

Robust Tracking Control Based on Neural Network for Nonholonomic Mobile Robot

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Abstract

To deal with the unknown factors of nonholonomic mobile robot, such as model uncertainties and external disturbances, a robust tracking controller with bounded estimation based on neural network is proposed. A neural network is to approximate the uncertainties terms, the interconnection weights of the neural network can be tuned online. And the robust controller is designed to compensate for the external uncertainties and the approximation error. Moreover, an adaptive estimation algorithm is employed to estimate the bound of the approximation error. The stability of the proposed controller is proven by Lyapunov function. The proposed robust tracking controller based on neural network can overcome the uncertainties and the disturbances. The simulation results demonstrate that the proposed method has good robustness.

Keywords: Robust Control; Neural Network; Bounded Estimation; Mobile Robot; Nonholonomic Constraint

1 Introduction

The tracking control of nonholonomic mobile robot has been a topic of research during recent years. The characteristic of the nonholonomic system is that the constraints, which are imposed on the motion, are not integratable, i.e., the constraints cannot be written as time derivatives of some functions of the generalized co-ordinates. It is a typical nonholonomic mechanical system with high nonlinearity and its control is very difficult. It is also a typical nonlinear uncertain system with both the parametric uncertainty in the dynamic model of the robot including motor dynamics and disturbances from the external environment or unmodelled dynamics.

For the tracking control problem of the mobile robot, lots of control methods have been applied. J. M. Yang and J. H. Kim [1] proposed a robust tracking controller for nonholonomic wheeled mobile robots using sliding mode technique. Y. Kanayama et al. [2] developed smooth static time invariant state feedback for a velocity-controlled mobile robot with nonholonomic constraint. In [3-8], the backstepping technique was used to design the adaptive and robust controller for the nonholonomic system. M.S.

Kim et al. [9] applied a robust adaptive dynamic controller for a nonholonomic mobile robot with modeling uncertainty and disturbances. In recent years, intelligent systems, such as fuzzy logic [10] and neural network [5, 11-13], have been applied to approximate the models or to deal with the disturbances and dynamic uncertainties of dynamic systems [14, 15]. F. M. Raimondi, M. Melluso [10] developed a new theoretical control method based on the dynamic behavior of a wheeled vehicle, where a mechanism of fuzzy inference for designing a robust control system was present. In [5], a robust motion controller based neural network and backstepping technique is proposed for a two-DOF low-quality mobile robot. In [11-13], the neural network controllers in the proposed control structure were to deal with unmodeled bounded disturbances and unstructured unmodeled dynamics in the vehicle.

In this paper, we proposed a robust tracking controller based on neural network for a mobile robot with nonholonomic constraints. The proposed controller can guarantee robustness to parametric and dynamics uncertainties and also rejects any bounded, immeasurable disturbances entering the system. The stability is proved by the Lyapunov theory.

The rest of this paper is organized as follows. In Section II, a mobile robot with nonholonomic constraints is introduced. An robust controller based on neural network with bounded estimation for the mobile robot is designed in Section III, and the stability is proven using the Lyapunov method the velocity tracking error, the neural network weights error and the bounded estimation error are all bounded. Section IV gives some simulation results and conclusions are given in section V.

2 Dynamic Model of a Nonholonomic Mobile Robot

2.1 Preliminary definitions

A mobile robot is shown in Fig. 1, which contains two driven wheels mounted on the same axis and a castor. It is a typical example of a nonholonomic mechanical system.

