Calibration and Control Experiments on Redundant Legs of a Stewart Platform based Machine Tool

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Abstract

In recent robotics research, a parallel-kinematic manipulator has been increasingly studied for possible use as a machine tool due to the advantages of high stiffness and accuracy over serial-kinematic manipulators. In general, a spatial parallel manipulator has some limitations for increasing the stiffness only with six actuators. In order to further increase the stiffness of a machine tool, the method to add more than one additional actuator may be considered, although it may cause some more cost and restrict the workspace to some extent. In this paper, a prototype Stewart platform based machine tool with two redundant legs is demonstrated. The passive force controller for the redundant legs is suggested and the kinematic calibration of the redundant legs is performed. Finally, cutting experiment result is presented to show the effectiveness of the redundant actuation method.

1 Introduction

It has been well recognized that the Gough-Stewart type parallel manipulator, or referred to here shortly as the Stewart platform, has some advantages over serial-type manipulators in view of positioning accuracy and stiffness [1]. Among all the possible applications, the Stewart platform interests many researchers especially in using it as a machine tool. In designing and evaluating a machine tool, stiffness may be one of the most important factors to be considered, since the stiffness directly affects accuracy in machining applications. Although a parallel manipulator is usually stiffer than a serial manipulator, it is made up of several serial chains. For example, a Stewart platform consists of 6 serial chains, which can be modeled as 6 linear springs connecting base to moving platform. If there are limitations to increase the stiffness of each serial chain, the remaining way to further increase the Cartesian stiffness is to add more serial chains, i.e., springs in parallel.

With this regards, the redundant actuation method to add more than one additional serial chain is suggested. In general, the stiffness of a Stewart platform along the X- and Y-axes is smaller that that along the Z-axis. Therefore, in this work, two redundant legs are placed on the XY plane, in order to further increase the stiffness along the X- and Y-axes; One (7th leg) is mounted along the X-axis and the other (8th leg) is along the Y-axis as shown in Figs. 1 and 2.

Since the six legs of a Stewart platform fully define the position and orientation of the end-effector, the lengths of the two redundant legs cannot be arbitrarily determined. When there exist some kinematic errors in the redundant legs, the legs need to have some compliance not to break the system. To give some compliance to the redundant legs and to make the redundant legs act like linear spring, a passive force control method is developed. In order to reduce undesired internal forces between the redundant legs and the Stewart platform, the initial lengths of the springs should be accurately determined. For that purpose, the kinematic calibration method of using constrained optimization is suggested. The experiment result of the calibration shows that the suggested algorithm is more robust to measurement noises that the previous ones [2-5].

This paper is organized as follows: First, a prototype Stewart platform based machine tool with two redundant legs is demonstrated. The passive force controller for the redundant legs is suggested and the kinematic calibration of the redundant legs is performed. Finally, cutting experiment result is presented to verify the effectiveness of the redundant actuation method.

2 System Configuration

The overall system of the Stewart platform based machine tool system with two redundant legs is shown in Fig. 1. The kinematic parameters of the manipulator are as follows (refer to Fig. 2 and [6]):

$r_b = 400$, $r_m = 150$,

$l_{i,\min} = 801$, $\Delta l_i = 364$, for $i = 1,2,\ldots,6$

where r_b and r_m denote the radii of the base and moving platforms, respectively, $l_{i,\min}$ and Δl_i denote the minimum length and stroke of a leg, and the unit of length is millimeter. The locations of the spherical joints of the machine tool with respect to each coordinate system can be expressed by

$$\boldsymbol{b}_i = r_b[\cos\Lambda_i, \sin\Lambda_i, 0]^T$$
$$\boldsymbol{M}_i = r_m[\cos\lambda_i, \sin\lambda_i, 0]^T, \text{ for } i = 1,2,\ldots,6$$

