Cooperative Manipulation of a Floating Object by Some Space Robots

– Application of a Tracking Control Method Using Transpose of Generalized Jacobian Matrix –

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Abstract

For free floating space robots with manipulators, we have proposed a digital tracking control method using the transpose of the Generalized Jacobian Matrix (GJM). In future space missions, it is considered that many tasks will be achieved by cooperative motions of some space robots. In this paper, the tracking control method using the transpose of the GJM is applied to cooperative manipulations of a floating object by some space robots. Simulation results show the effectiveness of the control method.

1 Introduction

Many control methods of space robots with manipulators have been proposed [1]. Most of them use the inverse of the Generalized Jacobian Matrix (GJM) which is a coefficient matrix between the end-effector's velocity and the joint velocity of the manipulator. Therefore, if the robot system becomes in a singular configuration, the manipulator is out of control because the inverse of the GJM does not exist. We have proposed discrete time control methods using the transpose of the GJM [2,3]. Since the control methods belong to a class of constant value control such as PID control, we have proposed a digital tracking control method based on the reference [3].

In future space missions, it is considered that many tasks will be achieved by cooperative motions of some space robots. We have studied on control problems for realizing cooperative manipulations and reported that a system consisting of some space robots with manipulators and a floating object can be treated as a kind of distributed system [5,6]. Using the distributed system representation each robot constituting the distributed system can be designed the control system individually.

In this paper, the tracking control method using the transpose of the GJM is applied to cooperative manipulations of a floating object by some space robots. To validate the control method computer simulations are done. Simulation results show the effectiveness of the control method.

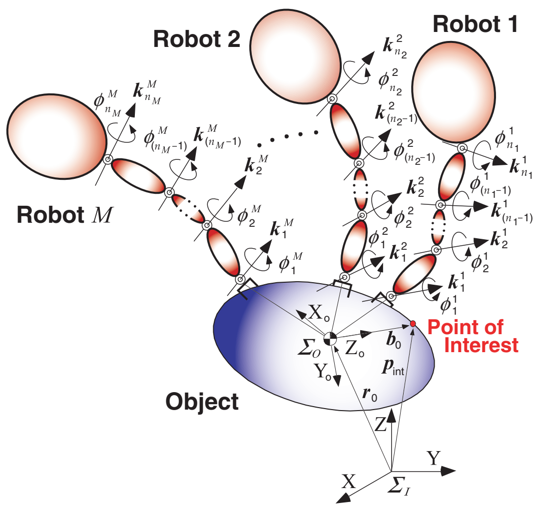

Fig. 1 Model of space robot system

2 Modeling

2.1 Robot system model

In this paper, we consider a space robot system consisting of M robots with manipulators and a floating object shown in Fig. 1. The h-th robot ($h = 1, \cdots, M$) is consisting of an uncontrolled base and n_h-DOF manipulator with revolute joints. Assumptions and symbols used in this paper are defined as follows:

[Assumptions]

A1) All elements of the space robot are rigid.

A2) The robot system is standing still at an initial state, i. e., the initial linear momentum and angular momentum of the space robots are zero.

A3) No external force acts on the robot system.

A4) Positions and attitude angles of robots and an object in inertial coordinate frame can be measured.

[Symbols]

Σ_I: inertial coordinate frame
$\Sigma_{\rm int}$: point of interest coordinate frame
Σ_T: target coordinate frame
i^h: number of link or joint i of robot h
$\boldsymbol{p}_{\rm int}$: position vector of point of interest
$\boldsymbol{p}_T$: position vector of origin of Σ_T
$\boldsymbol{r}_0$: position vector of mass center of object
$\boldsymbol{v}_*$: linear velocity vector of point of interest ($*=$ int) or mass center of object ($*=0$)
$\boldsymbol{\omega}_*$: angular velocity vector of point of interest ($*=$ int) or mass center of object ($*=0$)
$\boldsymbol{p}_i^h$: position vector of joint i^h
$\boldsymbol{r}_i^h$: position vector of mass center of link i^h
$\boldsymbol{k}_i^h$: unit vector indicating joint axis direction of joint i^h
$\boldsymbol{r}_g$: position vector of mass center of system
$\boldsymbol{r}_g^h$: position vector of mass center of robot h
$\boldsymbol{q}$: joint angle vector
${}^I\boldsymbol{A}_*$: rotation matrix from Σ_* ($*=$ int, T) to Σ_I
ϕ_i^h: relative angle of joint i^h
$\boldsymbol{\phi}^h$: joint angle vector of robot h
m_0: mass of object
m_i^h: mass of link i^h
$\boldsymbol{I}_0$: inertia tensor of object
$\boldsymbol{I}_i^h$: inertia tensor of link i^h
$\boldsymbol{E}$: identity matrix

