Accepting Powers of Four-Dimensional Alternating Turing Machines with Only Universal States

Youichirou NAKAMA[1], Makoto SAKAMOTO[1], Makoto SAITO[1], Shinya TANIGUCHI[1], Takao ITO[2], Katsushi INOUE[3], Hiroshi FURUTANI[1] and Susumu KATAYAMA[1]

[1]Dept. of Computer Science and Systems Engineering, Miyazaki University, Miyazaki 889-2192, JAPAN

[2]Dept. of Business Administration, Ube National College of Technology, Ube 755-8555, JAPAN

[3]Dept. of Computer Science and Systems Engineering, Yamaguchi University, Ube 755-8611, JAPAN

Abstract

During the past about forty years, many types of two- or three-dimensional automata have been proposed and investigated the properties of them as the computational model of pattern processing. On the other hand, recently, due to the advances in many application areas such as computer animation, motion image processing, and so on, the study of three-dimensional pattern processing with the time axis has been of crucial importance. Thus, we think that it is very useful for analyzing computation of three-dimensional pattern processing with the time axis to explicate the properties of four-dimensional automata. In this paper, we deal with four-dimensional alternating Turing machines, and investigate several accepting powers of four-dimensional alternating Turing machines which each sidelength of each input tape is equivalent.

KeyWords **:** alternation, configuration, four-dimensional input tape, space bound, Turing machine.

1 Introduction and Preliminaries

Blum et al. first proposed two-dimensional automata, and investigated their pattern recognition abilities in 1967 [1]. Since then, many researchers in this field have been investigating a lot of properties about automata on two- or three-dimensional tapes. In 1976, Chandra et al. introduced the concept of 'alternation'as a theoretical model of parallel computation [2]. After that, Inoue et al. introduced two-dimensional alternating Turing machines as a generalization of two-dimensional nondeterministic Turing machines and as a mechanism to model parallel computation [5]. Moreover, Sakamoto et al. presented three-dimensional alternating Turing machines in [7].

On the other hand, recently, due to the advances in many application areas such as computer animation, motion image processing, and so forth, it has become increasingly apparent that the study of four-dimensional pattern processing, i.e., three-dimensional automata with the time axis should be of crucial importance. Thus, we think that it is very useful for analyzing computation of four-dimensional pattern processing to explicate the properties of four-dimensional automata. From this viewpoint, we introduced some four-dimensional automata[6, 8].

In this paper, we continue the investigations about four-dimensional alternating Turing machines [6], and mainly investigate fundamental properties of four-dimensional alternating Turing machines with only universal states which each sidelength of each input tape is equivalent.

Let Σ be a finite set of symbols. A *four-dimensional input tape* over Σ is a four-dimensional rectangular array of elements of Σ. The set of all the four-dimensional input tapes over Σ is denoted by $\Sigma^{(4)}$. Given an input tape $x \in \Sigma^{(4)}$, for each $j(1 \leq j \leq 4)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = m_1$, $l_2(x) = m_2$, $l_3(x) = m_3$, and $l_4(x) = m_4$ is denoted by $\Sigma^{(m_1,m_2,m_3,m_4)}$. If $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq 4)$, let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4). Furthermore, we define $x\ [(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 4)$, as the four-dimensional input tape y satisfying the following:

(i) for each $j(1 \leq j \leq 4)$, $l_j(y) = i'_j - i_j + 1$;

(ii) for each r_1, r_2, r_3, r_4 $(1 \leq r_1 \leq l_1(y), 1 \leq r_2 \leq l_2(y), 1 \leq r_3 \leq l_3(y), 1 \leq r_4 \leq l_4(y))$, $y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 - 1, r_2 + i_2 - 1, r_3 + i_3 - 1, r_4 + i_4 - 1)$.

As usual, a four-dimensional input tape x over Σ is surrounded by the boundary symbols #'s ($\# \notin \Sigma$). Furthermore, four-dimensional tape is the sequence of three-dimensional rectangular arrays along the time axis. By $Cube_x(i)$ ($i \geq 1$), we denote the ith three-dimensional rectanglar array along the time axis in $x \in \Sigma^{(4)}$ which each sidelength is equivalent.

