Fault Detection by Evolution Strategies Based Particle Filters

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Abstract

Fault detection in dynamic systems has attracted considerable attention in designing systems with safety and reliability. Though a large number of methods have been proposed for solving the fault detection problem, it is hardly apply to nonlinear stochastic state space models. A novel filter called the Evolution Strategies based particle filter (ESP) proposed by recognizing the similarities and the difference of the processes between the particle filters and Evolution Strategies is applied here to fault detection of nonlinear stochastic state space models. Results of numerical simulation studies exemplify the applicability of this approach.

1 Introduction

The problem of fault detection in dynamic systems has attracted considerable attention in designing systems with safety and reliability. In the past two decades, a large number of methods have been proposed for solving the fault detection problem [1], [2], [3]. Among these, the model-based approaches using the quantitative analytical model of the system to be monitored are by nature the most powerful ones. For all model-based approaches, the decision of a fault is based on the innovations based on the state estimate obtained from the observed input-output data and a mathematical model of the system. Though the fault detection method can be easily constructed in linear/Gaussian state space models where the well-known Kalman filters [4], [5] can employ to evaluate the state estimate, the idea is generally difficult to apply to nonlinear systems with non-Gaussian noises. In this paper, a new fault detection method is proposed for nonlinear/non-Gaussian state space models using the idea of the backward sequential probability ratio test (BSPRT) [6] and the evolution strategies based particle filter (ESP).

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2 Fault detection

Consider here the following set of nonlinear state space models indexed by $m = 0, 1$.

$$x_{t+1} = f^{(m)}(x_t, u_t) + v_t \tag{1}$$

$$y_t = g^{(m)}(x_t) + w_t \qquad m = 0, 1 \tag{2}$$

where x_t, u_t, y_t are the state variable, input and observation, respectively, $f^{(m)}(\cdot)$ and $g^{(m)}(\cdot)$ are known possibly nonlinear functions, and v_t and w_t are independently identically distributed (i.i.d.) system noise and observation noise sequences, respectively. The system works normally and its behavior is governed by the given normal mode model described as in (1) and (2) indexed by $m = 0$, and then the model may change to the given fault mode model indexed by $m = 1$ at unknown time $t = \tau$. The problem here is to develop a detection procedure to find a model change as quickly as possible.

3 Evolution Strategies Based Particle Filters

Recent massive increase of the computational power leads to much attentions to particle filters, a simulation-based filter based on Bayesian sequential analysis, and a lot of particle filters have been developed. Recognizing that the importance sampling and resampling processes in Sampling importance resampling (SIR) particle filter [7] are corresponding to mutation and selection processes in Evolution Strategies (ES) [8], which is one of Evolutionary Computation approaches [9], we can develop a novel particle filter called Evolution strategies based particle filter (ESP) as in [10].

Resampling process in SIR filter is introduced to avoid the degeneracy phenomenon in Sequential importance sampling (SIS) particle filter [11], [12], where all but one of the normalized importance weights are very close to zero after a few iterations and a large computational effort is wasted to updating trajectories whose contribution to the final estimate is almost zero. It selects offspring with probability proportional to the importance weight $w_{t|t}^{(i)}$, and this corresponds to selection process in ES with the importance weight as fitness function. On the other hand, the importance sampling process in SIR filter samples $x_{t|t}^{(i)}$ according to the importance density $q(x_t|x_{t-1|t-1}^{(i)}, y_{1:t})$, and this corresponds to mutation process in ES from the viewpoint of generating offspring $x_{t|t}^{(i)}$ from the extrapolated parents $f(x_{t-1|t-1}^{(i)})$ with perturbation by v_t. The main difference is resampling in SIR is carried out probabilistically and the weights are reset as $1/n$, while the selection in ES is deterministic and the fitness function is never reset. Hence, by replacing the resampling process in SIR by the deterministic selection process in ES, we can derive a new particle filter called Evolution strategies based particle filter comma (ESP(,)) and Evolution strategies based particle filter plus (ESP(+)) [10]. As shown in [10], the ESP filters behave more stable than SIR both in squared estimation errors and processing time by their deterministic selection process, we will develop fault detection methods using the ESP.