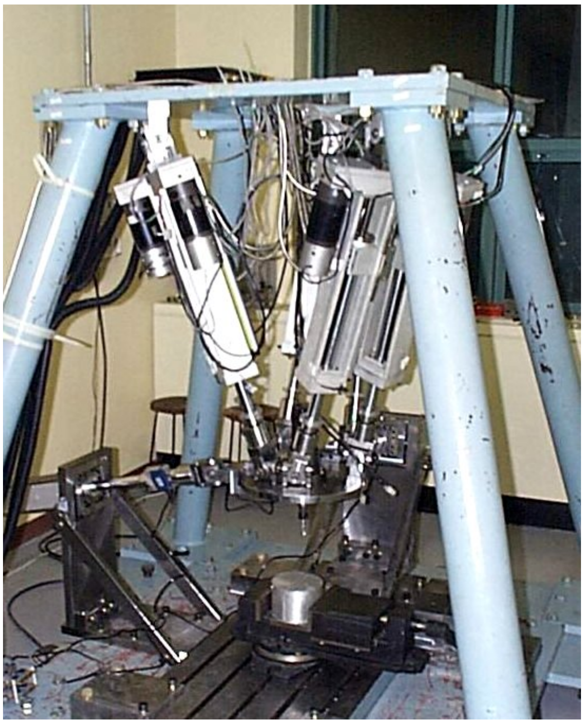

Fig. 1 Prototype Stewart platform based machine tool with two redundant actuators.

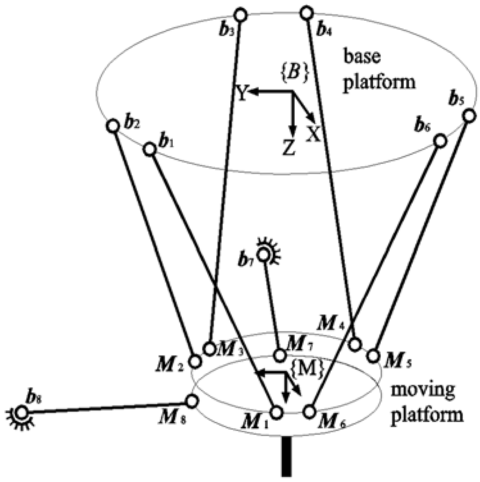

Fig. 2 Kinematic configuration of the manipulator.

where $\Lambda = [60^\circ - \phi_b, 60^\circ + \phi_b, 180^\circ - \phi_b, 180^\circ + \phi_b, -60^\circ - \phi_b, -60^\circ + \phi_b]$, $\lambda = [\phi_m, 120^\circ - \phi_m, 120^\circ + \phi_m, -120^\circ - \phi_m, -120^\circ + \phi_m, -\phi_m]$, $\phi_b = 10^\circ$ and $\phi_m = 16^\circ$.

The minimum length and stroke of a redundant actuator are given by

$$l_{i,\min} = 291,\ \Delta l_i = 364,\ \text{for}\ i = 7,8\ .$$

The locations of the spherical joints of the redundant actuators with respect to each coordinate system are given by

$$\boldsymbol{b}_7 = [-725, 0, 985]^T,\ \boldsymbol{b}_8 = [0, 725, 1045]^T$$

$$\boldsymbol{M}_7 = [-255, 0, 62]^T,\ \boldsymbol{M}_8 = [-255\sin 10^\circ, 255\cos 10^\circ, 62]^T$$

3 Passive Force Control

Since the six legs of the Stewart platform fully define the position and orientation of the end-effector, the lengths of the two redundant legs cannot be arbitrarily determined. If all the kinematic parameter values were perfectly known, the lengths of the redundant legs can be simply determined by the inverse kinematics. However, since it is almost impossible to know the exact kinematic parameter values, the redundant legs must have some compliance, otherwise, the manipulator may not move or very large internal forces may be generated, which could break some parts.

For better stability, the following passive force controller is suggested, which can provide two virtual linear springs to the moving platform in cutting as shown in Fig. 3. Therefore, the static and dynamic errors of the moving platform due to cutting force may be reduced. In this work, the passive force controller with the trajectory estimator is suggested by

$$\text{Control Law:}\quad f = k_p(\hat{x} - x) - k_d\dot{x} \qquad (1)$$

$$\text{Trajectory Estimator:}\quad \hat{x} = x_d + \frac{1}{k_f} f_s \qquad (2)$$

where x_d and x denote respectively desired and actual positions and $\dot{x}$ is the derivative of actual position with respect to time. f_s is the measured force from the load cell mounted at the end of a redundant leg. $\hat{x}$ is the estimated trajectory based on the information of the force sensor. k_p and k_d are the proportional and derivative gains, respectively, and k_f corresponds to the stiffness of a redundant leg. The block diagram of the suggested passive controller is shown in Fig. 4. The reason for using the trajectory estimator instead of the traditional stiffness controller is that the back-drivable force of redundant actuators is relatively large.

