Computer Simulation of an Augmented Automatic Choosing Control Designed by Hamiltonian and Absolute Anti-windup Measure

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Abstract

In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) for nonlinear systems with constrained input. Constant terms which arise from sectionwise linearization of a given nonlinear system are treated as coefficients of a stable zero dynamics. Parameters included in the control are suboptimally selected by Hamiltonian and absolute anti-windup measure with the aid of the genetic algorithm.

1 Introduction

A genetic algorithm (GA) is one of evolutionary computing algorithms in engineering sciences[1]. The GA has been used to solve such complicated tasks as nonlinear global optimization problems. The purpose of this paper is to present a nonlinear feedback control called AACC (Augmented Automatic Choosing Control), which is designed by making good use of the GA.

Generally, it is easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[2]∼[7]. One of most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion truncated at the first order and then the linear optimal control method. This is only effective in a small region around the steady state point or in almost linear systems. Controllers based on a change of coordinates in differential geometry are effective in wider region, but it is not easy to implement them to practical systems. Controllers based on Fuzzy linearization are more practical, but they usually need a lot of complicated divisions. In many physical problems of interest to the control engineer there are various constraints on the control vector. Recently many controls with constraints have been studied.

In this paper we present the AACC with constrained input using the GA for nonlinear systems and its design procedure is as follows. Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls. These LQ controls are smoothly united by sigmoid type automatic choosing functions to synthesize a single nonlinear feedback controller.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, and gradients of the automatic choosing function. These parameters must be selected optimally so as to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the GA suboptimally. The suboptimal values of these parameters are obtained by minimizing a performance function. It is made of the Hamiltonian and an absolute anti-windup measure which is a time-derivative of quadratic function.

This approach is applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results show that the new controller enables us to improve performance remarkably well.

2 Augmented Automatic Choosing Control

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbf{D} \tag{1}$$

subject to

$$u_{j,min} \leq u[j] \leq u_{j,max} \quad (j = 1, \cdots, r) \tag{2}$$

where $\cdot = d/dt$, $x = [x[1], \cdots, x[n]]^T$ is an n-dimensional state vector, $u = [u[1], \cdots, u[r]]^T$ is an

r-dimensional control vector, $f(x) : \mathbf{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x) : \mathbf{D} \to R^{n\times r}$ is a driving matrix with $g(0) \neq 0$ and is continuously differentiable, $\mathbf{D} \subset R^n$ is a domain , and T denotes transpose.

Considering the nonlinearity of the system (1), introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x[2]$ is the element which has the highest nonlinearity of (1), then

$$C(x) = x[2] \in D \subset R \quad (L = 1)$$

(see Section 4). The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \leq i \leq M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L}[a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \quad (x[n+1](0) \simeq 1,\ 0 < \sigma_i < 1) \tag{3}$$

where the value of σ_i shall be selected so that $\sigma_i = -\dot{x}[n+1]/x[n+1] \leq -\dot{x}[k]/x[k]$ holds for all $k (k = 1, \cdots, n)$. This tries to keep $x[n+1] \simeq 1$ for a good while when the system (1) is not on $C^{-1}(D_0)$.

Combine (1) with (3) to form an augmented system

$$\dot{\mathbf{X}} = \bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u \tag{4}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \in \mathbf{D} \times R$$

$$\bar{f}(\mathbf{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \quad \bar{g}(\mathbf{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

Let a cost function be

$$J = \frac{1}{2}\int_0^\infty \left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T\mathbf{R}u\right) dt \tag{5}$$

where

$$\mathbf{Q} = \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix}, \quad R \ni q > 0,$$

$Q = Q^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ which denote positive symmetrix matrices. Values of $\mathbf{Q}$ and $\mathbf{R}$ are properly determined based on engineering experience.