4 Fault Detection by Evolution Strategies Based Particle Filters

Then the fault detection problem to be considered here can be reduced to perform a hypothesis testing for the hypotheses:

H_0 (Normal mode) : System models are indexed by $m = 0$
H_1 (Fault mode) : System models are indexed by $m = 1$

Wald's sequential probability ratio test (SPRT) [13] is a common testing procedure, where the logarithm of likelihood ratio function (LLR) $\lambda_t = \log p(y_{1:t}|H_1)/p(y_{1:t}|H_0)$ is evaluated and compared with two threshold values $B^* < 0 < A^*$ until it exceeds these thresholds. It is known that the fault detection system based on the above mentioned Wald's SPRT formulation minimizes, on the average, the time to reach a decision for specified error probabilities if the system is either in the normal mode or the fault mode from the beginning of the test. However, the characteristics of the fault process differs from it; the system is initially operated in normal mode and then transition occurs to the fault mode at time instant τ during observations. To fit this situation, the idea of the backward SPRT (BSPRT) [6] is introduced.

Rewriting the hypotheses representing normal and fault modes in fault detection process as

H_0 (Normal mode) :
System models at time $t-k+1$ are indexed by $m = 0$
H_1 (Fault mode) :
System models at time $t-k+1$ are indexed by $m = 1$,
$t > \tau,\ k = 1, \ldots, t-\tau+1$

we can introduce a backward LLR (BLLR), where LLR is computed in reverse (*backward*) from the current observation to the past observations:

$$\lambda_{t,k}^B = \log \frac{p(y_t, y_{t-1}, \ldots, y_{t-k+1}|H_1)}{p(y_t, y_{t-1}, \ldots, y_{t-k+1}|H_0)} \tag{3}$$

We can express the BLLR approximately with the conventional LLR as

$$\lambda_{t,k}^B = \lambda_t - \lambda_{t-k}, \quad k = 1, 2, \ldots, n \tag{4}$$

with $\lambda_0 = 0$ by assuming $p(y_{1:t}) = p(y_{1:k})p(y_{k+1:t}|y_{1:k}) \approx p(y_{1:k})p(y_{k+1:t})$ ($y_{1:k}$ and $y_{k+1:t}$ are independent), and the decision rule for acceptance of the hypothesis that the system is in the fault mode can be restated as

$$\lambda_{t,k}^B = \lambda_t - \lambda_{t-k} > K \text{ for some } k = 1, 2, \ldots, t \tag{5}$$

or,

$$\lambda_t - \min_{1 \le k \le t} \lambda_k > K \tag{6}$$

Introducing the statistics called the maximum BLLR,

$$\begin{aligned} S_t &= \max[0, S_{t-1} + \ell_t], \quad t = 1, 2, \ldots \\ S_0 &= 0 \end{aligned} \tag{7}$$

with

$$\ell_t = \log \frac{p(y_t|y_{1:t-1}, H_1)}{p(y_t|y_{1:t-1}, H_0)} \quad t = 1, 2, \ldots \tag{8}$$

where $p(y_t|y_{1:t-1}, H_m)$ is the one step output prediction density of y_t under the hypothesis H_m, $(m = 0, 1)$, then the decision rule based on the BLLR can be expressed as

"*If $S_t > K$, where K is a suitable constant, one terminates observation with acceptance of the hypothesis that the system is in the fault mode. Otherwise, one continue observations as the system is likely not in the fault mode.*"