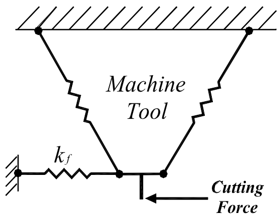

Fig. 3 Planar representation of the passive force controllers.

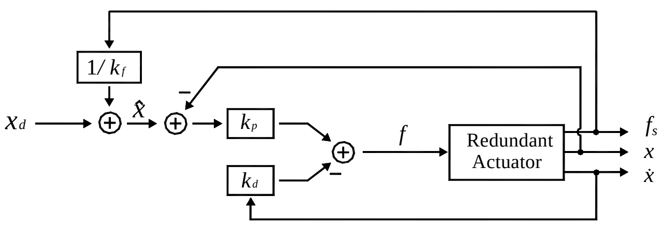

Fig. 4 Block diagram of the suggested passive force controller.

4 Kinematic Calibration

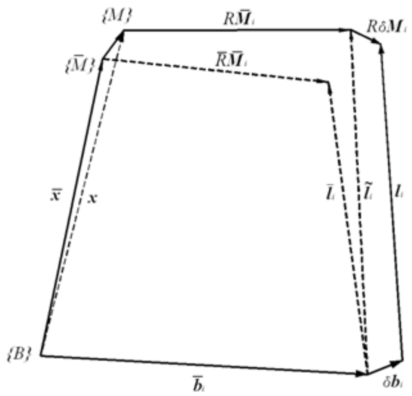

Fig. 5 Kinematic error model.

4.1 Kinematic Error Model

In Fig. 5, the outer vector loop represents the actual model to be estimated through calibration, and the inner vector loop indicates the nominal model based on the CAD data. The actual leg length and position vectors of the spherical joints can be written as

$$l_i = \bar{l}_i + \delta l_i\,,\ \boldsymbol{M}_i = \bar{\boldsymbol{M}}_i + \delta \boldsymbol{M}_i\,,\ \boldsymbol{b}_i = \bar{\boldsymbol{b}}_i + \delta \boldsymbol{b}_i \qquad (3)$$

where the kinematic parameters for a nominal model are expressed with an upper bar. δl_i , $\delta \boldsymbol{M}_i$ and $\delta \boldsymbol{b}_i$ are the kinematic errors of a leg length and locations of spherical joints at the moving and base plates, which will be estimated by the kinematic calibration.

Using the vector loop in Fig. 5, an actual leg can be expressed by

$$\boldsymbol{l}_i = \tilde{\boldsymbol{l}}_i + R\,\delta\boldsymbol{M}_i - \delta\boldsymbol{b}_i \qquad (4)$$

where $\tilde{\boldsymbol{l}}_i$ is defined as a virtual leg by [6-8]

$$\tilde{\boldsymbol{l}}_i = \boldsymbol{x} + R\bar{\boldsymbol{M}}_i - \bar{\boldsymbol{b}}_i \qquad (5)$$

where $\boldsymbol{x}$ and R denote the actual position vector and rotation matrix of the end-effector. The quadratic form of Eq. (4) can be written as the following:

$$\begin{aligned} l_i^2 &= (\bar{l}_i + \delta l_i)^2 = \tilde{l}_i^{\,2} + \|\delta\boldsymbol{M}_i\|^2 + \|\delta\boldsymbol{b}_i\|^2 \\ &+ 2(\tilde{\boldsymbol{l}}_i^{\,T} R\,\delta\boldsymbol{M}_i - \tilde{\boldsymbol{l}}_i^{\,T}\delta\boldsymbol{b}_i - \delta\boldsymbol{M}_i^{\,T} R\,\delta\boldsymbol{b}_i) \end{aligned} \qquad (6)$$

