# **Design of Tracing Type Jumping Robots**

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#### Abstract

This paper discusses a leg design for a tracing type jumping robot driven by an actuator with an extremely high output/weight. We found that there is a close relationship between the jump ratio(=h/l) and the leg length. Through analysis and simulation, we found that there exists the optimum design specification where the jump motion becomes maximum. After explaining an design orientation by considering joint torque and mechanical parameters, experiments are shown to verify the basic idea.

Keywords—Jumping Robot, Motor Based Actuation, Jump Ratio

## 1 Introduction

There have been many works discussing legged robots. Legged robots can be classified into two groups; static based locomotion where the center of gravity is always ensured within the support polygon constructed by the supporting legs, and dynamic based locomotion where the center of gravity is allowed to be away from the support polygon. Jumping robot can be categorized into the latter one. While various jumping robots have been proposed so far, accumulated energy using either spring or pneumatic actuator is utilized for most of them. This is because there have been no powerful actuator ensuring an enough jump height. With the increase of actuation technique, it has become possible to see the chance that a motor based robot can jump since the effect to gravitational force is reduced. For a given actuator, how much size is appropriate for achieving a jump? How to design a robot for achieving the maximum jump height?

To answer these questions, we consider a simple model where the robot is composed of one actuator and two legs, as shown in Fig.1. The open-close motion of both legs is controlled by one actuator. As a result, this robot realizes the jumping motion by tracing the surface of the ground by both tips of leg. For such a simple model, we obtain the analytical result of the jumping height ratio defined by the height normalized by the leg length. Although the result is available only under a couple of limited assumptions, it includes really useful information for determining the specification of a jumping robot.



Fig. 1: Model for tracing type jumping robot.

Through above analysis, we show an interesting observation is that there exists the optimum design where the jump ratio becomes maximum. For a given specification of actuator, we explain how to determine the robot specification leading to the optimum design. Along the proposed approach, we design a jumping robot which can achieve the maximum jump ratio. Finally, We develop the jumping robot with optimum design and show that experiments strongly support our design. Nice qualitative coincidences are observed between analysis and experiments.

# 2 Related Works

While there are many works discussing walking machine[1][2], most of them supposed that at least one leg makes contact on the ground. On the other hand, there exists a phase where all legs are away from the ground in jumping robots[3]. Raibert et al.[4] have developed one-legged hopping machine driven by a pneumatic actuator. Okubo et al.[5][6] have proposed a jumping machine with small output actuator where they utilized self-energizing spring system. The jump ratio which we utilize as the index of evaluation can be regarded as "Jumping height index" proposed in [5] by replacing the maximum distance that body can move with the leg length. Tsukagoshi et al.[7] have developed the jumping & rolling inspector as a rescue robot and discussed the control of the jumping height and energy

saving by using a pneumatic actuator. Arikawa and Mita[8] have developed the design of multi-DOF jumping robot. They have discussed planning parameters of robot motions for achieving jumping and somersault.

## 3 Design of Jumping Robots

## 3.1 Model for Analysis

Let us consider the two-legged model as shown in Fig.1, where  $h, l, m_b, \tau_0, \omega_{\text{max}}$ , and g are the maximum height of jump, the length of leg, mass of body, torque of actuator, the maximum angular velocity of actuator, and the gravitational acceleration, respectively. We suppose that the actuator operates with constant torque  $\tau_0$  with respect to angular velocity. While a regular DC servo motor has a decreasing characteristics with respect to angular velocity, there are a couple of AC servo motors approximately supporting this characteristics by supplying current depending upon the angular velocity. We also suppose that the actuator has the limitation of angular velocity with its maximum value of  $\omega_{\rm max}$  and each leg is controlled by a single motor. We ignore the effect of the friction between the tip of leg and the ground, since we regard it as a secondary factor for reducing the jumping height.

In this work, we utilize jump ratio  $J_c$  to evaluate the height of jump as follows;

$$J_c = \frac{h}{l} \tag{1}$$

Physical meaning of  $J_c$  is that it expresses the maximum height which the robot can jump in units of its leg length.

#### 3.2 Optimum Design on Leg Length

We now consider an extremely simple model as shown in Fig.1, where we assume that two rigid links without mass are connected to an actuator with the mass of  $m_b$ and the robot motion is symmetry with respect to the z-axis. Suppose that the robot is jumping as shown in Fig.1 where the link angle is  $\theta$  with respect to the horizontal line. Let the joint torque  $\tau_0$  is given until  $\theta > \theta_b$ .