To compute ℓ_t in the statistics (7), we can use the grid approximation

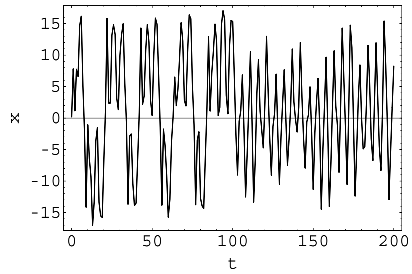
$$p(x_t|y_{1:t}, H_m) \approx \sum_{i=1}^{n} w_{t|t}^{(i,m)} \delta(x_t - x_{t|t}^{(i,m)}), \quad (m = 0, 1) \tag{9}$$

where the second superscript m is corresponding to the models. Then the pdf $p(x_t|y_{1:t-1}, H_m)$ $(m = 0, 1)$ can be approximated as

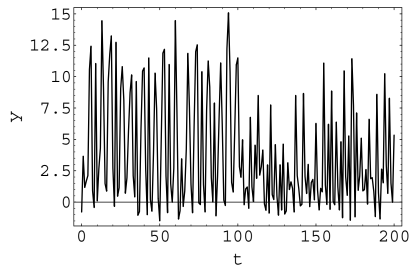
$$p(x_t|y_{1:t-1}, H_m) \approx \sum_{i=1}^{n} w_{t-1|t-1}^{(i,m)} p_v(x_t - f^{(m)}(x_{t-1|t-1}^{(i,m)}))$$

On the other hand, we can approximate the pdf $p(y_t|y_{1:t-1}, H_m)$ in (8) by

$$p(y_t|y_{1:t-1}, H_m) \approx \frac{1}{n} \sum_{i=1}^{n} p_w(y_t - g^{(m)}(x_{t|t-1}^{(i,m)}))$$



(a) State



(b) Observation

Figure 1: Sample behavior of state and observation processes

where $x_{t|t-1}^{(i,m)}$, $(i = 1, \ldots, n)$ are samples from the pdf $p(x_{t-1} \ |y_{1:t-1}, H_m)$. Thus, we can conduct the fault detection by evaluating the BLLR S_t with the pdf estimates obtained by two ESP filters under the system model H_m, $(m = 0, 1)$ and compare S_t with suitable threshold K.

5 Numerical Examples

To exemplify the applicability of the proposed ESP filters, we carried out a numerical simulation. We consider the following nonlinear state space model with known parameters.

$$
\begin{aligned}
x_t &= \frac{x_{t-1}}{2} + \frac{a^{(m)} x_{t-1}}{1 + x_{t-1}^2} + 8\cos(1.2t) + v_t \\
&= f^{(m)}(x_{t-1}) + v_t, \qquad (m = 0, 1) \\
y_t &= \frac{x_t^2}{20} + w_t = g^{(m)}(x_t) + w_t
\end{aligned}
\tag{10}
$$

with $a^{(0)} = 25$ for normal mode and $a^{(1)} = 12.5$ for fault mode, and v_t and w_t are i.i.d. zero-mean Gaussian random variates with variance 10 and 1, respectively. We assume that the fault occurs at $t = \tau = 101$. A sample behavior of the true state and corresponding observation processes is shown in Fig.1. Here the Gaussian distribution with mean $f(x_{t-1|t-1}^{(i)})$ and variance 10 is chosen as the importance density $q(x_t | x_{t-1|t-1}^{(i)}, y_{1:t})$.

Sample behaviors of state estimates by ESP(,) with $n = 10$, $r = 2$ based on the model H_m, $(m = 0, 1)$, and BLLR S_t and λ_t are given in Fig. 2 with corresponding results by EKF as well for comparison.

The test statistics BLLR S_t takes positive value and is growing up rapidly after the change point τ both in ESP and EKF, we can detect the model change when the BLLR exceeds the suitable threshold K. Moreover, it should be noted that, as shown in Fig. 2, the state estimate by EKF shows poor behavior and hence the behavior of test statistics sometimes provides poor detection result. Eventually, the rate of false alarm[1] and miss alarm[2] are higher by the detection procedure using EKF than by the procedure using ESP as shown in Table 1 that summarizes 10 simulation results of fault detection with the threshold $K = 25$. These results illustrate the applicability of the proposed approach for fault detection of nonlinear stochastic state space models. By introducing the other choice of evolution processes such as crossover and suitable choice of evolution parameters it is expected the improvement of the performance, and their better choice will be pursued.

