Cost Function Analysis of Optimizing Fuzzy Energy Regions in Control of Underactuated Manipulators

Kiyotaka Izumi, Keisuke Ichida, and Keigo Watanabe
Dept. of Advanced Systems Control Engineering,
Graduate School of Science and Engineering, Saga University,
1-Honjomachi, Saga 840-8502, Japan

Abstract

Underactuated manipulators have to be controlled by following a restricted way that the number of actuators is less than the number of generalized coordinates, because they have some passive joints.

In this paper, we propose a logic based switching mechanism using fuzzy energy region. Boundary curves to separate an energy region into some energy subregions are constructed by fuzzy reasoning. Therefore, fuzzy related parameters are optimized by genetic algorithm (GA). The present method is applied to an underactuated system with drift term such as an 2-DOF planar manipulator which has only one active joint. The effectiveness of the present method is illustrated with some simulations.

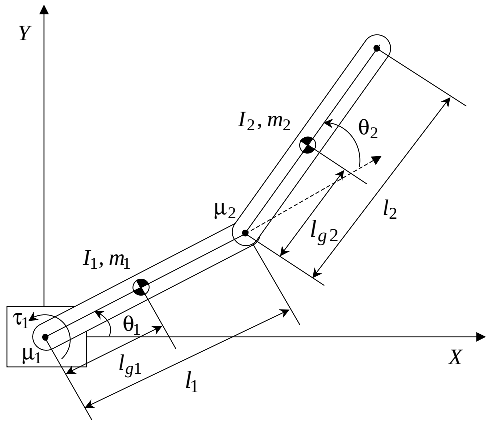

Figure 1: Model of 2-link underactuated manipulator

1 Introduction

Control of underactuated manipulators is an attractive research theme in robotics because of complex behaviors and difficulty of control [1, 2, 3]. Since underactuated manipulators have some passive joints, their energy efficiency can be better than full actuated manipulators.

As a control method for underactuated systems, authors have already proposed a switching control, in which some partly stable controllers were designed by computed torque method and the switching low was obtained as the index of controller directly by fuzzy reasoning [4]. The switching control is proposed to design simply a control low without any complex variable transformation for underactuated manipulators. The switching low is a key-point to obtain sufficient results in this method. A logic based switching method using energy regions [5] is also proposed for a nonholonomic system without drift term.

In this paper, we propose a logic based switching mechanism using fuzzy energy region. Boundary curves to separate an energy region into some energy subregions are constructed by fuzzy reasoning. Fuzzy related parameters and control gains of the partly stable controllers are optimized by genetic algorithm (GA). We prepare some cost functions of GA. The present method is applied to an underactuated system with drift term such as an 2-DOF planar manipulator which has only one active joint. The effectiveness of the present method is illustrated with some simulations.

2 Underactuated Manipulator

Figure 1 shows a two-link underactuated manipulator, in which the second joint is constructed of a passive joint. Here, τ_1 denotes the applying torque of 1st joint, θ_i denotes the angle of ith link, m_i denotes the mass of ith link, l_{gi} denotes the distance from the joint to the center of mass of ith link, I_i denotes the moment of inertia of ith link, and μ_i denotes the coefficient of viscous friction. The dynamical model of the underactuated manipulator is given as follows:

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) = \boldsymbol{\tau} \tag{1}$$

where

$$\begin{aligned} \boldsymbol{\theta} &= [\theta_1\ \ \theta_2]^T \\ \boldsymbol{\tau} &= [\tau_1\ \ 0]^T \end{aligned}$$

