposition state (vector sum) of the two vectors $|0\rangle$ and $|1\rangle$ with each complex probability amplitude.

$$q_{ij} = \alpha_{ij} |0\rangle + \beta_{ij} |1\rangle = \begin{bmatrix} \alpha_{ij} \\ \beta_{ij} \end{bmatrix}, \quad (1)$$

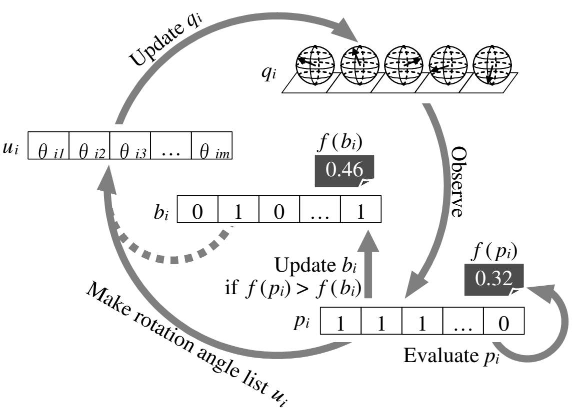

Figure 1: Evaluation of an individual.

Table 1: Lookup table of the rotation angle θ_{ik}

p_{ik}	b_{ik}	$f(p_i) \geq f(b_i)$	θ_{ik} $\alpha_{ik}\beta_{ik} > 0$	$\alpha_{ik}\beta_{ik} < 0$	$\alpha_{ik} = 0$	$\beta_{ik} = 0$
0	1	false	θ_C	-θ_C	—	$\pm\theta_C$
1	0	false	-θ_C	θ_C	$\pm\theta_C$	—
Otherwise			0	0	0	0

$|\alpha_{ij}|^2$ is the probability that the state of $|0\rangle$ is observed, and $|\beta_{ij}|^2$ is the probability that the state of $|1\rangle$ is observed. The binary string p_i is obtained by observing q_i. The fitness value $f(p_i)$ of the individual i can be calculated from p_i like CGA.

2.2 Procedures in QEA and QEAPS

The algorithm of QEAPS that we propose is shown in Figure 2. We first describe the common process of QEA and QEAPS (white part of Figure 2), and then we describe the different process of QEA and QEAPS (black part of Figure 2).

To begin with, the initialization is carried out by setting α_{ik} and β_{ik} to $1/\sqrt{2}$ in order to equally observe the states of $|0\rangle$ and $|1\rangle$ in the individual $i(=1)$. Next, the evolution of an individual with qubits and the exchange of the best solution information in the individual are repeated according to the following procedure, until a given termination condition is satisfied.

The procedure of the individual update is shown Figure 1 and as follows. First of all, p_i is obtained by observing q_i. And, the fitness $f(p_i)$ is calculated from p_i, and the fitness of the individual is decided.

Then, the rotation angle list $u_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{im}]$ is made from each value of p_{ij} and b_{ij} and the magnitude correlation of $f(p_i)$ and $f(b_i)$. This list is used to increase and decrease the observation probability of $|1\rangle$ and $|0\rangle$. How to decide the rotation angle $\theta_{ik}(k = 1, \ldots, m)$ is shown in Table 1[5, 6]. Unitary

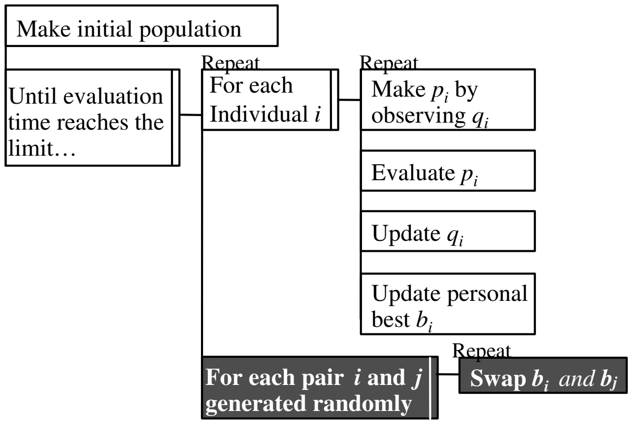

Figure 2: The algorithm of proposed QEAPS.