The tilde operator stands for a cross product such that $\tilde{\boldsymbol{r}}\boldsymbol{a} = \boldsymbol{r}\times\boldsymbol{a}$. All position and velocity vectors are defined with respect to the inertial reference frame.

2.2 Kinematic model

The robot system shown in Fig. 1 can be understood as one robot with M manipulators by regarding the object as a robot body, and M robot arms and robot bodies as M manipulators. The kinematic formulation of such space system has been derived by Yoshida et al. [7]. The relation obtained from its geometrical relationships, and the conservation laws of linear momentum and angular momentum under the above assumptions as follows:

$$\boldsymbol{\nu}_{\rm int} = \begin{bmatrix}\dot{\boldsymbol{p}}_{\rm int}\\ \boldsymbol{\omega}_{\rm int}\end{bmatrix} = \boldsymbol{J}_s\begin{bmatrix}\boldsymbol{v}_0\\ \boldsymbol{\omega}_0\end{bmatrix}, \quad \boldsymbol{H}_s\begin{bmatrix}\boldsymbol{v}_o\\ \boldsymbol{\omega}_o\end{bmatrix} + \boldsymbol{H}_m\dot{\boldsymbol{\phi}} = \boldsymbol{0} \quad (1)$$

where

$$\boldsymbol{J}_s = \begin{bmatrix}\boldsymbol{E} & \tilde{r}_0 - \tilde{\boldsymbol{p}}_{\rm int}\\ \boldsymbol{0} & \boldsymbol{E}\end{bmatrix}, \quad \boldsymbol{H}_s = \begin{bmatrix}w\boldsymbol{E} & w(\tilde{\boldsymbol{r}}_0 - \tilde{\boldsymbol{r}}_g)\\ w\tilde{\boldsymbol{r}}_g & \boldsymbol{I}_w\end{bmatrix},$$

$$\boldsymbol{H}_m = \begin{bmatrix}\boldsymbol{J}_{T_w}\\ \boldsymbol{I}_\phi\end{bmatrix}, \quad \boldsymbol{\phi} = \left[(\boldsymbol{\phi}^1)^T,\ (\boldsymbol{\phi}^2)^T,\ \cdots,\ (\boldsymbol{\phi}^M)^T\right]^T,$$

$$\boldsymbol{I}_w = \sum_{h=1}^{M}\boldsymbol{I}_w^h + \boldsymbol{I}_o, \quad \boldsymbol{J}_{T_w} = \sum_{h=1}^{M}\boldsymbol{J}_{T_w}^h, \quad \boldsymbol{I}_\phi = \sum_{h=1}^{M}\boldsymbol{I}_\phi^h,$$

$$\boldsymbol{I}_w^h = \sum_{i=1}^{n_h}\{\boldsymbol{I}_i^h - m_i^h\tilde{\boldsymbol{r}}_i^h(\tilde{\boldsymbol{r}}_i^h - \tilde{\boldsymbol{r}}_0^h)\}, \quad \boldsymbol{J}_{T_w}^h = \sum_{i=1}^{n_h} m_i^h\boldsymbol{J}_{T_i}^h,$$

$$\boldsymbol{I}_\phi^h = \sum_{i=1}^{n_h}(\boldsymbol{I}_i^h\boldsymbol{J}_{R_i}^h + m_i^h\tilde{\boldsymbol{r}}_i^h\boldsymbol{J}_{T_i}^h),$$

$$\boldsymbol{J}_{T_i}^h = \left[\boldsymbol{O}_a \quad \bar{\boldsymbol{J}}_{T_i}^h \quad \boldsymbol{O}_b\right], \quad \boldsymbol{J}_{R_i}^h = \left[\boldsymbol{O}_a \quad \bar{\boldsymbol{J}}_{R_i}^h \quad \boldsymbol{O}_b\right],$$