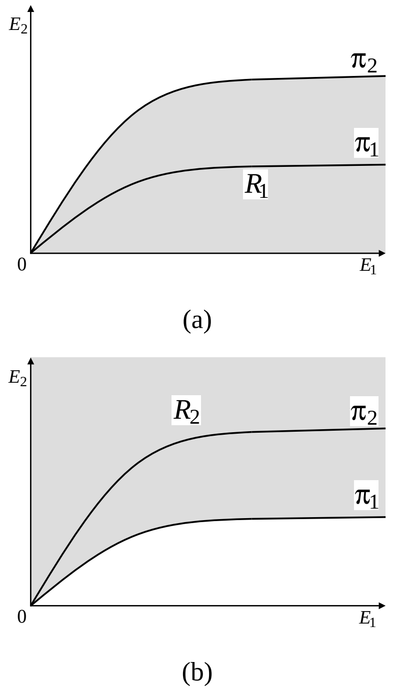

Figure 2: Subregions of energy

$$
\begin{aligned}
M(\boldsymbol{\theta}) &= \begin{bmatrix} M_{11}(\boldsymbol{\theta}) & M_{12}(\boldsymbol{\theta}) \\ M_{12}(\boldsymbol{\theta}) & M_{22}(\boldsymbol{\theta}) \end{bmatrix} \\
M_{11}(\boldsymbol{\theta}) &= (m_1 l_{g2}^2 + m_2 l_1^2 + I_1) + (m_2 l_{g2}^2 + I_2) \\
&\quad +2 m_2 l_1 l_{g2} \cos\theta_2 \\
M_{12}(\boldsymbol{\theta}) &= (m_2 l_{g2}^2 + I_2) + m_2 l_1 l_{g2} \cos\theta_2 \\
M_{22}(\boldsymbol{\theta}) &= m_2 l_{g2}^2 + I_2 \\
\boldsymbol{h}\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) &= \left[h_1\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)\ h_2\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)\right]^T \\
h_1\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) &= -(m_2 l_1 l_{g2})(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)\sin\theta_2 + \mu_1\dot{\theta}_1 \\
h_2\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) &= m_2 l_1 l_{g2}\dot{\theta}_1^2 \sin\theta_2 + \mu_2\dot{\theta}_2
\end{aligned}
$$

3 Fuzzy Region Based Switching Control

3.1 Partly stable controller

Equation (1) can be described by

$$
\begin{aligned}
\ddot{\theta}_1 &= -\frac{M_{22}(\boldsymbol{\theta})}{D} h_1\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) + \frac{M_{12}(\boldsymbol{\theta})}{D} h_2\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) \\
&\quad + \frac{M_{22}(\boldsymbol{\theta})}{D}\tau_1 \qquad (2) \\
\ddot{\theta}_2 &= \frac{M_{12}(\boldsymbol{\theta})}{D} h_1\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right) - \frac{M_{11}(\boldsymbol{\theta})}{D} h_2\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)
\end{aligned}
$$

$$
-\frac{M_{12}(\boldsymbol{\theta})}{D}\tau_1 \qquad (3)
$$

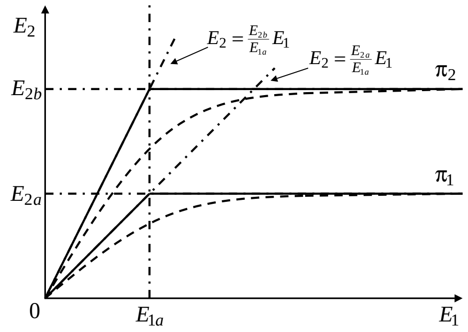

Figure 3: Approximation of regions for a logical switching

where

$$D = M_{11}(\boldsymbol{\theta}) M_{22}(\boldsymbol{\theta}) - M_{12}^2(\boldsymbol{\theta})$$

Here, it is found that we can design partly stable controllers for link 1 and link 2 using the computed torque method. The controller 1 to stabilize the link 1 is given by

$$
\begin{aligned}
\tau_1 &= \frac{D}{M_{22}(\boldsymbol{\theta})}\left(\ddot{\theta}_1^* + \frac{M_{22}(\boldsymbol{\theta})}{D} h_1\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)\right. \\
&\quad \left. -\frac{M_{12}(\boldsymbol{\theta})}{D} h_2\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)\right) \qquad (4) \\
\ddot{\theta}_1^* &= \ddot{\theta}_{d1} + K_{v1}\left(\dot{\theta}_{d1} - \dot{\theta}_1\right) + K_{p1}(\theta_{d1} - \theta_1)
\end{aligned}
$$

and the controller 2 to stabilize the link 2 is given by

$$
\begin{aligned}
\tau_1 &= -\frac{D}{M_{12}(\boldsymbol{\theta})}\left(\ddot{\theta}_2^* - \frac{M_{12}(\boldsymbol{\theta})}{D} h_1\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)\right. \\
&\quad \left. +\frac{M_{11}(\boldsymbol{\theta})}{D} h_2\left(\boldsymbol{\theta},\ \dot{\boldsymbol{\theta}}\right)\right) \qquad (5) \\
\ddot{\theta}_2^* &= \ddot{\theta}_{d2} + K_{v2}\left(\dot{\theta}_{d2} - \dot{\theta}_2\right) + K_{p2}(\theta_{d2} - \theta_2)
\end{aligned}
$$

where the desired vector of $\boldsymbol{\theta}$ is defined as $\boldsymbol{\theta}_d = [\theta_{d1}\ \ \theta_{d2}]^T$, in which the proportional gain of the controller i is K_{pi} and the derivative gain of the controller i is K_{vi}.

