Evaluating a solution of Tour Planning Problem based on the partially exhaustive exploration Monte Carlo Method

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Abstract

Researches on a planning, which includes the uncertainty that a result to a single solution cannot be determined uniquely, have been done. In such problems, a solution needs to be evaluated based on the uncertainty. In this paper, as the evaluation function based on such uncertainty, expected value estimated using the Monte Carlo Method is suggested. Moreover, the partially exhaustive exploration Monte Carlo Method is proposed to improve the accuracy of the estimation. As an example of the problem, the Tour Planning Problem is suggested and the proposed method is applied to this problem. From experimental results, it has been confirmed that the accuracy is improved by the proposed method. The proposed method can estimate the expected value as accurately as the standard Monte Carlo Method using the fewer samples. There is a probability of improving the effectiveness of searching because more solutions can be evaluated.

Keywords: Monte Carlo Method, uncertainty, Tour Planning Problem

1 Introduction

Researches on a planning, such as the automatic decision-making by the software or the combinatorial optimization problem containing probabilistic parameter, have been done [1][3][4]. These problems include the uncertainty that a result cannot be determined uniquely. In short, in such problems, a single solution may produce different results. Therefore, a solution needs to be evaluated based on the uncertainty. In the case where all possible results and each probability are known, the simplest evaluation function includes the expected value. However, if the number of possible results is large, it is difficult to calculate the expected value actually because of the time constraints.

In this paper, we suggest the expected value estimated using the Monte Carlo Method (MCM) [2] as the evaluation function based on the uncertainty and propose the partially exhaustive exploration Monte Carlo Method to improve the accuracy of the estimation using a few samples. Furthermore, we suggest the Tour Planning Problem as an example including such uncertainty. We apply the proposed method to this problem.

The remainder of the paper is organized as follows: In section 2, the Tour Planning Problem is formulated. In section 3, the partially exhaustive exploration Monte Carlo Method is proposed. In section 4, we apply the proposed method to the Tour Planning Problem and discuss the effectiveness. Finally, concluding remarks are given in Section 5.

2 Tour Planning Problem

2.1 Overview

The purpose is to plan a tour, which maximizes the total score within the time limit, when there are multiple tourist facilities. In traditional model, it is assumed that the each time cost involved with transferring between the facilities or staying at each facility is constant. We introduce the probability distributions as each time cost. It cannot be defined that whether a tour exceeds the time limit or not because each time cost is not defined uniquely.

We compared two evaluation functions about a tour in the preliminary experiment. One is the expected score. The other is the score, which is got when it is assumed that each cost is the average of given probability distribution. From experimental results, it is confirmed that using the expected value can search for a tour, which rarely exceeds the time limit and gets high score averagely. Therefore, we formulate the Tour Planning Problem as the combinatorial optimization problem aiming at maximizing the expected score.

2.2 Formulation

A set of node, which corresponds to a tourist facility, is denoted by V, and a set of time is denoted by T.

$$V = \{v_1, \cdots, v_n\}, \quad (1)$$
$$T = \{t_r | t_r = t_1 + (r-1)\cdot\delta t, r = 1, 2, \cdots\}, \quad (2)$$

where δt is unit time. The v_i has the opening time o_i and closing time e_i ($o_i, e_i \in T$). A set of time cost involved with moving from v_i to v_j and staying at v_i is denoted by W_{ij} and W_i ($v_i, v_j \in V$), respectively.

$$W_i = \{w_{ik} | w_{i,k+1} = w_{ik} + \delta t, k = 1, \cdots, m_i\}. \quad (3)$$

$$W_{ij} = \{w_{ijk} | w_{i,j,k+1} = w_{ijk} + \delta t, k = 1, \cdots, m_{ij}\}. \quad (4)$$