On each D_i, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$:

$$\begin{aligned} f(x) + g(x)u &\simeq A_i x + w_i + B_i u \\ &\simeq A_i x + w_i x[n+1] + B_i u \end{aligned} \tag{6}$$

where

$$\begin{aligned} A_i &= \partial f(x)/\partial x^T\big|_{x=\hat{X}_i},\ B_i = g(\hat{X}_i), \\ w_0 &= 0,\ w_i = f(\hat{X}_i) - A_i\hat{X}_i. \end{aligned}$$

That is, an approximation of (4) is

$$\dot{\mathbf{X}} = \bar{A}_i\mathbf{X} + \bar{B}_i u \quad \text{on} \quad C^{-1}(D_i) \times R \tag{7}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[3] to (5) and (7) yields

$$u_i(\mathbf{X}) = -\mathbf{R}^{-1}\bar{B}_i^T\mathbf{P}_i\mathbf{X} \tag{8}$$

where the $(n+1) \times (n+1)$ matrix $\mathbf{P}_i$ satisfies the Riccati equation :

$$\mathbf{P}_i\bar{A}_i + \bar{A}_i^T\mathbf{P}_i + \mathbf{Q} - \mathbf{P}_i\bar{B}_i\mathbf{R}^{-1}\bar{B}_i^T\mathbf{P}_i = 0. \tag{9}$$

Introduce an automatic choosing function of sigmoid type:

$$\begin{aligned} I_i(x) = \prod_{j=1}^{L}\Bigg\{1 &- \frac{1}{1+\exp\left(2N\left(C_j(x) - a_{ij}\right)\right)} \\ &- \frac{1}{1+\exp\left(-2N\left(C_j(x) - b_{ij}\right)\right)}\Bigg\} \end{aligned} \tag{10}$$

where N is positive real value, $-\infty \leq a_{ij}$ and $b_{ij} \leq \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Figure 1).

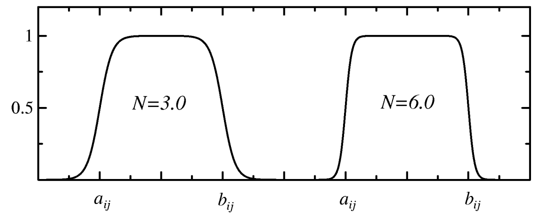

Figure 1: Automatic Choosing Function(N=3.0, 6.0)

Uniting $\{u_i(\mathbf{X})\}$ of (8) with $\{I_i(x)\}$ of (10) yields

$$\begin{aligned} \hat{u}(\mathbf{X}) &= [\hat{u}(\mathbf{X})[1], \cdots, \hat{u}(\mathbf{X})[j], \cdots, \hat{u}(\mathbf{X})[r]]^T \\ &= \sum_{i=0}^{M} u_i(\mathbf{X})I_i(x). \end{aligned}$$

We finally have an augmented automatic choosing control which is satisfied with the constraint condition (2) by

$$u(\mathbf{X}) = [u(\mathbf{X})[1], \cdots, u(\mathbf{X})[j], \cdots, u(\mathbf{X})[r]]^T \tag{11}$$

where

$$u(\mathbf{X})[j] = \begin{cases} u_{j,max} & \text{if} \quad \hat{u}(\mathbf{X})[j] \geq u_{j,max} \\ u_{j,min} & \text{if} \quad \hat{u}(\mathbf{X})[j] \leq u_{j,min} \\ \hat{u}(\mathbf{X})[j] & \text{otherwise} \end{cases}$$

$$(1 \leq j \leq r).$$

3 Parameter Selection by GA

The Hamiltonian for Eqs.(4) and (5) is given by

$$H(\mathbf{X},u,\lambda) = \frac{1}{2}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X}+u^T\mathbf{R}u\right) + \lambda^T\left(\bar{f}(\mathbf{X})+\bar{g}(\mathbf{X})u\right). \quad (12)$$