An inertial Cartesian frame $\{O, X, Y\}$ linked to the

world and $\{C, X_C, Y_C\}$ linked to the mobile platform are used here. It is assumed that the centre of mass of the mobile robot is local in C . The pose of the mobile robot is completely specified by:

$$\mathbf{q} = [x, y, \theta]^T \quad (1)$$

The nonholonomic constraint states that the mobile robot satisfies the conditions of pure rolling and non-slipping, i.e., the mobile robot can only move in the direction normal to the axis of the driving wheels:

$$\dot{y} \cos \theta - \dot{x} \sin \theta - d \dot{\theta} = 0 \quad (2)$$

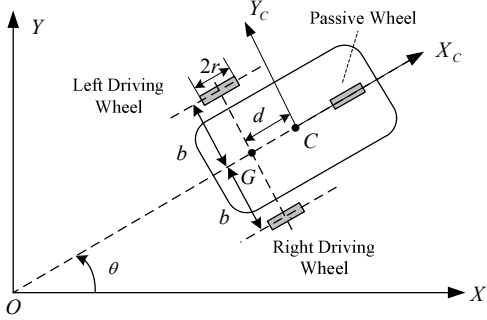


Fig. 1 A nonholonomic mobile robot

2.2 Dynamic model of a mobile robot

Consider a nonholonomic mobile robot system with n generalized coordinate \mathbf{q} and subject to m constrains can be described by [12]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) + \boldsymbol{\tau}_d = \mathbf{B}(\mathbf{q})\boldsymbol{\tau} - \mathbf{A}^T(\mathbf{q})\boldsymbol{\lambda} \quad (3)$$

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0} \quad (4)$$

where $\mathbf{M}(\mathbf{q}) \in \mathcal{R}^{n \times n}$ is a symmetric, positive definite inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathcal{R}^{n \times n}$ is the centripetal and coriolis matrix, $\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathcal{R}^n$ denotes the surface friction and the gravitational vector, $\boldsymbol{\tau}_d \in \mathcal{R}^n$ denotes bounded unknown disturbances including unstructured unmodeled dynamics, $\mathbf{B}(\mathbf{q}) \in \mathcal{R}^{n \times r}$ is the input transformation matrix, $\boldsymbol{\tau} \in \mathcal{R}^r$ is the input vector, $\mathbf{A}^T(\mathbf{q}) \in \mathcal{R}^{m \times n}$ is the matrix associated with the constrains, $\boldsymbol{\lambda} \in \mathcal{R}^m$ is the vector of constrain forces.

Let $\mathbf{S}(\mathbf{q}) = [s_1(\mathbf{q}), \dots, s_{n-m}(\mathbf{q})]$ be a set of smooth and linearly independent vector fields in $N(A)$, the null space of $\mathbf{A}(\mathbf{q})$, i.e.,

$$\mathbf{S}^T(\mathbf{q})\mathbf{A}^T(\mathbf{q}) = \mathbf{0} \quad (5)$$

It is possible to find a velocity vector $\mathbf{v}(t) \in \mathcal{R}^{n-m}$,

such that

$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\mathbf{v}(t) \quad (6)$$

Multiplying both sides by \mathbf{S}^T and using (5), we have

$$\begin{aligned} \mathbf{S}^T \mathbf{M} \mathbf{S} \dot{\mathbf{v}} + \mathbf{S}^T (\mathbf{M} \dot{\mathbf{S}} + \mathbf{C} \mathbf{S}) \mathbf{v} + \mathbf{S}^T \mathbf{F} + \mathbf{S}^T \boldsymbol{\tau}_d \\ = \mathbf{S}^T \mathbf{B} \boldsymbol{\tau} \end{aligned} \quad (7)$$

$$\bar{\mathbf{M}} \dot{\mathbf{v}} + \bar{\mathbf{C}} \mathbf{v} + \bar{\mathbf{F}} + \bar{\boldsymbol{\tau}}_d = \bar{\boldsymbol{\tau}} \quad (8)$$

Where $\mathbf{v} = [v, \omega]^T$, v is the velocity of mobile robot, ω is the angle velocity, $\bar{\mathbf{M}} = \mathbf{S}^T \mathbf{M} \mathbf{S}$, $\bar{\mathbf{C}} = \mathbf{S}^T (\mathbf{M} \dot{\mathbf{S}} + \mathbf{C} \mathbf{S})$, $\bar{\mathbf{F}} = \mathbf{S}^T \mathbf{F}$, $\bar{\boldsymbol{\tau}}_d = \mathbf{S}^T \boldsymbol{\tau}_d$, $\bar{\boldsymbol{\tau}} = \mathbf{S}^T \mathbf{B} \boldsymbol{\tau}$.