$$\bar{\boldsymbol{J}}_{T_i}^h = \left[\tilde{\boldsymbol{k}}_1^h(\boldsymbol{r}_i^h - \boldsymbol{p}_1^h),\ \cdots,\ \tilde{\boldsymbol{k}}_i^h(\boldsymbol{r}_i^h - \boldsymbol{p}_i^h),\ \boldsymbol{0},\ \cdots,\ \boldsymbol{0}\right],$$

$$\bar{\boldsymbol{J}}_{R_i}^h = \left[\boldsymbol{k}_1^h,\ \cdots,\ \boldsymbol{k}_i^h,\ \boldsymbol{0},\ \cdots,\ \boldsymbol{0}\right],$$

and $\boldsymbol{O}_a \in \boldsymbol{R}^{3\times n_a}$ ($n_a = \sum_{i=1}^{h-1} n_i$) and $\boldsymbol{O}_b \in \boldsymbol{R}^{3\times n_b}$ ($n_b = \sum_{i=h+1}^{M} n_i$) are zero matrices.

Form Eq. (1), the relation between velocity $\boldsymbol{\nu}_{\rm int}$ of the object and joint angular velocity $\dot{\boldsymbol{\phi}}$ of the manipulator can be derived as follows:

$$\boldsymbol{\nu}_{\rm int} = \boldsymbol{J}^*\dot{\boldsymbol{\phi}} \quad (2)$$

where $\boldsymbol{J}^* = -\boldsymbol{J}_s(\boldsymbol{H}_s)^{-1}\boldsymbol{H}_m$ is a GJM of the system shown in Fig. 1.

2.3 System partition

For the system shown in Fig. 1 control systems can be easily constructed by using Eq. (2). However, if the number of robots is changed, Eq. (2) must be recalculated. Furthermore, if the number of robots becomes increased, a large amount of calculation for the system is necessary. To solve the problems described above, This total robot system is regarded as a distributed system.

By examining parameters and variables included in the matrix $\boldsymbol{H}_s$ and vector $\boldsymbol{H}_m\dot{\boldsymbol{\phi}}$ in Eq. (1), the matrix and vector can be rewritten as follows:

$$\boldsymbol{H}_s = \boldsymbol{H}_s^0 + \sum_{h=1}^{M}\boldsymbol{H}_s^h, \quad \boldsymbol{H}_m\dot{\boldsymbol{\phi}} = \sum_{h=1}^{M}\boldsymbol{H}_m^h\dot{\boldsymbol{\phi}}^h \quad (3)$$

where

$$\boldsymbol{H}_s^0 = \begin{bmatrix}m_0\boldsymbol{E} & \boldsymbol{0}\\ m_0\tilde{r}_0 & \boldsymbol{I}_0\end{bmatrix}, \quad \boldsymbol{H}_s^h = \begin{bmatrix}m^h\boldsymbol{E} & m^h(\tilde{\boldsymbol{r}}_0^h - \tilde{\boldsymbol{r}}_g^h)\\ m^h\tilde{\boldsymbol{r}}_g^h & \boldsymbol{I}_w^h\end{bmatrix},$$

$$\boldsymbol{H}_m^h = \begin{bmatrix}\bar{\boldsymbol{J}}_{T_w}^h\\ \bar{\boldsymbol{I}}_\phi^h\end{bmatrix}, \quad m^h = \sum_{i=1}^{n_h} m_i^h,$$

$$\bar{\boldsymbol{J}}_{T_w}^h = \sum_{i=1}^{n_h} m_i^h\bar{\boldsymbol{J}}_{T_i}^h, \quad \bar{\boldsymbol{I}}_\phi^h = \sum_{i=1}^{n_h}\left(\boldsymbol{I}_i^h\bar{\boldsymbol{J}}_{R_i}^h + m_i^h\tilde{\boldsymbol{r}}_i^h\bar{\boldsymbol{J}}_{T_i}^h\right).$$

$\boldsymbol{H}_s^h$ and $\boldsymbol{H}_m^h$ are matrices including parameters the h-th robot only, and $\boldsymbol{H}_s^0$ is a matrix including parameters of the object only.