3.2 Logic based switching method

Energy of each link is defined by

$$E_i \triangleq e_i^2 + \dot{e}_i^2, \quad i = 1,\ 2 \qquad (6)$$

with

$$
\begin{aligned}
e_i &= \theta_{di} - \theta_i \\
\dot{e}_i &= \dot{\theta}_{di} - \dot{\theta}_i
\end{aligned}
$$

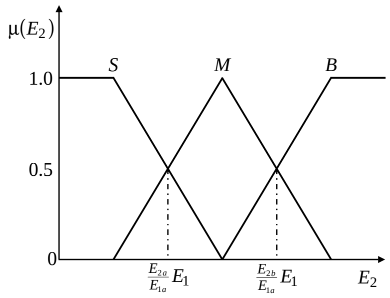

Figure 4: Membership functions for $E_1 \leq E_{1a}$

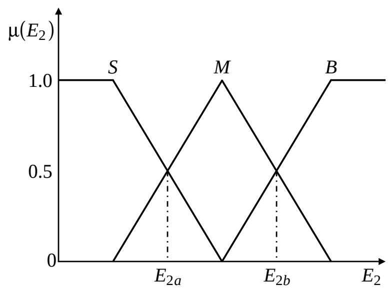

Figure 5: Membership functions for $E_1 > E_{1a}$

Energy plane is composed of E_i as shown in Fig. 2. In Fig. 2, π_i is a boundary curve which determines the subregion of energy to use a partly stable controller, and is plotted by an exponential curve. The region R_i with gray shadow is the subregion to which the controller i is applied.

3.3 Fuzzy energy region method

If a boundary curve comprises an exponential function, we can suitably design it with the amplitude and the time constant of a step-response. It is difficult to design such parameters of the function in advance, because we can't theoretically analyze them depending on the switching control. Therefore, we propose a fuzzy energy region based switching control method. At first, boundary curves are approximated by several straight-lines as shown in Fig. 3. After these approximations, fuzzy sets for E_2 can be defined for $E_1 \leq E_{1a}$ and $E_1 > E_{1a}$ cases as shown in Fig. 4 and Fig. 5. E_{1a}, E_{2a} and E_{2b} are design parameters of fuzzy sets. In order to realize ideal energy responses, fuzzy rules are given as follows:

Rule 1	:	If $E_2 = S$	Then $s_1 = 1$	
Rule 2	:	If $E_2 = M$	and $\phi_{t-1} = 1$	Then $s_2 = 1$
Rule 3	:	If $E_2 = M$	and $\phi_{t-1} = 2$	Then $s_3 = 2$
Rule 4	:	If $E_2 = B$	Then $s_4 = 2$	

Table 1: GA operations and methods

GA operations	Method
Selection for crossover	Tournament strategy with 3 individuals
Crossover	Uniform crossover with probability 0.6
Probability of mutation	1/96
Alternation	Elite strategy with 10 individuals

Table 2: Setting parameters of simulations

Conditions	Setting value
Simulation time	30 [s]
Sampling interval	0.01 [s]
Mass of each link	$m_1 = 0.582$ [kg], $m_2 = 0.079$ [kg]
Length of each link	$l_1 = 0.4$ [m], $l_2 = 0.22$ [m]
Distance between center of gravity and each joint	$l_{g1} = 0.2$ [m] $l_{g2} = 0.11$ [m]
Coefficient of viscous friction of each joint	$\mu_1 = 0$ [Ns/m^2] $\mu_2 = 0.02$ [Ns/m^2]
Desired state vector	$[0\ 0\ 0\ 0]^T$
1st initial state vector	$[0\ \pi/4\ 0\ 0]^T$
2nd initial state vector	$[\pi\ \pi/6\ 0\ 0]^T$

Note that, a parameter ϕ_{t-1} which means the index of controller for one-step delay, is introduced, because one-step delayed controller must be retained in the overlapped energy region according to ideal energy response. s_i is the index of controller that must be used in the fuzzy rule i.

The advantage of the present method is to set design parameters roughly, comparing to the logic based switching method, because the boundary curves have fuzziness to use the present fuzzy reasoning.

4 Optimizing Fuzzy Energy Regions by GA

The present method has the same difficulty to design parameters in advance as the logic based switching method. Here, we discuss about the design parameters of fuzzy rules using GA. These parameters are E_{1a}, E_{2a}, E_{2b}, K_{p1}, K_{v1}, K_{p2} and K_{v2}. Each parameter is encoded by 32 [bit], then the size of an individual is 224 [bit]. The searching domain of E_{1a}, E_{2a} and E_{2b} is set from 0.1 to 15. The searching domain of K_{pi} and K_{vi}, $i = 1, 2$ is set from 0.01 to 100. Each parameter is decoded using gray code. The size of a population is 100. The maximum number of generations is 1000. GA operations used here are shown in Table 1.