We now recall the definition of a *four-dimensional alternating Turing machine* (4-ATM), which can be considered as an alternating version of a four-dimensional Turing machine (4-TM) [8].

4-ATM M is defined by the 7-tuple

$$M = (Q,\, q_0,\, U, F,\, \Sigma,\, \Gamma,\, \delta), \text{ where}$$

(1) Q is a finite set of *states*;

(2) $q_0 \in Q$ is the *initial state*;

(3) $U \subseteq Q$ is the set of *universal states*;

(4) $F \subseteq Q$ is the set of *accepting states*;

(5) Σ is a finite input alphabet ($\# \notin \Sigma$ is the *boundary symbol*);

(6) Γ is a finite *storage-tape alphabet* ($B \in \Gamma$ is the *blank symbol*), and

(7) $\delta \subseteq (Q \times (\Sigma \cup \{\#\}) \times \Gamma) \times (Q \times (\Gamma - \{B\}) \times$ {east, west, south, north, up, down, future, past, no move} $\times$ {right, left, no move}) is the *next-move relation.*

A state q in $Q - U$ is said to be *existential.* As shown in Fig. 1, the machine M has a read-only four-dimensional input tape with boundary symbols #'s and one semi-infinite storage tape, initially blank. Of course, M has a finite control, an input head, and a storage-tape head. A *position* is assigned to each cell of the read-only input tape and to each cell of the storage tape, as shown in Fig. 1. The *step* of M is similar to that of a two- or three-dimensional Turing machine [3–5, 7], except that the input head of M can move in eight directions. We say that M *accepts* the tape x if it eventually enters an accepting state. Note that the machine cannot write the blank symbol. If the input head falls off the input tape, or if the storage head falls off the storage tape (by moving left), then the machine M can make no further move.

A *seven-way four-dimensional alternating Turing machine* (SV4-ATM) is a 4-ATM whose input head can move in seven directions – east, west, south, north, up, down, or future, and an alternating version of a seven-way four-dimensional Turing machine (SV4-TM).

Let $L(m)$: $\mathbf{N} \rightarrow \mathbf{R}$ be a function with one variable m, where $\mathbf{N}$ is the set of all positive integers and $\mathbf{R}$ is the set of all nonnegative real numbers. With each 4-ATM (or SV4-ATM) M we associate a space complexity function $SPACE$ that takes configurations to

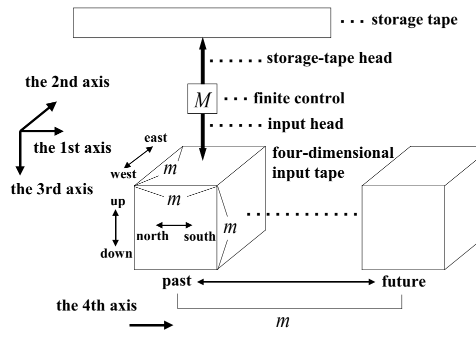

Fig. 1: Four-dimensional alternating Turing machine.

natural numbers. That is, for each configuration $c = (x,\, (i_1,\, i_2,\, i_3,\, i_4),\, (q,\, \alpha,\, j))$, let $SPACE(c) = |\alpha|$. M is said to be $L(m)$ *space-bounded* if for each $m \geq 1$ and for each x with $l_1(x) = l_2(x) = l_3(x) = l_4(x) = m$, if x is accepted by M, then there is an accepting computation tree of M on input x such that for each node v of the tree, $SPACE(L(v)) \leq \lceil L(m) \rceil$[1]. We denote an $L(m)$ space-bounded 4-ATM (SV4-ATM) by 4-ATM ($L(m)$) [SV4-ATM ($L(m)$)].

A 4-$ATM(0)$ [SV4-$ATM(0)$] is called a four-dimensional alternating finite automaton (seven-way four-dimensional alternating finite automaton), which can be considered as an alternating version of a four-dimensional finite automaton (4-FA) (seven-way four-dimensional finite automaton (SV4-FA)), and is denoted by 4-AFA (SV4-AFA).