Eqs. (1) and (3) make the following relation:

$$\left(\boldsymbol{H}_s^0 + \sum_{h=1}^{M} \boldsymbol{H}_s^h\right) \boldsymbol{J}_s^{-1}\boldsymbol{\nu}_{\rm int} + \sum_{h=1}^{M} \boldsymbol{H}_m^h \dot{\boldsymbol{\phi}}^h = \boldsymbol{0}. \qquad (4)$$

It is clear that the following set of equations is one of solutions of Eq. (4), when a constant and diagonal matrix $\boldsymbol{A}_h$ is introduced.

$$\bar{\boldsymbol{H}}_s^h \boldsymbol{J}_s^{-1}\boldsymbol{\nu}_{\rm int} + \boldsymbol{H}_m^h \dot{\boldsymbol{\phi}}^h = \boldsymbol{0} \quad (h{=}1,\cdots,M) \qquad (5)$$

where

$$\bar{\boldsymbol{H}}_s^h = \boldsymbol{H}_s^h + \boldsymbol{A}_h \boldsymbol{H}_s^0, \qquad \sum_{h=1}^{M} \boldsymbol{A}_h = \boldsymbol{E}.$$

Then, the following relation can be derived from Eq. (5).

$$\boldsymbol{\nu}_{\rm int} = -\boldsymbol{J}_s(\bar{\boldsymbol{H}}_s^h)^{-1}\boldsymbol{H}_m^h \dot{\boldsymbol{\phi}}^h \quad (h{=}1,\cdots,M). \qquad (6)$$

Therefore, for each robot of the system the control system can be designed individually.

3 Digital Control

Eq. (6) can be rewritten:

$$\boldsymbol{v}_{\rm int}(k) = \boldsymbol{J}_L^h \dot{\boldsymbol{\phi}}^h(k), \quad \boldsymbol{\omega}_{\rm int}(k) = \boldsymbol{J}_A^h \dot{\boldsymbol{\phi}}^h(k) \qquad (7)$$

where

$$\begin{bmatrix} \boldsymbol{J}_L^h \\ \boldsymbol{J}_A^h \end{bmatrix} = -\boldsymbol{J}_s(\bar{\boldsymbol{H}}_s^h)^{-1}\boldsymbol{H}_m^h.$$

For Eq. (7) the following digital tracking control law using the transpose of the GJM [4] is utilized:

$$\begin{aligned} \boldsymbol{\tau}_d^h(k) = (\boldsymbol{J}_L^h)^T(k)\left[\hat{k}_p(k)\boldsymbol{e}_{PI}(k) - \hat{\boldsymbol{K}}_{LV}(k)\boldsymbol{v}_{\rm int}(k)\right] \\ + (\boldsymbol{J}_A^h)^T(k)\left[\hat{k}_o(k)\boldsymbol{e}_{OI}(k) - \hat{\boldsymbol{K}}_{AV}(k)\boldsymbol{\omega}_{\rm int}(k)\right] \end{aligned} \qquad (8)$$

where $\boldsymbol{\tau}_d^h(k)$ is the joint torque input vector and

$$\boldsymbol{e}_{PI}(k) = \boldsymbol{p}_T(k) - \boldsymbol{p}_{\rm int}(k),$$

$$\boldsymbol{e}_{OI}(k) = -\frac{1}{2}\boldsymbol{E}_X^T(k)\boldsymbol{E}_{OI}(k),$$

$$\boldsymbol{E}_{OI}(k) = \begin{bmatrix} \boldsymbol{n}_T(k) - \boldsymbol{n}_{\rm int}(k) \\ \boldsymbol{s}_T(k) - \boldsymbol{s}_{\rm int}(k) \\ \boldsymbol{a}_T(k) - \boldsymbol{a}_{\rm int}(k) \end{bmatrix}, \quad \boldsymbol{E}_X(k) = \begin{bmatrix} \tilde{\boldsymbol{n}}_{\rm int}(k) \\ \tilde{\boldsymbol{s}}_{\rm int}(k) \\ \tilde{\boldsymbol{a}}_{\rm int}(k) \end{bmatrix},$$