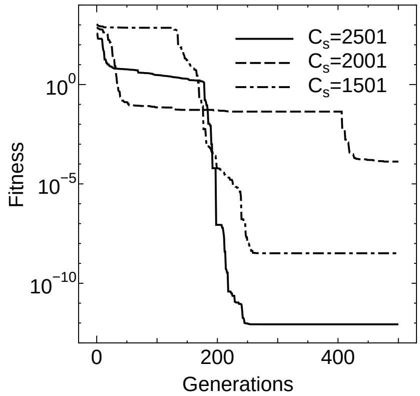

Figure 6: Optimizing history of cost functions

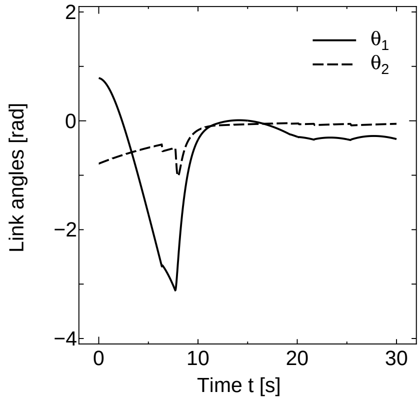

Figure 7: Simulation result with untrained initial value

Table 3: Obtained design parameters of fuzzy energy region method

	$C_s = 2501$	$C_s = 2001$	$C_s = 1501$
E_{1a}	11.5097	12.6989	14.0461
E_{2a}	0.2906	0.2978	1.0417
E_{2b}	12.4336	14.9812	8.1119
K_{p1}	10.5593	5.2048	10.4885
K_{v1}	11.5647	5.7413	11.0411
K_{p2}	5.2400	2.6096	24.2865
K_{v2}	56.3331	50.7481	4.0119

A cost function is given by

$$f_c = \sum_{i=1}^{2} \sum_{j=C_s}^{3000} \sum_{k=1}^{2} E_k(j) \tag{7}$$

where i is the index of simulation trials, j is the index of discrete times, k is the index of energy of each link and C_s is the starting index of discrete time to evaluate the response of an underactuated manipulator. Simulation conditions to train fuzzy parameters are shown in Table 2. Note that, the fitness function is not evaluated during a transition segment due to the dynamic characteristics of an underactuated system.

Training history in cost function is shown in Fig. 6. It is found from Fig. 6 that the case of $C_s = 2501$ is smaller than other cases. Obtained parameters are shown in Table 3. The obtained parameters in the case of $C_s = 2501$ are applied to the cases of untrained initial state vectors such as $\boldsymbol{\theta}(0) = [\pi/4\ \ -\pi/4\ 0\ 0]^T$. Response of link angles is shown by Fig. 7. The small steady state error is found in θ_1 from Fig. 7. Differential gain of partly stable controllers is high, so that a response has such error.

5 Conclusions

We have proposed a logic based switching method using fuzzy energy region, in which fuzzy design parameters and gains of partly stable controllers were trained by genetic algorithm. A cost function was tried to use in optimizing parameters. Obtained differential gains are a little bit high, so that a simulation result using untrained initial states has a steady state error. However, each link converges near the desired value.

References

[1] Y. Nakamura, T. Suzuki, and M. Koinuma, “Nonlinear behavior and control of a nonholonomic free-joint manipulator,” *IEEE Trans. on Robotics and Automation*, vol. 13, no.6, pp. 853–862, 1997.

[2] H. Arai, K. Tanie, and N. Shiroma, “Nonholonomic control of a three-DOF planar underactuated manipulator,” *IEEE Trans. on Robotics and Automation*, vol. 14, no. 5, pp. 681–695, 1998.

[3] I. Fantoni, R. Lozano, and M. W. Spong, “Energy based control of the pendubot,” *IEEE Trans. on Automatic Control*, vol. 45, no. 4, pp. 725–729, 2000.

[4] L. Udawatta, K. Watanabe, K. Izumi, and K. Kiguchi, “Control of underactuated manipulators using fuzzy logic based switching controller,” *J. of Intelligent and Robotic Systems*, vol. 38, pp. 155–173, 2003.

[5] J. P. Hespanha and A. S. Morse, “Stabilization of nonholonomic integrators via logic-based switching,” *Automatica*, vol. 35, pp. 385–393, 1999.