transformation can be used to change the ratio of the probability amplitudes α_{ik} and β_{ik} of the superposition state.

$$\begin{bmatrix} \alpha'_{ik} \\ \beta'_{ik} \end{bmatrix} = \begin{bmatrix} cos(\theta_{ik}) & -sin(\theta_{ik}) \\ sin(\theta_{ik}) & cos(\theta_{ik}) \end{bmatrix} \begin{bmatrix} \alpha_{ik} \\ \beta_{ik} \end{bmatrix}. \quad (2)$$

q_i is upgrade following the rotation angle list u_i and the rotation matrix.

If $f(p_i) > f(b_i)$, then the best solution of the new individual is replaced by the currently observed binary information.

2.3 Migration Method of QEA

Migration strategy of QEA involves local migration and global migration. The local migration is the process of distributing the best solution information of an individual with the highest fitness in each group, to all other individuals in each group, and repeated in every generations. The global migration is the process of distributing the best solution information of an individual with the highest fitness in all groups, to all other individuals in all groups, and repeated in every fixed generations. QEA shows the centralization of the search, but must determine two parameters of the number of groups and the timing of global migration by considering problem characteristics and scale, convergence speed in a group, rotation angle as QEA fundamental parameter[5, 6].

2.4 Pair Swap Method of QEAPS

QEAPS utilizes pair swap operation instead of global and local migration of QEA. To begin with, two individuals are randomly selected as a pair from all individuals in the whole group. Then, $n/2$ pairs are generated by selecting two individuals from n (even number) individuals with no overlaps in the group. Only each best solution information is exchanged in each pair with out carrying out any operation on the qubits in the individual.

3 Conputational Experiments

3.1 Experiment Preparation

The 0-1 KP is used for the evaluation experiments in order to prove the effectiveness of the proposed QEAPS. The KP in the paper [9] is used as a benchmark problem. The number of items N is 100(the first 100 items are used in the benchmark problem). The weight limit in the KP is set to be 50% of the total weight of all items. Parameters such as the population and the number g of groups in QEA are followed by the previous researches[6], respectively. Parameters used in QEAPS are followed by QEA as shown in Table 2.

When the evaluation times, the number of fitness calculation time, reach the preset value, the search stops. We perform the same experiments 30 times using each technique for each problem.

3.2 Constraint Handling Methods

We compare the performances of QEAPS and that QEA in the following three methods to handle constraint violation.

1. **Fitness = 0 (Zero)**
   If the condition is not satisfied, then $f = 0$.

2. **Penalty Function (Penalty)**
   The penalty function shown by the equation 3 is used.

$$f(p_i) = \sum_{j=1}^{n} a_j p_{ij} - \alpha \, max \left\{ 0, \sum_{j=1}^{n} w_j p_{ij} - C \right\}, \quad (3)$$

   where, p_{ij} is the value of the j-th gene in the chromosome of the i-th individual, a_j is the profit of item j, w_j is the weight of item j, and C is the capacity of the knapsack, and $\alpha = max_{j=1\ldots n} \{a_j/w_j\}$.

3. **Random Repair (Repair)**
   Random Repair [5] that consists of the following procedures in applied. Step 2 is applied even when weight limits are satisfied.

Table 2: Parameter configurations.

Parameter names	Values used QEA	QEAPS
Number of individuals	10, 20, 30, . . . , 100	
Number of subpopulations (groups) (g)	5	–
Number of individuals in a subpopulation	2, 4,. . . , 20	–
Rotation angle (θ_C)	0.01π	0.01π
Number of observations	1	1
Interval of global migration	100	–

Step 1: One item is randomly selected and removed until the knapsack capacity is filled.