$$\hat{k}_p(k) = k_p\{1 + \alpha_L \nu_L(k)\}, \quad \hat{k}_o(k) = k_o\{1 + \alpha_A \nu_A(k)\},$$

$$\hat{\boldsymbol{K}}_{LV}(k) = \boldsymbol{K}_{LV}\{1 - \beta_L \nu_L(k)\},$$

$$\hat{\boldsymbol{K}}_{AV}(k) = \boldsymbol{K}_{AV}\{1 - \beta_A \nu_A(k)\},$$

$$\nu_L(k) = \frac{||\boldsymbol{v}_{{\rm int}_d}(k)||}{v_{d_{\max}}}, \quad \nu_A(k) = \frac{||\boldsymbol{\omega}_{{\rm int}_d}(k)||}{\omega_{d_{\max}}}.$$

Table 1 Physical parameters of robots and object

	Length m	Mass kg	Moment of inertia kg·m^2
Base	3.5	2000	3587.9
Link 2	2.5	50	26.2
Link 1	2.5	50	26.2
hand	0.5	5	0.23
Object	4.0	1200	2400.0

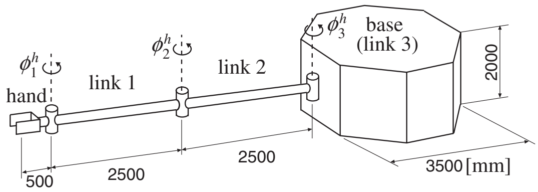

Fig. 2 Space robot

The vectors $\boldsymbol{n}_*$, $\boldsymbol{s}_*$ and $\boldsymbol{a}_*$ ($* = T$, int) are unit vectors along the axes of Σ_* with respect to Σ_I, i. e., ${}^I\!\boldsymbol{A}_* = [\boldsymbol{n}_*(k)\ \boldsymbol{s}_*(k)\ \boldsymbol{a}_*(k)]$. $\boldsymbol{v}_{{\rm int}_d}(k)$ and $\boldsymbol{\omega}_{{\rm int}_d}(k)$ are the desired velocities of $\boldsymbol{v}_{\rm int}(k)$ and $\boldsymbol{\omega}_{\rm int}(k)$, $v_{d_{\max}}$ and $\omega_{d_{\max}}$ are the maximum values of the norm of $\boldsymbol{v}_{{\rm int}_d}(k)$ and $\boldsymbol{\omega}_{{\rm int}_d}(k)$, $\alpha_\dagger$ ($\alpha_\dagger \geq 0$) and $\beta_\dagger$ ($0 \leq \beta_\dagger \leq 1$) ($\dagger = L,\ A$) are setting parameters. Furthermore, k_p and k_o are positive scalar gains for position and orientation, and $\boldsymbol{K}_{LV}$ and $\boldsymbol{K}_{AV}$ are symmetric and positive definite gain matrices for linear and angular velocities of the point of interest on the object.

To examine the performance of the tracking control law Eq. (8), simulations are performed by using three and same horizontal planar 3-DOF robots shown in Fig. 2 and an object.

Physical parameters of the robots and object are shown in Table 1. Simulations are carried out under the following condition. A point of interest on the object moves along a straight path from the initial position to the target position and the object angle is set up the initial value. The sampling period is $T = 0.01$s and the coefficient matrices are $\boldsymbol{A}_1 = \boldsymbol{A}_2 = 0.33\boldsymbol{E}$ and $\boldsymbol{A}_3 = 0.34\boldsymbol{E}$. The feedback gains are $k_p = k_o = 2 \times 10^5$, $\boldsymbol{K}_{LV} = {\rm diag}\{2 \times 10^4,\ 2 \times 10^4\}$ and $\boldsymbol{K}_{AV} = 2 \times 10^4$. The setting parameters are $\alpha_\dagger = 2$ and $\beta_\dagger = 0.2$ ($\dagger = L,\ A$).

Fig. 3 shows the motion of the robot system. From this figure, the object is successfully moved by three robots. Fig. 4 shows the time history of the simulation. This figure also shows the case of constant gains, i. e., $\alpha_\dagger = \beta_\dagger = 0$. From Fig. 4, it can be seen that good control performance can be achieved using the tracking control law.