Property 1. $\bar{\mathbf{M}}$ is a symmetric positive definite matrix.

Property 2.

$$\bar{M}_{\min} \leq \|\bar{\mathbf{M}}(\mathbf{q})\| \leq \bar{M}_{\max}, \quad \|\bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\| \leq \bar{C}_b \|\dot{\mathbf{q}}\| \quad (9)$$

where \bar{M}_{\min} , \bar{M}_{\max} , \bar{C}_b are some positive constants that assumed to be unknown. and $\|\cdot\|$ denotes Euclid norm.

Property 3. The matrix $(\dot{\bar{\mathbf{M}}}(\mathbf{q}) - 2\bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}))$ is skew-symmetric.

Assumption 1. The friction and gravity are bounded by $\|\bar{\mathbf{F}}(\mathbf{q}, \dot{\mathbf{q}})\| \leq \xi_0 + \xi_1 \|\dot{\mathbf{q}}\|$, where ξ_0 and ξ_1 are some positive constants.

Assumption 2. Disturbance is bounded by $\|\bar{\boldsymbol{\tau}}_d\| \leq \bar{\tau}_D$, where $\bar{\tau}_D$ is a positive constant.

For a two-wheeled mobile robot, the kinematic model can be given as [2]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & -d \sin \theta \\ \sin \theta & d \cos \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (10)$$

In order to simplify the problem formulation, it is assumed that $d = 0$. The alternative formulations can be readily deduced when $d \neq 0$ [5].

Suppose the mobile robot is required to follow a reference trajectory, with position and velocity are

$$\begin{cases} \mathbf{q}_r = [x_r, y_r, \theta_r]^T \\ \mathbf{v}_r = [v_r, \omega_r]^T \end{cases} \quad (11)$$

Then the tracking error expressed with respect to a frame fixed on the mobile robot are given as [2]

$$\mathbf{e}_q = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (12)$$

neural network weights in vectors \mathbf{W} that are needed to best approximate the given function f are difficult to determine. All one needs to know for control purposes is that, for a specified value of E some ideal approximating neural network weights exist. Then, an estimate value of f can be given by

$$\hat{f} = \hat{\mathbf{W}}^T \sigma(\mathbf{V}^T x) \quad (22)$$

where $\hat{\mathbf{W}}$ is the estimated value of \mathbf{W} .

Choose the tracking control law as

$$\mathbf{U} = -\hat{\mathbf{W}}^T \sigma(\mathbf{V}^T x) - \mathbf{U}_R \quad (23)$$

where \mathbf{U}_R is robust controller.

Then, equation (19) can be rewritten as

$$\dot{\mathbf{r}} = \tilde{\mathbf{W}} \sigma(\mathbf{V}^T x) - \mathbf{U}_R + \varepsilon_d \quad (24)$$

where $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$ is the estimation error, $\varepsilon_d = \varepsilon - \bar{\mathbf{M}} \bar{\tau}_d$ is the uncertain term of the approximation error and the external disturbances.

According to (21) and Assumption 2, we can know that the uncertain term is bounded, that is,

$$\|\varepsilon_d\| \leq \|\varepsilon\| + \|\bar{\mathbf{M}}\| \|\bar{\tau}_d\| \leq \Delta + \bar{M}_{\max} \bar{\tau}_D = E \quad (25)$$

Theorem: Given the system (8), choose the velocity control law (15) the tracking control law (23), and the adaptation law of the neural network as

$$\dot{\hat{\mathbf{W}}} = -\dot{\tilde{\mathbf{W}}} = \Gamma \mathbf{r} \sigma(\mathbf{V}^T x) \quad (26)$$

where $\Gamma > 0$ is the learning rate of the neural network.