The v_i has the probability distribution denoted by $p_i(w_{ik})(\sum_k p_i(w_{ik})=1,\ p_i(w_{ik})\geq 0)$, which indicates the probability of staying at v_i for w_{ik}. Moreover, the probability distribution denoted by $p_{ij}(w_{ijk})(\sum_k p_{ij}(w_{ijk})=1,\ p_{ij}(w_{ijk})\geq 0)$, which indicates the probability of taking w_{ijk} to move from v_i to v_j, is defined. Given starting point v_s, goal v_g, departure time D_s and time limit D_g, the objective of Tour Planning Problem is to maximize

$$\sum_{b \in B} (S(A, b) \cdot p(b)) = E\left[S(A, b)\right]. \quad (5)$$

$$A = (a_{ui})_{u=1,\cdots,n,i=1,\cdots,n} \ (a_{ui} \in \{0, 1\}). \quad (6)$$

$$S(A, b) = \begin{cases} \sum_{u=2}^{L-1} s_{a(u)}(t_u) : t_L \leq D_g \\ 0 \quad : t_L > D_g \end{cases}. \quad (7)$$

$$s_i(t) = \begin{cases} s_i : o_i \leq t \leq e_i \\ 0 : otherwise \end{cases}. \quad (8)$$

$$B = W_{a(1)} \times \cdots \times W_{a(L)} \times W_{a(1),a(2)} \times \cdots \times W_{a(L-1),a(L)}. \quad (9)$$

$$b = (c_{a(1)}, \cdots, c_{a(L)}, c_{a(1),a(2)}, \cdots, c_{a(L-1),a(L)}). \quad (10)$$

$$p(b) = \prod_{u=1}^{L} p_{a(u)}(c_{a(u)}) \times \prod_{u=1}^{L-1} p_{a(u),a(u+1)}(c_{a(u),a(u+1)}). \quad (11)$$

$$t_\xi = D_s + \sum_{u=2}^{\xi-1} c_{a(u)} + \sum_{u=1}^{\xi-1} c_{a(u),a(u+1)}. \quad (12)$$

$$L = \sum_{u=1}^{n} \sum_{i=1}^{n} a_{ui}. \quad (13)$$

$$a(u) = \sum_{i=1}^{n} (a_{ui} \cdot i). \quad (14)$$

subject to

$$a_{1,s} = a_{L,g} = 1. \quad (15)$$

$$\sum_{u=1}^{n} a_{ui} \leq 1. \quad (16)$$

$$\sum_{i=1}^{n} a_{u+1,i} \leq \sum_{i=1}^{n} a_{ui}. \quad (17)$$

Equation (5) represents the expected score. The matrix (6) represents a tour. If $a_{ui} = 1$, the u-th node that a client visits is v_i. Equation (7) represents the total score. In this model, the score for the case of exceeding the time limit is set to zero because we assume the time limit as the departure time of the airplane or the train that a client will take. Therefore, if to exceed the time limit is allowed somewhat, it is possible that not zero but the function of a certain penalty is used as such score. Equation (8) is a score of v_i in the time t. Equation (9) represents a direct product consisted of combination of time cost. As (10) shown, b ($\in B$) represents the combination of arbitrary costs. Where c_i and c_{ij} are arbitrary time costs determined according to $p_i(w_{ik})$ and $p_{ij}(w_{ijk})$. The value of (11) is occurrence probability of b. In short, $p(b)$ is a probability that arbitrary c_i and c_{ij} are selected at the same instant. Equation (12) represents the time that a client arrives at ξ-th node. The value of (13) is the total number of node that a client visits. The value of (14) represents the number of u-th node that a client visit. Equations (15)-(17) are the constrained conditions.

3 Proposed method

3.1 Estimation of expected score

Because it is difficult to search for the optimum solution, we search the approximate solution by the heuristic search. However, if the expected score is calculated actually, it takes time granted that heuristic search is used. Therefore, we propose the expected score estimated by the MCM as the evaluation function. Some combinations of c_i and c_{ij}, which is b, is selected randomly according to $p_i(w_{ik})$ and $p_{ij}(w_{ijk})$ as samples. Then, the MCM estimates the expected score. The number of samples is denoted by M. The M has to be set a low value because of the time constraints. Therefore, we propose the partially exhaustive exploration Monte Carlo Method to improve the accuracy of the estimation using a few samples.

3.2 Partially exhaustive exploration Monte Carlo Method

This method uses the feature that the b with the high occurrence probability can be enumerated without exploring all combinations of time costs. As above, the $p(b)$ is calculated as a product of $p_i(c_i)$ and $p_{ij}(c_{ij})$. Therefore, a certain threshold is set when the $p(b)$ is calculated. If a product is below the threshold in the course of calculation of $p(b)$, the rest of all the calculation about the combination is omitted. By this tree pruning, the cost for enumeration can be reduced.