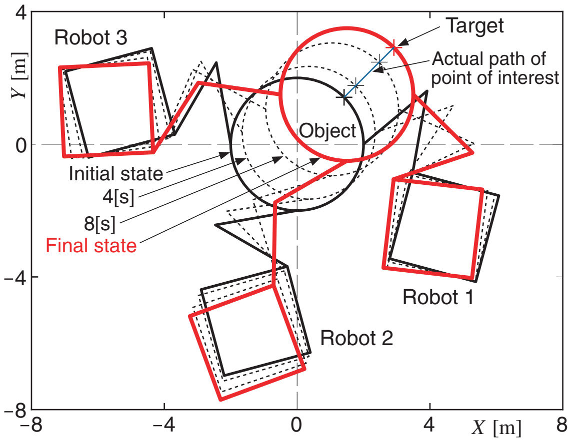

Fig. 3 Motion of the robot system

4 Conclusion

In this paper, we propose a tracking control method using the transpose of the GJM for handling a floating object cooperatively by some space robots. Simulation results show the effectiveness of the control method.

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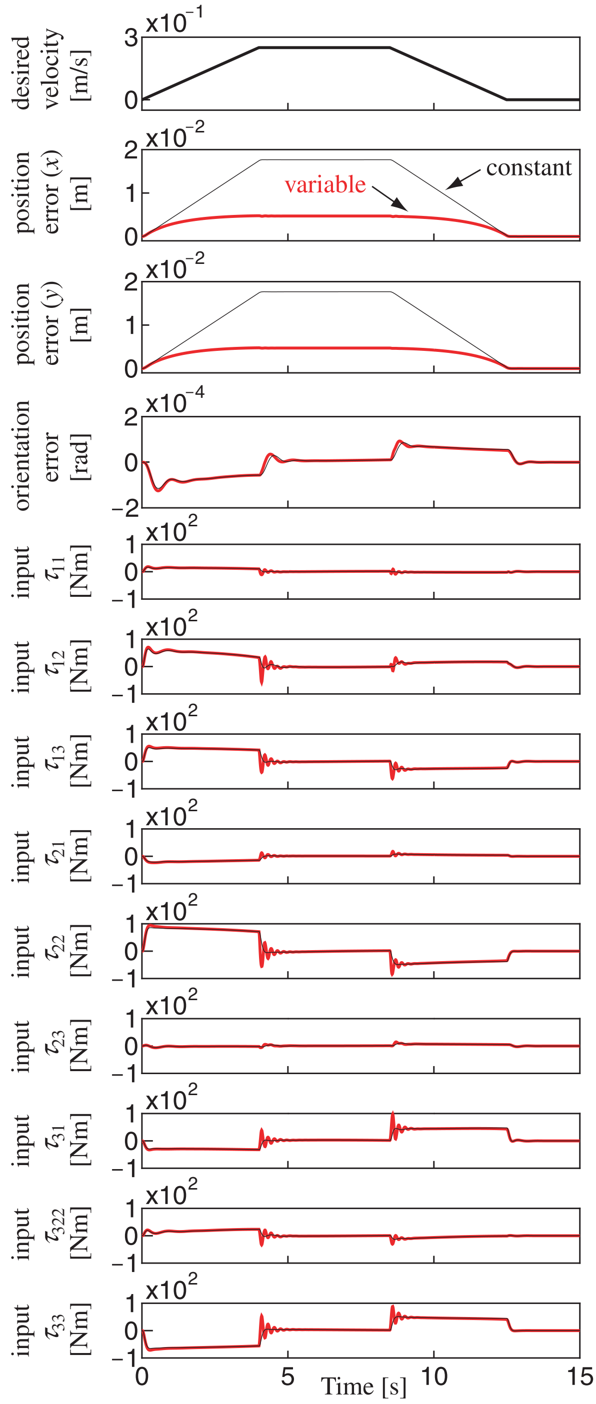

Fig. 4: Time history (variable: $\alpha_\dagger = 2$ and $\beta_\dagger = 0.2$, constant: $\alpha_\dagger = 0$ and $\beta_\dagger = 0$ ($\dagger = L,\ A$))

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