In order to distinguish among determinism, nondeterminism, and alternation, we denote a deterministic 3-TM [nondeterministic four-dimensional Turing machine (4-TM), deterministic seven-way four-dimensional Turing machine (SV4-TM), nondeterministic SV4-TM, deterministic 4-TM ($L(m)$), nondeterministic 4-TM ($L(m)$), deterministic SV4-TM ($L(m)$), nondeterministic SV4-TM ($L(m)$), deterministic 4-FA, nondeterministic 4-FA, deterministic SV4-FA, nondeterministic SV4-FA] by 4-DTM [4-NTM, SV4-DTM, SV4-NTM, 4-DTM ($L(m)$), 4-NTM ($L(m)$), SV4-DTM ($L(m)$), SV4-NTM ($L(m)$), 4-DFA, 4-NFA, SV4-DFA, SV4-NFA].

Let M be an automaton on a three-dimensional tape. We denote by $T(M)$ the set of all three-dimensional tapes accepted by M. As usual, for each $X \in \{D,\, N,\, A\}$, we denote, for example, by £[3-XTM] the class of sets of all the four-dimensional tapes accepted by 4-XTM's. That is, £[4-XTM] $= \{T \mid T = T(M)$ for some 4-XTM $M\}$. £[SV4-

[1] $\lceil r \rceil$ means the smallest integer greater than or equal to r.

XTM], £[4-*XTM* ($L(m)$)], £[*SV*4-*XTM* ($L(m)$)], £[4-*XFA*], and £[*SV*4-*XFA*] also have analogous meanings.

2 Accepting Powers of *SV*4-*UTM*'s

We denote by *SV*4-*UTM* (*SV*4-*UFA*) an *SV*4-*ATM* (*SV*4-*AFA*) which has only universal states. For any function L: $\mathbf{N} \to \mathbf{R}$, we denote by *SV*4-*UTM* ($L(m)$) an $L(m)$ space-bounded *SV*4-*UTM*, and let £[*SV*4-*UTM*($L(m)$)] $= \{T \mid T = T(M)$ for some *SV*4-*UTM* ($L(m)$) $M\}$. £[*SV*4-*UFA*] is defined in a similar way.

In this section, we investigate the relationship between the accepting powers of *SV*4-*UTM*'s and *SV*4-*ATM*'s (*SV*4-*NTM*'s or *SV*4-*DTM*'s).

The following lemma says that there exists a set accepted by an *SV*4-*NFA*, but not accepted by any *SV*4-*UTM* ($L(m)$) for any L such that $L(m) = o(m^3)$.

Lemma 2.1. *Let* $T_1 = \{x \in \{0,\ 1\}^{(4)} \mid \exists\ m \geq 2\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = m]\ \&\ Cube_x(1) = Cube_x(2)\}$. *Then*

(1) $\overline{T}_1 \in$ £[*SV*4-*NFA*],[2] *and*

(2) $\overline{T}_1 \notin$ £[*SV*4-*UTM* ($L(m)$)] *for any* L: $\mathbf{N} \to \mathbf{R}$ *such that* $L(m) = o(m^3)$.

Proof: The set $\overline{T}_1$ is accepted by an *SV*4-*NFA* which, given an input $x \in \{0,\ 1\}^{(4)}$, simply checks by using nondeterministical states that $Cube_x(1) \neq Cube_x(2)$. It is obvious that part (1) of the lemma holds. Here, we only prove (2). Suppose that there exists an *SV*4-*UTM* ($L(m)$) M accepting $\overline{T}_1$, where $L(m) = o(m^3)$. Let s and r be the numbers of states (of the finite control) and storage tape symbols of M, respectively. For each $m \geq 3$, let

$$V(m) = \{x \in \{0,\ 1\}^{(4)} \mid l_1(x) = l_2(x) = l_3(x) = l_4(x) = m\ \&\ Cube_x(1) = Cube_x(2)\ \&\ x\ [(1,\ 1,\ 1,\ 3),\ (m,\ m,\ m,\ m)] \in \{0\}^{(4)}\}.$$

For each x in $V(m)$, let $S(x)$ and $C(x)$ be sets of semi-configurations of M defined as follows:

$S(x) = \{((i_1,\ i_2,\ i_3,\ 2),\ (q,\ \alpha,\ j)) \mid$ there exists a computation path of M on x, $I_M(x) \vdash_M^* (x,\ ((i_1,\ i_2,\ i_3,\ 1),\ (q',\ \alpha',\ j'))) \vdash_M (x,\ ((i_1,\ i_2,\ i_3,\ 2),\ (q,\ \alpha,\ j)))$ (that is, $(x,\ ((i_1,\ i_2,\ i_3,\ 2),\ (q,\ \alpha,\ j)))$ is a configuration of M just after the input head reached $Cube_x(2)$)$\}$,

$C(x) = \{\sigma \in S(x) \mid$ when, starting with the configuration $(x,\ \sigma)$, M proceeds to read the segment $Cube_x(2)$, there exists a sequence of steps of M in which M never enters an accepting state$\}$.

(Note that, for each x in $V(m)$, $C(x)$ is not empty since x is not in $\overline{T}_1$, and so not accepted by M.) Then the following proposition must hold.

Proposition 2.1. *For any two different tapes* x, y *in* $V(m)$, $C(x) \cap C(y) = \phi$.
[**Proof:** This proposition can be proved by the well-known technique [7]. □]

Proof of Lemma 2.1(*continued*) **:** Clearly, $|V(m)| = 2^{m^3}$ and $p(m) \leq s(m+2)^3 L(m) r^{L(m)}$, where $p(m)$ denotes the number of possible semi-configurations of M just after the input head reached the second plane of tapes in $V(m)$. Since $L(m) = o(m^3)$, we have $|V(m)| > p(m)$ for large m. Therefore, it follows that for large m there must be two different tapes x, y in $V(m)$ such that $C(x) \cap C(y) \neq \phi$. This contradicts Proposition 2.1 and completes the proof of (2). □

We need the following three lemmas. The proof of the following lemmas is omitted here since it is similar to that of Lemma 2.1.

Lemma 2.2. *Let* $T_2 = \{x \in \{0,\ 1\}^{(4)} \mid \exists\ m \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2m\ \&\ x\ [(1,\ 1,\ 1,\ 1),\ (2m,\ 2m,\ 2m,\ m)] = x\ [(1,\ 1,\ 1,\ m+1),\ (2m,\ 2m,\ 2m,\ 2m)]]\}$. *Then*

(1) $\overline{T}_2 \in$ £[*SV*4-*NTM* ($\log m$)], *and*

(2) $\overline{T}_2 \notin$ £[*SV*4-*UTM* ($L(m)$)] *for any* L: $\mathbf{N} \to \mathbf{R}$ *such that* $L(m) = o(m^4)$.

Lemma 2.3. *Let* T_2 *be the set described in Lemma* 2.1. *Then*

(1) $T_1 \in$ £[*SV*4-*UFA*], *and*

(2) $T_1 \notin$ £[*SV*4-*NTM* ($L(m)$)] *for any* L: $\mathbf{N} \to \mathbf{R}$ *such that* $L(m) = o(m^3)$.

Lemma 2.4. *Let* T_2 *be the set described in Lemma* 2.2. *Then*

(1) $T_2 \in$ £[*SV*4-*UTM* ($\log m$)], *and*

(2) $T_2 \notin$ £[*SV*4-*NTM* ($L(m)$)] *for any* L: $\mathbf{N} \to \mathbf{R}$ *such that* $L(m) = o(m^4)$.

From Lemmas 2.1–2.4, we can get

Theorem 2.1. *Let* L: $\mathbf{N} \to \mathbf{R}$ *be a function such that* (i) $L(m) = o(m^2)$, *or* (ii) $L(m) \geq \log m$ $(m \geq 1)$ *and* $L(m) = o(m^4)$. *Then*

(1) £[*SV*4-*UTM* ($L(m)$)] $\subsetneq$ £[*SV*4-*ATM* ($L(m)$)],

[2]If $T \subseteq \Sigma^{(4)}$, then define $\overline{T} = \Sigma^{(4)} - T$.

(2) $\pounds[SV4\text{-}UTM\ (L(m))]$ *is incomparable with* $\pounds[SV4\text{-}NTM\ (L(m))]$, *and*

(3) $\pounds[SV4\text{-}DTM\ (L(m))] \subsetneq \pounds[SV4\text{-}UTM\ (L(m))]$.