The procedure to estimate the expected score about an A is as follows. First, a certain threshold is determined and the set B is divided into two sets. One is the set of b, which has the higher $p(b)$ than the threshold. The other is the set of other b. The former is denoted by $B^{(1)}$ and the latter is denoted by $B^{(2)}$. The number of samples used for estimation about $B^{(1)}$ is M_1 and about $B^{(2)}$ is M_2. M is the sum of M_1 and M_2. Secondly, all b contained in $B^{(1)}$ are explored exhaustively. Therefore, M_1 equals $|B^{(1)}|$. The E_1, which is answer of exhaustive exploration about $B^{(1)}$, is calculated.

$$E_1 = \sum_{b \in B^{(1)}} (S(A, b) \times p(b)) \ . \tag{18}$$

Thirdly, the MCM estimates the expected score about $B^{(2)}$. The M_2 samples are generated randomly according to $p_i(c_i)$ and $p_{ij}(c_{ij})$. They are denoted by ξ_q. However, the $p(\xi_q)$ may be higher than the threshold. Such sample cannot be used. Therefore, the number of the samples, which can actually be used, may be less than M_2. The number of samples, which can be used, is denoted by M_2^{true}. The E_2, which is expected score estimated about $B^{(2)}$, is calculated.

$$E_2 = \frac{1}{M_2^{true}} \sum_{q=1}^{M_2^{true}} S(A, \xi_q). \quad (M_2^{true} \le M_2) \tag{19}$$

Finally, the expected score E is calculated.

$$E = E_1 + \sum_{b \in B^{(2)}} p(b) \cdot E_2 \ . \tag{20}$$

In the preliminary experiment, it is confirmed that the larger there is deviation between the occurrence probabilities, the more this method is effective. When there is not deviation so much, the effect cannot be expected. However, because it is difficult to define that the Tour Planning Problem is contained in which, it is necessary to investigate.

4 Experiment

4.1 Experimental setup

We design the model based on fourteen tourist facilities in Sapporo city, Japan. The $p_{ij}(w_{ijk})$ is determined as follows. First, standard time cost involved with moving from v_i to v_j, which is denoted by μ_{ij}, is calculated using the store-bought map software. Secondly, σ_{ij} is calculated as μ_{ij} divided by K. This K is parameter to set the variation of the time cost. Finally, $p_{ij}(w_{ijk})$ is determined by dividing a normal distribution $N(\mu_{ij}, \sigma_{ij})$ according to δt as Figure 1. The $p_i(w_{ik})$ is determined in the same way as $p_{ij}(w_{ijk})$. The standard time cost μ_i is determined referring to some guidebooks. Each o_i and e_i is the actual value. In this experiments, $\delta_t = 5$, and $K = 15$ are used.

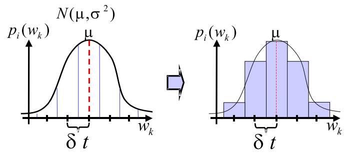

Figure 1: The calculation of the occurrence probability

Table 1: The three kinds of setups about client

	v_s	v_g	D_s	D_g
Client 1	13	0	840	1380
Client 2	0	0	480	1080
Client 3	0	13	540	1260

4.2 Comparison with the standard MCM

To compare the accuracy of estimation, the expected score is estimated about one thousand solutions each ten thousand times by each MCM. Then, to evaluate the accuracy, a error rate to a true expected score is calculated. Three kinds of client are prepared as shown in Table 1. In the setting of client 1, some nodes will be closed depend on the time. In the setting of client 2, some nodes do not stay open at first. In the setting of client 3, more time can be used than other settings. Furthermore, two kinds of settings about M, M=200 and M=500 are used. In this paper, we used the M as an indicator of the calculation cost. Because it is thought that the computation time depends on the feature of the problem or efficiency of programs, we fixed not the computation time but M.

4.3 Results of the first experiment

Figure 2 - 7 show the average error rate. The "1.0" in figures represents the standard MCM. About clients 1 and 2, the proposed method could estimate the expected score more accurately than the standard MCM. In the case of M=500 of client 1, accuracy was improved about one to two percent. In view of the fact that the Standard MCM can estimate the expected score comparatively accurately, it is thought that the accuracy was improved considerably. In other words, the proposed method can estimate the expected score as accurately as the standard MCM using the fewer samples. Furthermore, the accuracy differs depending on the balance between the threshold and the number of samples M. Therefore, the threshold is very important. About client 3, there is not so much of a difference between the proposed method and the Standard MCM. Because client 3 can visit more nodes, the combination of time cost increases. Each $p(b)$ becomes low and almost all $p(b)$ becomes lower than a threshold. As a result, the proposed method is not so different from the standard MCM. Next, client 1 that the accuracy is most improved is considered selectively.