Step 2: One item selected randomly is put in the knapsack until capacity are exceeded. When capacity are exceeded, the item put at the end is removed.

3.3 Experimental Result

Regarding evaluation criteria, we focus on the optimal solution discovery rate per trial number *Opt*[%], the mean fitness m_f. The upper limit of evaluation times is set to $N \times 10^3$ as a termination condition of the search.

As a function of the individual total numbers, *Opt*, m_f are shown in Figure 3 and 4. First, when paying attention to *Opt*, The performance of QEAPS is better than that of QEA in the same number of individuals in all constraint handling methods. In *Opt* of QEA, Repair is the highest, and Zero and Penaly are lower. The difference is seen for all number of individuals between the methods. On the other hand, such difference is not so seen at QEAPS. Moreover, *Opt* is almost 100% in any method when more than 40 individuals are used.

The result of m_f in Figure 3 and 4 also indicate that QEAPS has higher-performance than QEA. In m_f of QEA, there is obvious difference between that of Repair and these of Zero and Penalty. In QEAPS, the highly qualified solutions are obtained in all constraint handling methods. It should be noted that m_f of Repair, Penalty and Zero of QEAPS is higher than that of Repair in QEA.

In QEAPS, the difference between Repair, Zero, and Penalty is quite small. Even though Zero and Penalty are simple and not specialized in the knapsack problem, they can search for optimal solutions, consequently QEAPS is more robust against constraint

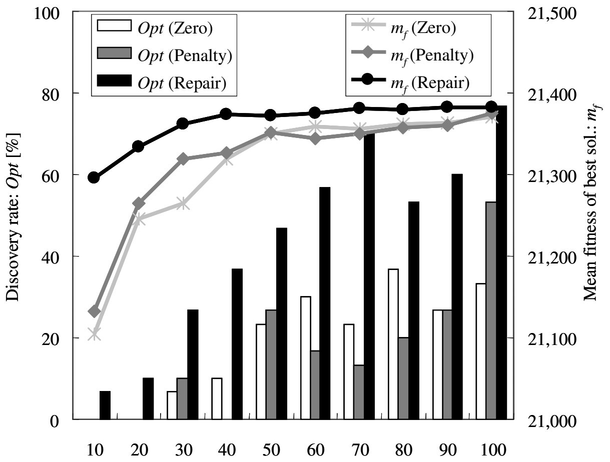

Figure 3: Discovery rate and evaluation time (QEA)

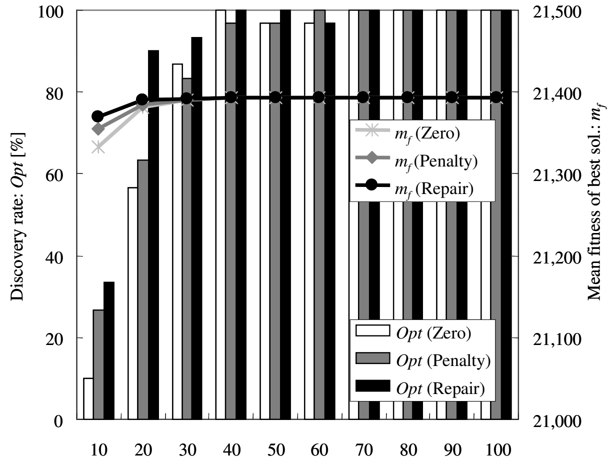

Figure 4: Discovery rate and evaluation time (QEAPS)

methods.

4 Conclusion

In this paper, we focused on evaluating the performance and robustness of QEAPS to improve QEA. We compared the performane between the constraint handling method in 0-1 KP. Experimental result show that QEAPS is able to discover the optimal solution at the higher probability compared with QEA, and that the solutions found by QEAPS are of even quality, and that QEAPS is robust against constrain handling methods.

We plan to verify the search performance in a larger-scale problem, improve the algorithm, examine application to other combination optimization problems, and clarify the characteristic of the problem to which QEAPS is effective.

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