The work where the torque executes during  $0 \le \theta \le \theta_b$  is given as follows;

$$W = \int_0^{2\theta_b} \tau_0 d\theta \tag{2}$$

$$= 2\tau_0 \theta_b \tag{3}$$

Since W is fully transmitted to the potential energy when the robot reaches the highest position by jumping



**Fig. 2**: Relationship among l,  $l_{lim}$ , and  $J_c$ .

motion, we can obtain the jump ratio with the function of  $\tau_0$  as follows;

$$J_c = \frac{2\theta_b \tau_0}{m_b g l} \tag{4}$$

Eq.(4) is the closed form solution exhibiting the relationship between  $J_c$  and mechanical parameters, where both legs maintain contact with the ground during  $0 \leq \theta \leq \theta_b$ . Now, we would note that the leg cannot rotate with more than  $\omega_{\text{max}}$ . The above constraint for angular velocity leads to the following inequality.

$$\theta_{\max} \le \omega_{\max}$$
 (5)

The maximum angular velocity of joint  $\theta_{\text{max}}$  is achieved at the moment of  $\theta = \theta_b$  and the energy balance at this moment is given as follows;

$$\frac{1}{2}\left(\dot{\theta}_{\max}l\cos\theta_b\right)^2 + m_b g l\sin\theta_b = 2\tau_0\theta_b \tag{6}$$

From eq.(6), we can obtain  $\theta_{\text{max}}$  as follows;

$$\dot{\theta}_{\max} = \dot{\theta}\Big|_{\theta=\theta_b}$$
 (7)

$$= \frac{1}{l\cos\theta_b}\sqrt{2\left(\frac{2\tau_0\theta_b}{m_b} - gl\sin\theta_b\right)}$$
(8)

Now, we would focus on the leg length from the viewpoint of the design for robots. Substituting eq.(8) into ineq.(5) yields,

$$l \ge l_{\lim}$$
 (9)

where

$$l_{\rm lim} = \frac{-q + \sqrt{q^2 + 4m_b \tau_0 \theta_b p^2}}{m_b p^2}$$
(10)

$$p = \omega_{\max} \cos \theta_b \tag{11}$$

$$q = m_b g \sin \theta_b \tag{12}$$

 Table 1:
 Specification of the lightweight high-speed motor

motor type	AC
max torque [Nm]	1.71
max speed [r/min]	300
time response to a step input [msec]	30
weight [g]	59.6



Fig. 3: The lightweight high-speed motor and the developed jumping robot.

In the case where  $l < l_{\text{lim}}$ , eq.(4) is not guaranteed since each tip of the leg cannot trace the ground and is away from it. Fig.2 shows the relationship among l,  $l_{\text{lim}}$ , and  $J_c$ . While we can suppose that jump ratio decrease less than the that of eq.(4), we would note that  $l_{\text{lim}}$  may be the optimum leg length when the specification of actuator is given.

## 4 Experiments

Table.1 shows the specification of the lightweight powerful motor (Harmonic Drive Systems, Inc.)[9][10]. This motor has the maximum torque of 1.71[Nm], the maximum rotational speed of 300[r/min] and the mass of 59.6[g]. Fig.3(a) and (b) show an overview of the motor and a photo of the developed robot, respectively. Fig.4 shows an overview of the developed robot where one leg is connected to the axis of the motor and the other one is fixed to the outer case of the motor.

Fig.5 shows the map showing experimental results with respect to l. The circles shown in Fig.5 are points obtained by experiments. For reference, the results obtained by eq.(4) and eq.(10) are also provided in Fig.5, where  $\theta_b = 0.5$ . A really interesting observation is that we can find the maximum jump ratio between l = 200[mm] and 300[mm] for analysis, which supports the idea of this work. Fig.6 and Fig.7 show series of photos during a jump motion of the robot with the leg length of 100[mm] and 200[mm], respectively. The robot with the length of 200[mm] achieved a big jump with 550 [mm] with  $J_c = 2.75$ .



Fig. 4: An overview of the developed robot.



Fig. 5: Experimental results.

## 5 Concluding Remarks

We discussed the design of tracing type jumping robots by focusing on the leg length. The main results are as follows:

- (1) Jump ratio was chosen as the evaluation index for designing a jumping robot.
- (2) We obtained the relationship between the jump ratio and the leg length by using a simple model.
- (3) It was shown experimentally that there exists the optimum design point leading to the maximum jump ratio.

One of features for using motors for a jumping robot is that we can expect the capability of grasping as well as jumping. Fig.8 shows an example where the robot is capturing an object suspended in air after jumping up. We believe that a kind of dexterity can be anticipated through an AC motor based jumping robot.

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Fig. 6: A series of photos during a jumping motion (l = 100 [mm]).

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Fig. 7: A series of photos during a jumping motion (l = 200 [mm]).



Fig. 8: A series of photos during a jumping and grasping motion.