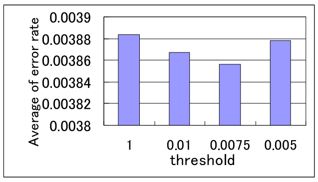

Figure 2: The average error rate about client 1 (M=200)

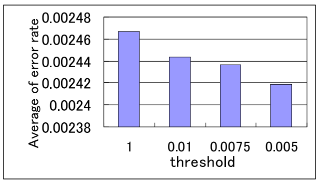

Figure 3: The average error rate about client 1 (M=500)

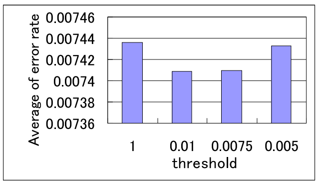

Figure 4: The average error rate about client 2 (M=200)

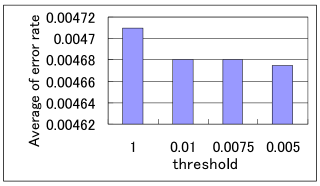

Figure 5: The average error rate about client 2 (M=500)

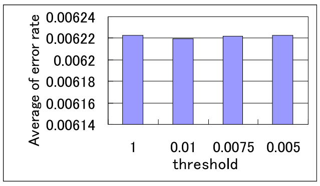

Figure 6: The average error rate about client 3 (M=200)

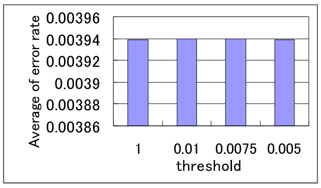

Figure 7: The average error rate about client 3 (M=500)

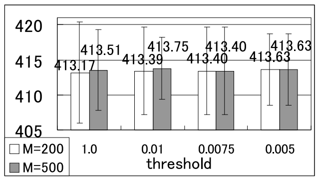

Figure 8: The average of true expected scores of 500 tours

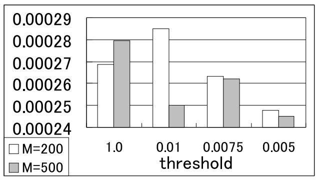

Figure 9: The average of probabilities of breach to time limit

4.4 Comparison of effectiveness of searching

To discuss the effectiveness of searching by each method, we search for a tour 500 times actually about the setting of client 1 and compare the tours, which are selected finally. We use the simulated annealing (SA) because of the ease of implementation. The parameters are determined in the preliminary experiments. A tour is expressed as permutation of the visited node number.

4.5 Results of the second experiments

Figure 8 shows the average of true expected scores of 500 tours selected by 500 trials of SA. There is not so much of a difference between the two methods. It is thought that a tour, which finally is selected, is much the same because the expected score is estimated comparatively accurately whichever method. However, as stated in section 4.3, the proposed method can estimate the expected score as accurately as the standard MCM using the fewer samples. As a result, more solutions can be evaluated. Therefore, there is a probability of improving the effectiveness of searching by the proposed method. Figure 9 shows the average of probability that a selected tour exceed the time limit. The selected tours rarely exceed the time limit. The computation time of the proposed method is similar to that of the standard MCM within four seconds.

5 Conclusion

In this paper, we suggest the expected value estimated using the MCM as the evaluation function based on the uncertainty. Moreover, the partially exhaustive exploration Monte Carlo Method is proposed to improve the accuracy of the estimation. Furthermore, the Tour Planning Problem is suggested as an example of the problem including the uncertainty. From results of experiments, it has been confirmed that the accuracy is improved by the proposed method. In other words, the proposed method can estimate the expected value as accurately as the standard MCM using the fewer samples. There is a probability of improving the effectiveness of searching because more solutions can be evaluated. Moreover, a reasonable tour, which rarely exceeds the time limit and gets high score averagely, can be searched by using the expected score as the evaluation function. As the future works, we have to design the means to adjust a threshold automatically. Moreover, we have to discuss a case that it is difficult to estimate an expected value.

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