## THE WORK OF

#### SUEO UENO, ROBERT KALABA, AND RICHARD BELLMAN

Harriet H. Natsuyama Los Angeles, CA 90066 hhnatsu@yahoo.com

#### Abstract

This paper recalls the critical developments of three exceptional individuals in the fields of nonlinear analysis, systems modeling, computational solution of initial value problems, system identification and optimal control.

### 1 Introduction

This paper was suggested by Masanori Sugisaka who initiated the AROB symposia ten years ago and who attributes their success to the inspiration and contributions of three giants: Sueo Ueno, Robert Kalaba, and Richard Bellman. They are special for their great talents, productivity and personalities. I am privileged to have worked with them since the early 1960s.

# 2 Decades of Research

My relationship with these three giants of modern analysis and computing is very special to me. It goes back to the early days of the Rand Corporation, the quasi-governmental think tank in Santa Monica. In the decade of the 1950s, after their World War II efforts had ceased, many of the top scientists from Los Alamos National Laboratories joined the exciting new work at Rand.

John von Neumann developed the general-purpose digital computer (as opposed to special purpose military computers) in the basement and it was fondly called the Johnniac. Programmers used machine language to code paper tapes. The field of computer science had yet to be named, and the mathematicians using these computers called themselves numerical analysts.

When I joined Rand in 1961, IBM had introduced their mainframe computers and I learned Fortran by reading the manual written by McCracken. Fresh out of graduate school with a masters degree in physics, I had been hired to assist Bob Kalaba and Dick Bellman in their computational experiments. Dick Bellman had already introduced the principle of optimality and the concept of dynamic programming for nonlinear optimization problems. Some numerical examples had been worked out and more computations were needed. Dynamic programming, I learned during my later years, applies to life as well as manmade systems

Bob Kalaba had created the technique called quasilinearization to solve nonlinear two-point boundary value problems, such as those that arise when solving Euler equations from the calculus of variations. One of the first problems that we attacked together was that of orbit determination. Remember this was going on during the early days of the space program. We formulated the problem of estimating the orbit of a moving body whose angular position has been observed at various instants of time. In other words, a complete set of initial conditions would be sought such that the theoretical orbit explained the observations in a least squares sense. This is a multipoint boundary value problem.

We extended Bob's quasilinearization to handle the multiple conditions. We were delighted to see that solutions converged quadratically and we could determine the missing initial conditions even when the initial estimates were rather crude. We also introduced noise into the measurements and it still worked. Then we tried estimating the unknown mass of the moving object, letting its dynamical equation be that the time derivative of mass is zero and requiring that its initial condition be determined along with the other initial conditions for the equations of motion. This experiment was successful also. The results were published in the Proceedings of the National Academy of Sciences, and there were many, many requests from aerospace companies for reprints of this paper.

A couple of years later, Sueo Ueno from Kyoto University arrived at Rand. This distinguished astrophysicist had been in correspondence with Dick Bellman for a number of years. Dick wanted to learn more about the multiple scattering of radiation through slabs of finite thickness. Dick saw that there was a resemblance with the problems of neutron transport which he had dealt with at Los Alamos with Milt Wing using the new method of invariant imbedding. But the physical aspects seemed more complex. He wanted Prof. Ueno to spend some time at Rand so they could apply invariant imbedding to radiative transfer.

Prof. Ueno, during these correspondences, was spending a year with Jacqueline Lenoble and other French scientists in Paris, with side trips to visit Ida Busbridge in Cambridge, Ambarzumian in Leningrad, and Sobolev in Moscow. Then he had to return to Kyoto University for another year before he could go abroad again. Thus it was that he arrived in Santa Monica after I had been there for a while.

Prof. Ueno taught us the physical principles of multiple scattering so that we could "count photons" for invariant imbedding and derive initial value problems for the Riccati equations of reflection functions. We, or rather it was I, who wrote the Fortran programs for large systems of ordinary differential equations with initial conditions. When we integrated these equations using fourth-order Runge-Kutta, the solutions agreed with those previously published for very thin or very thick slabs. Furthermore, we obtained solutions for all thicknesses between zero and effectively infinity. We saw reflection functions that no one else had seen before!

This research program in which I was involved led to Prof. Ueno becoming my advisor for the doctoral degree in *uchu butsuri* in the Department of Astrophysics of Kyoto University. In particular I investigated and demonstrated various techniques for solving inverse problems of atmospheric physics as system identification problems.

While the approach of invariant imbedding was regarded by Bob and Dick as dynamic programming without the optimization, it led to a nonlinear filter, a powerful extension of the Kalman filter. This filter was developed in collaboration with R. Sridhar, and further developed by M. Sugisaka. Indeed, Prof. Sugisaka was introduced to this nonlinear filter by Sueo Ueno, and that is how we came to meet each other and how I spent three delightful months in his department at Oita University in the fall of 1995.

In subsequent years, I had the opportunity to spend time on various occasions with Prof. Ueno and learn about his life and his work. By this time, he had retired from Kyoto University and Kanazawa Institute of Technology, and he was head of information systems at Kyoto Computer Gakuin. We produced the Springer book on terrestrial radiative transfer with Alan Wang. I compiled Prof. Ueno's collected works and deposited them in the new library of Kyoto University where they are available for researchers and students alike. Prof. Ueno lives quietly in Yokohama.

In the late sixties both Dick and Bob left Rand to become professors at the University of Southern California. Dick, who established a program in biomathematics, passed away in 1984.

Robert Kalaba held a joint appointment at the University of Southern California in the departments of economics and biomedical engineering. There, he prepared a new generation of students and was well-loved by students and staff alike. Bob passed away at the end of September, 2004. Dr. Yueyue Fan, the last of Bob's doctoral students, and I are in the process of organizing Bob's papers so that a library can be established in his memory at USC.

### 3 Summary

Richard Bellman, Robert Kalaba, and Sueo Ueno – to these pioneers we owe great thanks. And I thank you for your interest in continuing these explorations.

### 4. Bibliography

Richard E. Bellman: see *Memorial Tributes*, National Academy of Engineering, Vol. 3, pp 22-29, 1989, and *The Bellman Continuum*, Robert S. Roth (ed.), World Scientific Pub. Co., 1986.

Robert E. Kalaba: see the special issue, *Applied Mathematics and Computation*, Vol. 45, n. 2, September 1991.

Sueo Ueno: see the special issue, *Applied Mathematics* and *Computation*, Vol. 116, n. 1-2, November 2000, and *Terrestrial Radiative Transfer: Modeling, Computation,* and Data Analysis, H. H. Natsuyama, S. Ueno, A.P. Wang, Springer-Verlag Telos, 1998.