4.2 Kinematic Calibration Method

When there exist some measurement errors not to be negligible, the calibrated kinematic values may be updated to an undesirable direction. Although the exact information about the kinematic errors cannot be known without the well-organized calibration method, the bounds of the kinematic errors may be estimated. When the updated kinematic parameters will go to an unexpected point due to measurement noises, it may be required or useful to impose the inequality constraints on the kinematic errors. Furthermore, even in absence of noise in the measurement, the previous calibration algorithms may lead to up to 20 different values for the kinematic parameter [5]. With this regards, the optimization with inequality constraints is suggested for the kinematic calibration. The constrained optimization problem for the kinematic calibration can be stated as follows: [8]

$$\begin{aligned} &\text{Minimize} \quad : H = \sum_{k=1}^{m} F_k^{\,2} \\ &\text{Subject to} \quad : |\delta l_i| \le e_l\,, \|\delta\boldsymbol{M}_i\| \le e_m\,, \|\delta\boldsymbol{b}_i\| \le e_b \end{aligned} \qquad (7)$$

where the subscript $k = 1, \dots, m$ denotes the number of each measurement, the bounds of kinematic parameter errors can be obtained from the information on tolerances of parts and assembling errors, and the objective function is given by

$$\begin{aligned} F_k \equiv\ & (\bar{l}_i + \delta l_i)^2 - \tilde{l}_{i,k}^{\,2} - \|\delta\boldsymbol{M}_i\|^2 - \|\delta\boldsymbol{b}_i\|^2 \\ & - 2(\tilde{\boldsymbol{l}}_i^{\,T} R_k\delta\boldsymbol{M}_i - \tilde{\boldsymbol{l}}_i^{\,T}\delta\boldsymbol{b}_i - \delta\boldsymbol{M}_i^{\,T} R_k\delta\boldsymbol{b}_i) \end{aligned} \qquad (8)$$

4.3 Experiment Results

In this section, the effectiveness of the constrained optimization is verified through the calibration experiments on two redundant actuators of the Stewart platform in Fig. 1. Since the proposed passive force controller for redundant legs is basically based on a position control, the accurate kinematic information becomes one of the most important factors in control. Since the lengths of six non-redundant legs are given, the position and orientation of the end-effector can be obtained from the forward kinematics, which will be used for the measurement positions and orientations in the calibration of redundant legs.

In order to show the effectiveness of the proposed constrained optimization method, the results from the unconstrained and constrained optimization methods have been compared. The bounds of the kinematic errors are assumed as

$$|\delta l_{7,8}| < 0.05\,,\ \|\delta\boldsymbol{b}_{7,8}\| < 50\,,\ \|\delta\boldsymbol{M}_{7,8}\| < 1\ [\text{mm}]\,.$$

It is noted that the bounds of the constraints are based on the information about tolerances of parts and assembling errors. It can be seen that the kinematic errors from the constrained optimization are obtained within the ranges of constraints.

In Fig. 6, the errors between the measured and the calculated lengths before and after calibration are plotted at the sixteen measured points. From these plots, it can be said that the proposed calibration algorithm using constrained optimization is more effective than the previous algorithm using unconstrained optimization. The length errors from the constrained optimization are within ± 1 mm.

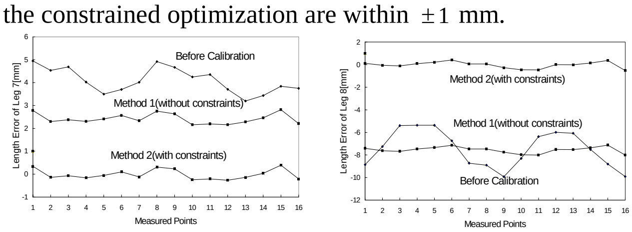

Fig. 6 Calibration results of the redundant legs.

5 Cutting Experiment

In order to show the effectiveness of the redundant actuation scheme, the dynamic errors in cutting have been measured both for the non-redundant and redundant cases. Some of the cutting conditions for the machining experiment are as follows:

Tool: flat end-mill with 2 flutes and $\phi 10$ [mm],
Feedrate: 30 [mm/min],
Spindle speed: 2000 [rpm],
Material: Al 2024,
Measurement device: resolution with 1[μm].

The cutting direction is along the X-axis, and using the linear encoder, the dynamic error in the cutting operation is measured along the X- and Y-axes for both non-redundant and redundant cases as shown in Fig. 7. In Fig. 8(a) and (c), the dynamic errors along the X- and Y-axes are plotted when no redundant legs are used. In Fig. 8(b) and (d), the dynamic errors are plotted when redundant legs are used. From Fig. 8, it can be seen that the dynamic error along the Y-axis, i.e., perpendicular to the cutting direction is larger than that along the X-axis, i.e., the cutting direction. For the cutting experiment with 2mm depth, the maximum dynamic error along the Y-axis is reduced about from 120 μm (without redundant actuation) to 60 μm (with redundant actuation). However, it is noticed that the static error in the redundant case is much larger than that in the non-redundant case. The major source of relatively large static error in the redundant case is the inaccurate kinematic information.