Assume that the adjoint vector $\lambda(\mathbf{X}) \in R^{n+1}$ is defined by

$$\lambda(\mathbf{X}) = [\lambda^I(\mathbf{X})^T, \lambda^{II}(\mathbf{X})^T]^T \quad (13)$$

where $\lambda^I(\mathbf{X}) = [\lambda[1],\cdots,\lambda[r]]^T = -(G^T(x))^{-1}\mathbf{R}u(\mathbf{X})$, $\lambda^{II}(\mathbf{X}) = [\lambda[r+1],\cdots,\lambda[n+1]]^T = [\mathbf{0},E]\hat{\lambda}$,

$$\hat{\lambda} = \sum_{i=0}^{M}\{(\bar{B}_i-\bar{g}(\mathbf{X}))\bar{g}(\mathbf{X})^\dagger + E\}^T\mathbf{P}_i\mathbf{X}I_i(x) \quad \in R^{n+1},$$

$\bar{g}(\mathbf{X})^\dagger\bar{g}(\mathbf{X}) = E$, E is an appropriate-dimentional unit matrix, and † denotes pseudo inverse.

There are two necessary conditions of the optimality. One of them is $\partial H/\partial u = 0$ or $u = -\mathbf{R}^{-1}\bar{g}(\mathbf{X})^T\lambda = -\mathbf{R}^{-1}G^T(x)\lambda^I(\mathbf{X})$, which is satisfied with Eq.(11) from Eq.(13). By it, Eq.(12) becomes

$$H(\mathbf{X},u,\lambda) = \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u^T\mathbf{R}u + \bar{f}^T(\mathbf{X})\lambda. \quad (14)$$

The other one is $\dot{\lambda} = -\partial H/\partial\mathbf{X}$.

Moreover, as a measure of suppressness of output windup which might arize by constraint inputs, we consider a swiftly decreasing motion of the following quadratic function:

$$V(t) = \frac{1}{2}\|\mathbf{X}(t)\|^2_{\widetilde{P}} = \frac{1}{2}\mathbf{X}^T(t)\widetilde{P}\mathbf{X}(t)$$

where $\widetilde{P}$ is $(n+1)\times(n+1)$ matrix. That is, the measure about the movement of $V(t)$ is assumed to be described by the absolute mean of time-derivative of $V(t)$:

$$\frac{1}{2}\int\left|\dot{V}(t)\right|/\mathbf{X}^T\mathbf{X}d\mathbf{X} = \int\left|\mathbf{X}^T(t)\widetilde{P}\dot{\mathbf{X}}(t)\right|/\mathbf{X}^T\mathbf{X}d\mathbf{X}$$
$$= \int\left|\mathbf{X}^T\widetilde{P}\left(\bar{f}(\mathbf{X})+\bar{g}(\mathbf{X})u\right)\right|/\mathbf{X}^T\mathbf{X}d\mathbf{X}.$$

Therefore in this paper we define the performance

$$PI = \omega_1\int_{\mathbf{D}}|H(\mathbf{X},u,\lambda)|/\mathbf{X}^T\mathbf{X}d\mathbf{X} + \omega_2\int_{\mathbf{D}}\left\|\dot{\lambda}+\partial H(\mathbf{X},u,\lambda)/\partial\mathbf{X}\right\|/\mathbf{X}^T\mathbf{X}d\mathbf{X} + \omega_3\int_{\mathbf{D}}\left|\mathbf{X}^T\widetilde{P}\left(\bar{f}(\mathbf{X})+\bar{g}(\mathbf{X})u\right)\right|/\mathbf{X}^T\mathbf{X}d\mathbf{X}, \quad (15)$$

where $\omega_i \geq 0 (i = 1,2,3)$ are weights. A set of parameters included in the control of Eq.(11) is $\bar{\Omega} = \{M,N,\hat{X}_i,a_{ij},b_{ij},\cdots\}$ which is suboptimally selected by minimizing PI with the aid of GA[1] as follows.