The robust controller is designed as

$$\mathbf{U}_R = -\hat{E} \operatorname{sgn}(\mathbf{r}) \quad (27)$$

where \hat{E} is the estimated value of E , $\operatorname{sgn}(\cdot)$ is a standard sign function. And the bound estimation law is choose as

$$\dot{\hat{E}} = -\dot{\tilde{E}} = \eta \mathbf{r} \operatorname{sgn}(\mathbf{r}) \quad (28)$$

where $\tilde{E} = E - \hat{E}$ is the estimation error, η is a positive constant.

Then, the closed-loop system (8) and (23) is asymptotically stable, the filtered error \mathbf{r} , the neural network weights error $\tilde{\mathbf{W}}$ and the bounded estimation error \tilde{E} are all bounded.

Proof: Choose Lyapunov function candidate as

$$L = L_1 + \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \tilde{\mathbf{W}}^T \Gamma^{-1} \tilde{\mathbf{W}} + \frac{1}{2} \tilde{E}^T \eta^{-1} \tilde{E} \quad (29)$$

Differentiating yields

$$\dot{L} = \dot{L}_1 + \mathbf{r} \dot{\mathbf{r}} + \tilde{\mathbf{W}}^T \Gamma^{-1} \dot{\tilde{\mathbf{W}}} + \tilde{E}^T \eta^{-1} \dot{\tilde{E}} \quad (30)$$

Substituting (16), (24), and (25)-(28) into (30), we can obtain:

$$\begin{aligned} \dot{L} &= \dot{L}_1 + \mathbf{r} \left(-\tilde{\mathbf{W}}^T \sigma(\mathbf{V}^T x) - \mathbf{U}_R + \varepsilon_d \right) \\ &\quad + \tilde{\mathbf{W}}^T \Gamma^{-1} \dot{\tilde{\mathbf{W}}} - \tilde{E}^T \mathbf{r} \operatorname{sgn}(\mathbf{r}) \\ &= \dot{L}_1 + \mathbf{r} \left(\varepsilon_d - \hat{E} \operatorname{sgn}(\mathbf{r}) \right) - \tilde{E}^T \mathbf{r} \operatorname{sgn}(\mathbf{r}) \\ &\quad - \tilde{\mathbf{W}}^T \left(\mathbf{r} \sigma(\mathbf{V}^T x) + \Gamma^{-1} \dot{\tilde{\mathbf{W}}} \right) \\ &\leq \mathbf{r} \varepsilon_d - \hat{E} \mathbf{r} \operatorname{sgn}(\mathbf{r}) - (E - \hat{E}) \mathbf{r} \operatorname{sgn}(\mathbf{r}) \\ &\leq -\alpha \|\mathbf{r}\| (E - \|\varepsilon_d\|) = -\alpha \|\mathbf{r}\| \leq 0 \end{aligned} \quad (31)$$

where $\alpha = E - \|\varepsilon_d\| > 0$ is a small positive constant.

Since $\dot{L} \leq 0$, it can be inferred that the filtered error \mathbf{r} , the neural network weights error $\tilde{\mathbf{W}}$ and the bounded estimation error \tilde{E} are all bounded. Let function $\Xi(t) = -\dot{L} = \alpha \|\mathbf{r}\|$, and integrate function $\Xi(t)$ with respect to time [14, 15]

$$\int_0^t \Xi(\tau) d\tau \leq L(\mathbf{r}(0), \tilde{\mathbf{W}}, \tilde{E}(0)) - L(\mathbf{r}(t), \tilde{\mathbf{W}}, \tilde{E}(t)) \quad (32)$$

Because $L(\mathbf{r}(0), \tilde{\mathbf{W}}, \tilde{E}(0))$ is bounded, and $L(\mathbf{r}(t), \tilde{\mathbf{W}}, \tilde{E}(t))$ is nonincreasing and bounded, the following result is obtained

$$\lim_{t \rightarrow \infty} \int_0^t \Xi(\tau) d\tau \leq 0 \quad (33)$$

In addition, $\dot{\Xi}(t)$ is bounded, by Barbalat's Lemma, it can be show that $\lim_{t \rightarrow \infty} \Xi(\tau) = 0$. That is, $\mathbf{r}(t) \rightarrow 0$ as $t \rightarrow \infty$. As a result, the closed-loop system (8) and (23) is asymptotically stable.