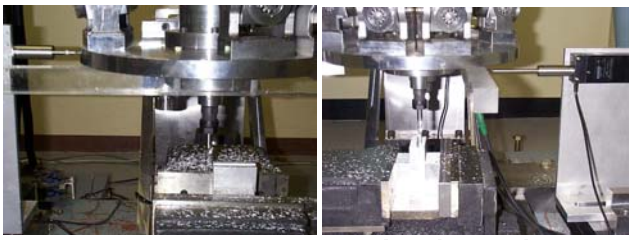

Fig. 7 Dynamic error measurement along the X- and Y-axes.

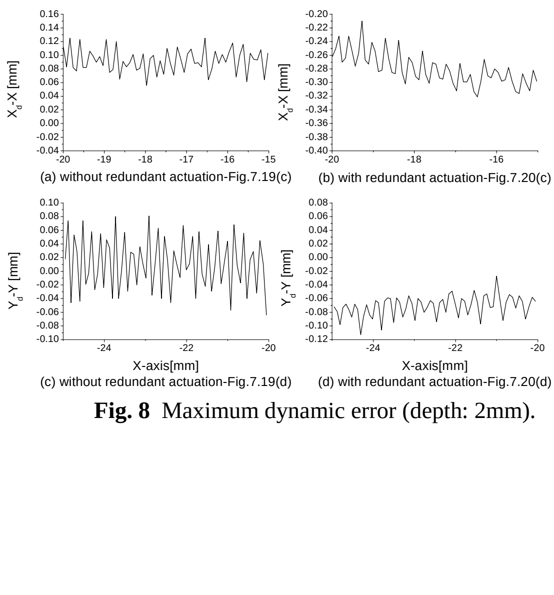

Fig. 8 Maximum dynamic error (depth: 2mm).

6 Conclusions

The prototype Stewart platform based machine tool with two redundant legs is developed. For the redundant legs, the passive force controller is suggested, which provides virtual linear springs to the cutting tool so as to increase the stiffness along the X- and Y-axes and to reduce the dynamic error in cutting. The kinematic calibration method of using constrained optimization is suggested, which is robust to measuring noise. The calibration experiment on the two redundant legs shows that the constrained optimization method provides more reasonable solution than the unconstrained one. Using the updated kinematic parameters of the redundant legs and the suggested controller, the cutting experiment is performed. It shows that the redundant actuation scheme can increase the overall stiffness and reduce the dynamic error. However, it may yield larger static error if the kinematic calibration is not perfect. In the further works, we will focus on reducing the static error.

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References

[1] A. J. Wavering, "Parallel Kinematic Machine Research at NIST: Past, Present, and Future," *Parallel Kinematic Machines*, pp. 17-32, Milano, Italy, June, 1999.

[2] H. Zhuang and Z. S. Roth, "Method for Kinematic Calibration of Stewart Platforms," *Journal of Robotics Systems*, vol. 10, no. 3, pp. 391-405, 1993.

[3] Z. J. Geng and L. S. Haynes, "An Effective Kinematic Calibration Method for Stewart Platforms, " *ISRAM*, pp. 87-92, 1994.

[4] T. Roppenen and T. Arai, "Accuracy Analysis of a Modified Stewart Platform Manipulator," *IEEE International Conference on Robotics and Automation*, pp. 521-525, Nagoya, Aichi, Japan, 1995.

[5] C. Innocenti, "Algorithms for Kinematic Calibration of Fully-Parallel Manipulator," In J.-P. Merlet B. Ravani(eds.), *Computational Kinematics*, pp. 241-250, 1995.

[6] H. S. Kim, *Design and Control of a Stewart Platform based Machine Tool*, Ph. D. Thesis, Yonsei Univ., Korea, 2000.

[7] H. S. Kim and Y. J. Choi, "The Kinematic Error Bound Analysis of the Stewart Platform," *Journal of Robotic Systems*, vol.17, no.1 pp. 63-73, 2000.

[8] H. S. Kim, B. J. Kim and Y. J. Choi, "Kinematic Calibration for Redundant Legs of a Stewart Platform," *Proceedings of the 32nd International Symposium on Robotics*, Seoul, Korea, 19-21 Apr., 2001.