<ALGORITHM>

step1:A-priori: Set values $\bar{\Omega}_{apriori}$ appropriately.

step2:Parameter: Choose a subset $\Omega\subset\bar{\Omega}$ to be improved and rewrite it by $\Omega = \{M,N,\hat{X}_i\cdots\} = \{\alpha_k : k = 1,\cdot\cdot,K\}$.

step3:Coding: Represent each α_k with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.

step4:Initialization: Randomly generate an initial population of $\widetilde{q}$ strings $\{\Omega_p : p = 1,\cdot\cdot,\widetilde{q}\}$.

step5:Decoding: Decode each element α_k of Ω_p by $\alpha_k = (\alpha_{k,max}-\alpha_{k,min})A_k/\left(2^{\widetilde{L}}-1\right)+\alpha_{k,min}$ where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and A_k:decimal value of α_k.

step6:Control: Design $u = u(\mathbf{X})_p \quad (p = 1,\cdots,q)$ for Ω_p by using Eq.(11).

step7:Adjoint: Make $\lambda = \lambda(\mathbf{X})_p \quad (p = 1,\cdots,q)$ for Ω_p by using Eq.(13).

step8:Fitness value calculation: Calculate PI_p by Eqs.(14) and (15), or fitness $F_p = -PI_p$. Integration of PI_p is approximated by a finite sum.

step9:Reproduction: Reproduce each of individual strings with the probability of $F_p/\sum_{j=1}^{\widetilde{q}}F_j$.

step10:Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability P_c.

step11:Mutation: Alter a bit of string (0 or 1) by a mutation probability P_m.

step12:Repetition: Repeat step5∼step12 until prespecified G-th generation. If unsatisfied, go to step2.

As a result, we have a suboptimal control $u(\mathbf{X})$ for the string with the best performance over all the past generations.

4 Numerical Example

Consider a field excitation control problem of power system which is described[6][7] by

$$\widetilde{M}\frac{d^2\delta}{dt^2} + \widetilde{D}(\delta)\frac{d\delta}{dt} + P_e(\delta) = P_{in}$$
$$P_e(\delta) = E_I^2Y_{11}\cos\theta_{11} + E_I\widetilde{V}Y_{12}\cos(\theta_{12}-\delta)$$
$$E_I + T'_{d0}\frac{dE'_q}{dt} = E_{fd}$$
$$E_I = E'_q + (X_d - X'_d)I_d(\delta)$$
$$I_d(\delta) = -E_IY_{11}\sin\theta_{11} - \widetilde{V}Y_{12}\sin(\theta_{12}-\delta)$$

$$\widetilde{D}(\delta)=\widetilde{V}^2\left\{\frac{T''_{d0}(X'_d-X''_d)}{(X'_d+X_e)^2}\sin^2\delta + \frac{T''_{q0}(X_q-X''_q)}{(X_q+X_e)^2}\cos^2\delta\right\},$$

where δ: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, P_{in}: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, E_I: open circuit voltage, and E_{fd}: field excitation voltage. Put $x=[x[1],x[2],x[3]]^T=[E_I-\hat{E}_I,\delta-\hat{\delta}_0,\dot{\delta}]^T$ and $u=E_{fd}-\hat{E}_{fd}$, so that

$$\begin{bmatrix}\dot{x}[1]\\ \dot{x}[2]\\ \dot{x}[3]\end{bmatrix}=\begin{bmatrix}f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix}+\begin{bmatrix}g_1(x)\\ 0\\ 0\end{bmatrix}u \tag{16}$$

where $f_1(x)=-\frac{1}{kT'_{d0}}\left(x[1]+\hat{E}_I\right)+\frac{(X_d-X'_d)\widetilde{V}Y_{12}}{k}x[3]\cdot\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right)$, $f_2(x)=x[3]$, $f_3(x)=-\frac{\widetilde{V}Y_{12}}{\widetilde{M}}\cdot\left(x[1]+\hat{E}_I\right)\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right)-\frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)^2-\frac{\widetilde{D}(x)}{\widetilde{M}}x[3]+\frac{P_{in}}{\widetilde{M}}$, $\widetilde{D}(x)=\widetilde{V}^2\left\{\frac{T''_{d0}(X'_d-X''_d)}{(X'_d+X_e)^2}\cdot\sin^2\left(x[2]+\hat{\delta}_0\right)+\frac{T''_{q0}(X_q-X''_q)}{(X_q+X_e)^2}\cos^2\left(x[2]+\hat{\delta}_0\right)\right\}$, $g_1(x)=\frac{1}{kT'_{d0}}$, $k=1+(X_d-X'_d)\,Y_{11}\sin\theta_{11}$.

