Improvement of the Algorithm for the Search of Periodic Gaits of a Passive Dynamic Walker

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Abstract

Periodic gaits of passive dynamic waking are found by solving nonlinear dynamic equations, and the convergence speed of numerical solving of the equations is greatly affected by the choice of initial values. This paper investigates the iterative searching algorithm for periodic gaits by studying the simplest passive dynamic walking model. An improved algorithm for choosing the initial values is proposed based on the work of Garcia etc. The effectiveness of the proposed algorithm is demonstrated by numerical simulations.

1 Introduction

Why can human walk? One may focus on the control of nerve and muscle but relatively neglect other important factors. Passive dynamic walking proposed by McGeer [1] presented a novel idea.

Passive dynamic walker is a simple mechanism normally composed of some solid links connected by frictionless hinges. Without motors and controllers, it can walk down a small slope stably by gravity and have a human-like gait.

The key to analyze passive dynamic walking is to find periodic gaits, normally by numerically solving nonlinear differential equations with iterative method like Newton-Raphson method [2]. The iterative method first selects a set of initial values, then searches directly in the solution space whose dimension is relatively high for the fixed points of the equations. The choice of the initial values is quite important since it directly affects the convergence and the convergence speed of the iterative method, but how to choose the initial values is still a problem. Garcia et al. [3] had developed a method to solve this problem. They first got a low-order approximation to the equations, solved them for the analytical solutions which could be used as the initial values of the iterative method. This method works well when the equations are simple but involves some complicated symbolic manipulation. This article proposes an improved algorithm which searches in the parameter space whose dimension is relatively low instead of the solution space. The effectiveness of the algorithm is demonstrated by numerical simulation and comparison with the original algorithm.

2 Model

The model used here is the simplest walking model proposed by Garcia et al. [3], which is shown in Figure 1.

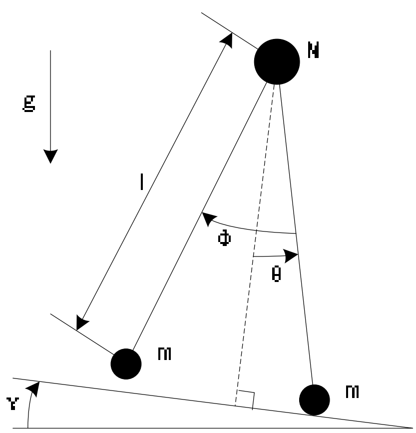

Figure 1: The simplest model of passive dynamic walking

Assumptions of the model are:

1 Mass: The only mass is at the hip and the feet, and the foot mass m is much smaller than the hip mass M, satisfying $\beta = \frac{m}{M} \to 0$.

2 Actuator: No actuators are used.

3 Collision: When the swing foot hits the ground, the collision is completely inelastic (no slip or bounce) and the double support phase is instantaneous (the stance foot leaves the ground when the swing foot hits the ground).

4 Ground: The swing foot may be below the ground level during the swing phase. To solve this problem, we allow the swing foot to move below the ground in numerical simulation and use a chessboard-like ground for real-world experiments.

According to Lagrange function, equations of the motion for the swing phase are

$$\left[\begin{array}{cc} \bar{M} & \left(\frac{\partial g}{\partial q}\right)^T \\ \frac{\partial g}{\partial q} & 0 \end{array}\right]\left[\begin{array}{c} \ddot{q} \\ \lambda \end{array}\right]=\left[\begin{array}{c} \bar{f} \\ -\frac{\partial^2 g}{\partial q \partial q}\dot{q}\dot{q} \end{array}\right] \quad (1)$$

and the relation of the velocities before and after the collision can be written as follows according to the angular momentum conservation

$$\left[\begin{array}{cc} \bar{M} & \left(\frac{\partial g}{\partial q}\right)^T \\ \frac{\partial g}{\partial q} & 0 \end{array}\right]\left[\begin{array}{c} \dot{q}^+ \\ \rho \end{array}\right]=\left[\begin{array}{c} \bar{M}\dot{q}^- \\ 0 \end{array}\right] \quad (2)$$

where $q = (u, v, \theta, \phi)^T$ is the configuration of the model, $x = F(q)$ the coordinates of the hip and the feet, $g(q) = 0$ the constraints of the ground, M the general mass matrix, f the force vector, λ, ρ the Lagrange multipliers, and $\bar{M} = \left(\frac{\partial F}{\partial q}\right)^T M\frac{\partial F}{\partial q}$, $\bar{f} = \left(\frac{\partial F}{\partial q}\right)^T\left[f - M\frac{\partial^2 F}{\partial q \partial q}\dot{q}\dot{q}\right]$.

We define the start of a stride as the moment the swing foot hits the ground and the stance foot leaves the ground, which is also the end of the last stride. The state at this moment is the initial state of the model, which is $v = \left(\theta, \dot{\theta}, \dot{\phi}\right)^T$. A Stride Function $v_{k+1} = S(v_k)$ is a map from the state of the k^{th} stride to the state of the $(k+1)^{th}$ stride.