4 Simulation Results

In order to verify the validity of the proposed controller, a nonholonomic mobile robot is used for illustration in this paper, as shown in Fig. 1. The dynamical equations of the mobile robot can be expressed in (1) where [12]

$$M(q) = \begin{bmatrix} m & 0 & md \sin \theta \\ 0 & m & -md \cos \theta \\ md \sin \theta & -md \cos \theta & I \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & md \dot{\theta} \cos \theta \\ 0 & 0 & md \dot{\theta} \sin \theta \\ 0 & 0 & 0 \end{bmatrix},$$

$$B(q) = \frac{1}{r} \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \\ b & -b \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}, \quad A^T(q) = \begin{bmatrix} -\sin \theta \\ \cos \theta \\ -d \end{bmatrix},$$

$$\lambda = -m(\dot{x}_c \cos \theta + \dot{y}_c \sin \theta)\dot{\theta}$$

where $m = 10\text{kg}$, $I = 5\text{kg} \cdot \text{m}^2$, $b = 0.25\text{m}$, $r = 0.05\text{m}$, and $v_r = 0.5\text{m/s}$. The external disturbance $|\tau_{d_i}| \leq 3.0$ is a random noise with the magnitude bounded.

The initial values of neural network weights W are selected randomly in $[-1, 1]$, and the estimations are $\hat{E}(0) = [0, 0]^T$. The controller gains are $k_1 = 10$,

$k_2 = 5$, $k_3 = 4$, and $k_4 = \text{diag}\{10, 10\}$.

Defining a straight line, starting from

$$q_r(0) = [x_r(0) \quad y_r(0) \quad \theta_r(0)]^T = [0 \quad 1 \quad 45^\circ]^T$$

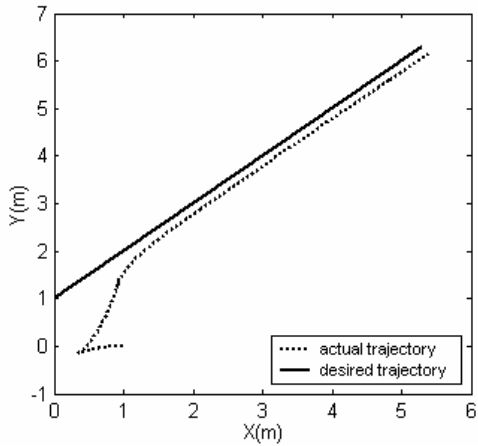
The mobile robot, however, is initially at

$$q(0) = [x(0) \quad y(0) \quad \theta(0)]^T = [1 \quad 0 \quad 0^\circ]^T$$

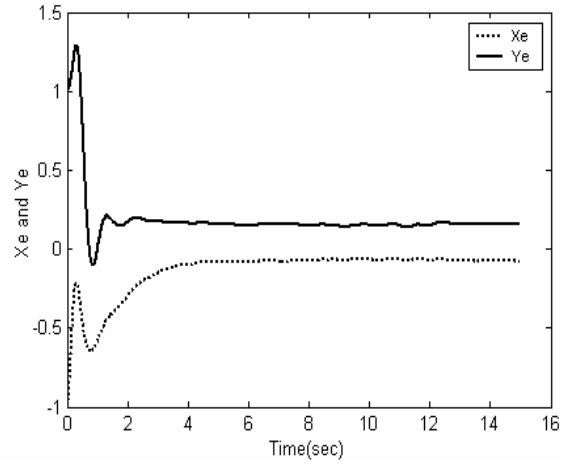
where $\theta(0) = 0^\circ$ indicates that the robot is heading toward positive direction of x .

Fig. 3 shows the simulation results for tracking a straight line using computed torque method. Since there are the uncertainties and disturbance, the mobile robot cannot track the trajectory and exhibit a steady state error.