Table 1: Fault detection result

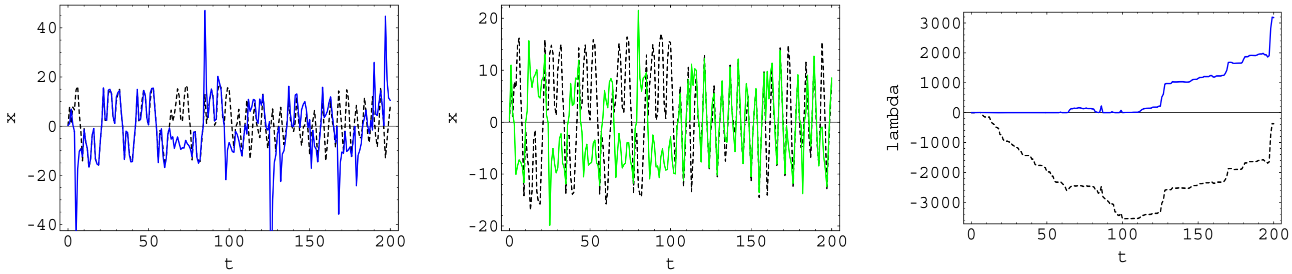
	False alarm rate	Miss alarm rate
Fault detection by ESP	1/20	0/20
Fault detection by EKF	5/20	1/20

6 Conclusions

Fault detection in dynamic systems has attracted considerable attention in designing systems with safety and reliability. Though a large number of methods have been proposed for solving the fault detection problem, it is hardly apply to nonlinear stochastic state space models. A novel filter called the Evolution Strategies based particle filter (ESP) proposed by recognizing the similarities and the difference of the processes between the particle filters and Evolution Strategies is applied here to fault detection of nonlinear stochastic state space models. Numerical simulation studies have been conducted to exemplify the applicability of this approach.

[1] The test statistics exceeds the threshold, i.e., the decision that system model has changed is made even when the system model does not change.

[2] The test statistics never exceeds the threshold, i.e., decision that the system model does not change is made even when model changes.

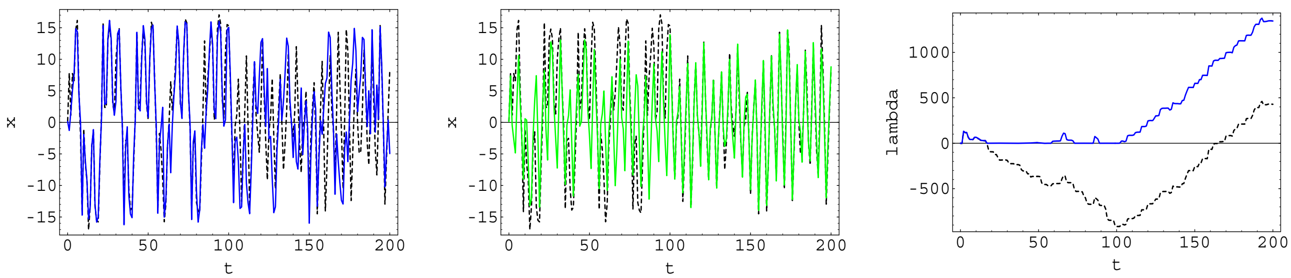


State estimate with model H_1 (solid line : estimate; dotted line : true state)

State estimate with model H_2 (solid line : estimate; dotted line : true state)

BLLR S_t and conventional LLR λ_t (solid line : S_t; dotted line : λ_t)

(a) EKF



State estimate with model H_1 (solid line : estimate; dotted line : true state)

State estimate with model H_2 (solid line : estimate; dotted line : true state)

BLLR S_t and conventional LLR λ_t (solid line : S_t; dotted line : λ_t)

(b) ESP

Figure 2: Sample behaviors of state estimates and test statistics by ESP and EKF

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