Assume that the constrained input is subject to $u_{min}+\hat{E}_{fd}\le E_{fd}\le u_{max}+\hat{E}_{fd}$.

Parameters are $\widetilde{M}=0.016095[pu]$, $T'_{d0}=5.09907[sec]$, $\widetilde{V}=1.0[pu]$, $P_{in}=1.2[pu]$, $X_d=0.875[pu]$, $X'_d=0.422[pu]$, $Y_{11}=1.04276[pu]$, $Y_{12}=1.03084[pu]$, $\theta_{11}=-1.56495[pu]$, $\theta_{12}=1.56189[pu]$, $X_e=1.15[pu]$, $X''_d=0.238[pu]$, $X_q=0.6[pu]$, $X''_q=0.3[pu]$, $T''_{d0}=0.0299[pu]$, $T''_{q0}=0.02616[pu]$.

Steady state values are $\hat{E}_I=1.52243[pu]$, $\hat{\delta}_0=48.57°$, $\hat{\dot{\delta}}_0=0.0[deg/sec]$, $\hat{E}_{fd}=1.52243[pu]$. Set $\mathbf{X}=[x^T,x[4]]^T=[x[1],x[2],x[3],x[4]]^T$, $n=3$, $\hat{X}_0=\hat{\delta}_0=48.57°$, $C(x)=x[2]$, $L=1$, $\mathbf{Q}=\text{diag}(1,1,1,1)$, $\mathbf{R}=1$, $\omega_1=\omega_2=1$, $\widetilde{P}=\mathbf{I}$ and $x[4](0)=1$, where $\mathbf{I}$ is $(n+1)\times(n+1)$ unit matrix. Experiments are carried out for the new control(AACC) and the ordinary linear optimal control(LOC)[2][3].

Table 1: Performances $\widetilde{J}$

Method	$x^T(0)$		
	$[0,0.6,0]$	$[0,1.0,0]$	$[0,1.6,-3]$
LOC	2.587	×	×
AACC($\omega_3=0.5$)	2.101	2.740	2.937
AACC($\omega_3=1$)	2.099	2.738	2.929

× : very large value

1) AACC($\omega_3=0.5$):
The parameters are suboptimally selected along the algorithm of section 3. $\omega_3=0.5$, $u_{max}=-u_{min}=0.5$, $\Omega=\{M,N,\hat{X}_i,a_{ij},b_{ij}\}$, $\widetilde{G}$=100, $\widetilde{q}$=100, $\widetilde{L}$=8, P_c=0.8, P_m=0.03, $\mathbf{D}$=[-1,1]×[-1,1.5]×[-5,5]×[0,1.5]. It results that $M=1$, $N=1.0$, $\hat{X}_1=95°$ and $a_1=b_0=49.0°$.

2) AACC($\omega_3=1$):
The parameters are suboptimally selected by using a similar way of the **AACC($\omega_3=0.5$)**. $\omega_3=1$. It results that $M=1$, $N=1.0$, $\hat{X}_1=95°$ and $a_1=b_0=49.7°$.

Table1 shows performances by the AACC and the LOC. The cost function of Table1 is $\widetilde{J}=\frac{1}{2}\int_0^{20}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X}+u^T\mathbf{R}u\right)dt$. These results indicate that the AACC is better than the LOC.

5 Conclusions

We have studied an augmented automatic choosing control using zero dynamics for nonlinear systems with constrained input. This approach have been applied to a field excitation control problem of power system. Simulation results have shown that this controller using the GA can improve performance remarkably well.

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