3 Search of the Fixed Points of the Equation

A periodic gait corresponds to a fixed point of the Stride Function, which is normally found by iterative method. If an initial value v satisfies the equation $v = S(v)$, it is a fixed point of the Stride Function S, otherwise the iterative method should modify the initial value by Δv to make the modified value satisfy $v + \Delta v = S(v + \Delta v)$. Since we can't explicitly write the expression of $S(v + \Delta v)$, we use the first-order approximation

$$S(v+\Delta v) \approx S(v) + J\Delta v, where\ \mathrm{J} = \frac{\partial \mathrm{S}}{\partial \mathrm{v}} \quad (3)$$

then we get

$$\Delta v = (I-J)^{-1}(S(v) - v) \quad (4)$$

The iterative method can be described as

$$\begin{array}{l} repeat \\ \Delta v = (I-J)^{-1}(S(v)-v) \\ v = v + \Delta v \\ until\, |S(v) - v| < \varepsilon \end{array} \quad (5)$$

This is the Newton-Raphson method.

4 Improvement of the Search of the Fixed Points of the Equation

Since we use the first-order approximation to the Stride Function, the initial value v should be near the fixed point. In a stride, the swing foot first moves below the ground level, then moves above it, and then moves downwards and hits the ground. If the initial value is far from the fixed point, the swing foot may not be able to move above the ground level, and the collision may not occur. Since we can't get the state after heel-strike, $S(v)$ and Jacobian matrix J can't be obtained. In a word, the initial value should guarantee that at least one heel-strike can happen.

To solve this problem, we propose an improved method. We assume that if the parameters of the model change slightly, the fixed point will not change too much. Based on this assumption, a fixed point for one set of parameters can be used as the initial value for another set of parameters if these two sets of parameters are near together. On the other hand, an initial value not suitable for one set of parameters may be a good choice for another set of parameters. Based on the above observation, we propose an improved method as follows:

1. Given a slope angle γ, choose an initial value v;
2. Search for the fixed point using Newton-Raphson method; if not found, jump to 3;
3. Keep the initial value v unchanged, let $\gamma = \gamma + \Delta\gamma$, and use Newton-Raphson method again; modify $\Delta\gamma$ until the fixed point is found. Let the found slope angle be γ^* and the fixed point v^*;
4. Use v^* as the initial value, let $\gamma^* = \gamma^* + \Delta\gamma$, search for the fixed point. Modify the slope angle by

$\Delta\gamma$ at a time, and search for the fixed point using the fixed point with the slope angle unchanged as the initial value. Repeat until $\gamma^* = \gamma$.

The dimension of the parameter space is 1 while the dimension of the state space is 3. The improved method searches in the parameter space instead of the state space, making the search easier.

5 An Example of the Algorithm

To demonstrate the effectiveness of the algorithm, we first give a plot of the fixed points versus the slope angles, see Figure 2. The same result was shown by Garcia et al. [4] too.

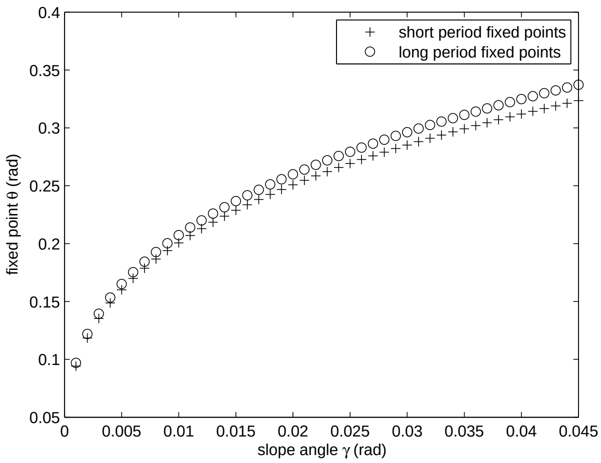

Figure 2: Fixed points of the stride function

We give an example here to demonstrate that the improved method can find the fixed point even when the initial value is not suitable.

1. Given a slope angle $\gamma = 0.016[rad]$, choose an initial vale $v = \left(\theta, \dot{\theta}, \dot{\phi}\right)^T = (0.1, -0.01, -0.002)^T$;

2. Search for the fixed point using Newton-Raphson method; we can't find the fixed point after 80 iterations;

3. Keep the initial value v unchanged, let $\gamma = \gamma - 0.001[rad]$, and use Newton-Raphson method again; we find the fixed point

$$v = \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0.2259204891316 \\ -0.22237603365099 \\ -0.022316588434412 \end{bmatrix}$$

when $\gamma = 0.013[rad]$;

4. Use the fixed point when $\gamma = 0.013[rad]$ as the initial value, search for the fixed point when $\gamma = 0.014[rad]$; Use the fixed point when $\gamma = 0.014[rad]$ as the initial value, search for the fixed point when $\gamma = 0.015[rad]$; finally we get the fixed point when $\gamma = 0.016[rad]$ using the fixed point when $\gamma = 0.015[rad]$ as the initial value

$$v = \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0.2259204891316 \\ -0.22237603365099 \\ -0.022316588434412 \end{bmatrix}$$

.

The procedure is shown in Table 1.

slope angle $\gamma\,(rad)$	initial value $\theta\,(rad)$	fixed point $\theta\,(rad)$
0.016	0.1	not found
0.015	0.1	not found
0.014	0.1	not found
0.013	0.1	0.2259204891316
0.014	0.2259204891316	0.23144913545946
0.015	0.23144913545946	0.23671218331329
0.016	0.23671218331329	0.24173855862323

Table 1: An Example of the Algorithm

6 Conclusion

This paper investigates the iterative searching algorithm for periodic gaits by studying the simplest passive dynamic walking model. An improved algorithm for choosing the initial values is proposed based on the work of Garcia etc. The effectiveness of the proposed algorithm is demonstrated by numerical simulations.

References

[1] T. McGeer, “Passive dynamic walking,” *Intern. J. Robot. Res.*, Vol. 9, pp. 62-82, 1990.

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