Corollary 2.1. (1) $\pounds[SV4\text{-}UFA] \subsetneq \pounds[SV4\text{-}AFA]$. (2) $\pounds[SV4\text{-}UFA]$ *is incomparable with* $\pounds[SV4\text{-}NFA]$. (3) $\pounds[SV4\text{-}DFA] \subsetneq \pounds[SV4\text{-}UFA]$.

It is natural to ask how much space is necessary and sufficient for $SV4\text{-}DTM$'s and $SV4\text{-}NTM$'s to simulate $SV4\text{-}UFA$'s. The following theorem answers this question.

THEOREM 2.2. (1) $\pounds[SV4\text{-}UFA] \subsetneq \pounds[SV4\text{-}DTM\ (m^3)]$. (2) m^3 *space is necessary and sufficient for* $SV4\text{-}DTM$'s *and* $SV4\text{-}NTM$'s *to simulate* $SV4\text{-}UFA$'s.

Moreover, by using a technique similar to that in the proof of Theorem 3.2 in [2], we can get the following theorem.

THEOREM 2.3. m^4 *space is necessary and sufficient for* $SV4\text{-}DTM$'s *to simulate* $SV4\text{-}AFA$'s *and* $4\text{-}AFA$'s.

3 Accepting Powers of 4-UTM's

We denote by $4\text{-}UTM$ ($4\text{-}UFA$) a $4\text{-}ATM$ ($4\text{-}AFA$) which has only universal states. For any function L: $\mathbf{N} \rightarrow \mathbf{R}$, we denote by $4\text{-}UTM\ (L(m))$ an $L(m)$ space-bounded $4\text{-}UTM$, and let $\pounds[4\text{-}UTM\ (L(m))] = \{T \mid T = T(M)$ for some $4\text{-}UTM\ (L(m))\ M\}$. $\pounds[4\text{-}UFA]$ is defined in a similar way. This section first investigates a relationship between the accepting powers of $4\text{-}UTM$'s and $4\text{-}ATM$'s ($4\text{-}NTM$'s or $4\text{-}DTM$'s).

From Lemma 5.2 in [7], we can get the following results.

Theorem 3.1. *Let* L: $\mathbf{N} \rightarrow \mathbf{R}$ *be a function such that* $L(m) = o(\log m)$. *Then,* $\pounds[4\text{-}DTM\ (L(m))] \subsetneq \pounds[4\text{-}UTM\ (L(m))] \subsetneq \pounds[4\text{-}ATM\ (L(m))]$.

Corollary 3.1. $\pounds[4\text{-}DFA] \subsetneq \pounds[4\text{-}UFA] \subsetneq \pounds[4\text{-}AFA]$.

We then investigate relationships between the accepting powers of eight-way and seven-way four-dimensional machines. By using the same way as in the proof of Theorems 2.1–2.3, we can get the following results.

Theorem 3.2. *Let* L: $\mathbf{N} \rightarrow \mathbf{R}$ *be a function such that* (i) $L(m)^3 = o(m^3)$, *or* (ii) $L(m) \geq \log m$ $(m \geq 1)$ and $L(m) = o(m^4)$. *Then,* $\pounds[SV4\text{-}UTM\ (L(m))] \subsetneq \pounds[4\text{-}UTM\ (L(m))]$.

Corollary 3.2. $\pounds[SV4\text{-}UFA] \subsetneq \pounds[4\text{-}UFA]$.

Theorem 3.3. (1) $\pounds[4\text{-}UFA] \subsetneq \pounds[SV4\text{-}DTM\ (m^4)]$, *and* (2) m^4 *space is necessary and sufficient for* $SV4\text{-}DTM$'s *to simulate* $4\text{-}UFA$'s.

4 Conclusion

In this paper, we investigated the accepting powers of four-dimensional alternating Turing machines with only universal states which each sidelength of each input tape is equivalent.

Let T_c be the set of all the four-dimensional connected tapes. If T_c is accepted by four-dimensional alternating Turing machines with only universal states, it will be interesting to investigate how much space is necessary and sufficient for four-dimensional alternating Turing machines with only universal states to accept T_c.

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