Under the same conditions, Fig. 4 shows the results for tracking a straight line using the proposed method. As it can be seen from the figure, the mobile robot can reach the line quickly and continues to track it.

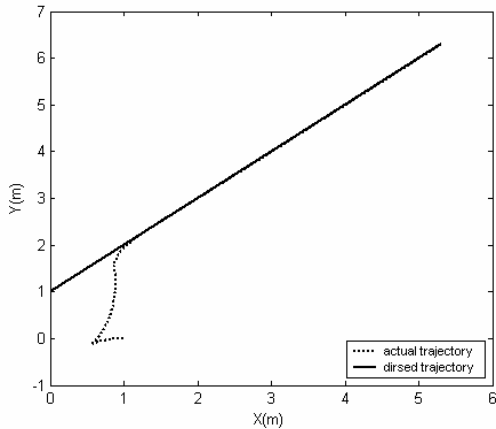


(a) Trajectory in the (x, y) plane

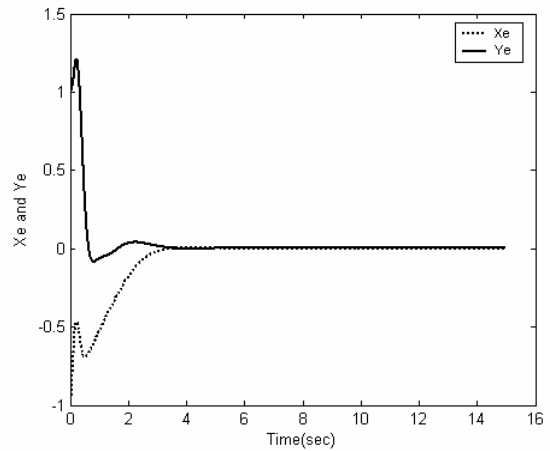


(b) Position errors

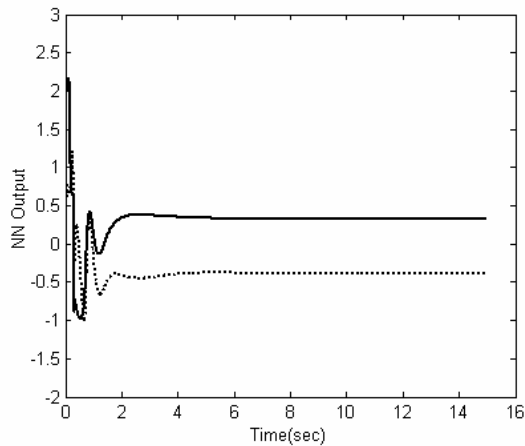
Fig. 3 Results by computed torque controller



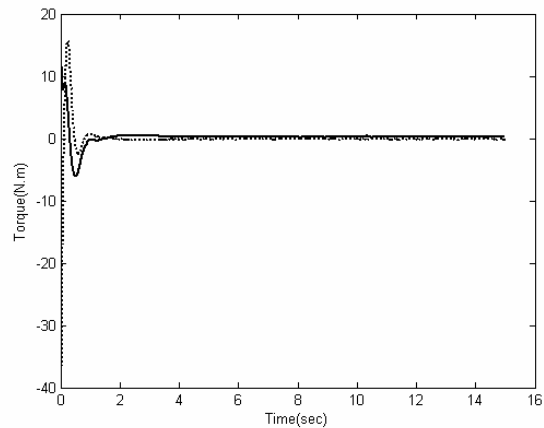
(a) Trajectory in the (x, y) plane



(b) Position errors



(c) Neural network outputs



(d) Torques

Fig. 4 Results by the proposed method

5 Conclusions

Using the robust and neural network methods, a robust tracking controller with bounded estimation based on neural network is proposed for a nonholonomic mobile robot. This controller can guarantee robustness to parametric and dynamics uncertainties and also rejects any bounded, immeasurable disturbances entering the system. The stability is proven using the Lyapunov method. The velocity error, the neural network weights error and the bounded estimation error are all bounded. Finally, some simulation examples are utilized to illustrate the control performance.

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