Impulsive noise reduction using M-transform and wavelet with applications to AFM signals

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Abstract

The authors propose here a new method for reducing both impulsive noise and white noise by use of M-transform and wavelet shrinkage. M-transform is a new signal transformation proposed by the authors, and any periodic time signal can be considered as the output of a filter whose input is an M-sequence. By using the properties of M-tarnsform, it is shown that both impulsive noise and white noise can be eliminated by use of first M-transform and then wavelet shrinkage. This method is applied to the signal obtained in Atomic Force Microscope(AFM) signal which usually contains many impulsive noise and withe noise. The result of experiments show that this method can be widely applied to practical situations for reducing both impulsive and white noise.

1 Introduction

In this paper, the authors propose a new method for impulsive noise reduction by using M-transform and wavelet shrinkage. Various filters such as Wavelet shrinkage have been proposed for the removal of white noise included in a signal[1]. However, these methods are based on assumption that the removed noise is a Gaussian white noise. Therefore, when a white noise and an impulsive noise are included in the observed signal, the impulsive noise can not be removed.

The authors have recently proposed a new signal processing technique called M-transform[2]. By using M-transform, both impulsive noise and a white Gaussian noise are converted into a small-amplitude random signal. By combining M-transform and Wavelet shrinkage, a new method is proposed here to remove both impulsive noise and white noise simultaneously in Atomic Force Microscope(AFM) signal. From the results of computer simulation, the proposed method is shown to be very efficient to remove both impulsive noise and white noise in AFM signal.

2 M-transform

M-transform is a new signal processing technique proposed by the authors[2]. This transform is based on the pseudo-orthogonal property of a pseudo-random M-sequence. Just like in case of Fourier transform where any time signal can be expressed as a sum of sinusoidal signals by use of Fourier transform, any periodic time function can be considered to be a weighted sum of M-sequences.

Let $\{a_i\}$ ($a_i = 1$ *or* 0) be an n-th order M-sequence. Then, we provide a new sequence $\{m_i\}$ as is defined in Eq.(1).

$$m_i = 1 - 2a_i \ (0 \leq i \leq N-1) \tag{1}$$

Here, $N = 2^n - 1$ is a period of the M-sequence.

A matrix M_i of $N \times N$ degree is defined by the next equation.

$$M_i = \begin{bmatrix} m_i, m_{i-1}, \cdots, m_{i-N+1} \\ m_{i+1}, m_i, \cdots, m_{i-N+2} \\ . \\ . \\ m_{i+N-1} \cdots\cdots, m_i \end{bmatrix} \tag{2}$$

Let X_i be an arbitrary periodic discrete time signal represented as

$$\begin{aligned} X_i &= (x(i), x(i+1), \cdots, x(i+N-1))^T \quad (3) \\ x(i) &\triangleq x(i\Delta t) \end{aligned}$$

where Δt is a sampling period. Then, M-transform A of the signal X_i is uniquely determined as

$$X_i = M_i A \tag{4}$$
$$A = (M_i^T M_i)^{-1} M_i^T X_i \tag{5}$$

The definition of M-transform is shown in Fig.1.

Any periodic time signal X_i can be considered as the output of a filter whose input is an M-sequence.

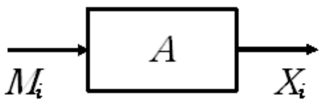

Figure 1: M-transform

Impulsive noise is converted into small-amplitude random signals through M-transform. Let P be a periodic time signal, in which a single impulse is included.

$$P(i) = (0, 0, \cdots, p_l, 0, \cdots, 0)^T \quad (6)$$

Here, l means a position of the impulse and p_l is the amplitude of the impulse. Substituting Eq.(6) into Eq.(5), M-transform A_p of the signal P is given as

$$A_p = (\alpha_p(0), \alpha_p(1), \cdots, \alpha_p(N-1)) \quad (7)$$

$$\alpha_p(i) = \frac{1}{N+1}(m_{i+l} - 1)p_l \quad (8)$$

Since the amplitude p_l is a constant, it is clear that the impulsive noise is converted into a small amplitude M-sequence. M-transform of a time signal is equivalent to calculating the cross-correlation between the time signal and M-sequence. Since white noise does not correlate with an M-sequence, M-transform of a white signal also becomes a random signal having small amplitude. Thus, these two different kinds of noise become the small-amplitude random signal through M-transform.

3 Wavelet Shrinkage

In the method called wavelet shrinkage, the noise included in the signal is removed according to the following procedures[1]. Let $u(i)$ be a discrete time original signal and $e(i)$ be a Gaussian white noise. Then, the observed signal $x(i)$ is given by the next equation.

$$x(i) = u(i) + e(i) \quad (i = 0, 1, \cdots, L-1) \quad (9)$$

Here, L is a length of the signal. Computing a level J_L orthogonal wavelet transform of the observed signal $x(i)$, the wavelet coefficient $W_x^{(j)}(k)$ is given by the next expression.

$$\begin{array}{l} W_x^{(j)}(k) = W_u^{(j)}(k) + W_e^{(j)}(k) \\ (j = 1, \cdots, J_L; k = 0, 1, \cdots, L_j - 1) \end{array} \quad (10)$$

Here, L_j is a length of the wavelet coefficient at level j. Since wavelet transform is a linear transformation, the wavelet coefficients $W_e^{(j)}(k)$ of the white noise become also white noise. On the other hand, when the original signal $u(i)$ does not contain high-frequency component, the coefficients $W_u^{(j)}(k)$ become 0. Thus, the white noise can be removed by removing the coefficient $W_x^{(j)}(k)$ below a threshold level λ. For the thresholding, Donoho[1] used the following equation.

$$W_x^{'(j)}(k) = \begin{cases} sgn(w_x^{(j)}(k)(|w_x^{(j)}(k)| - \lambda) \cdots \\ \qquad \cdots (if\ |w_x^{(j)}(k)| > \lambda) \\ 0 \cdots\cdots\cdots (if\ |w_x^{(j)}(k)| \leq \lambda) \end{cases} \quad (11)$$

Here, $sgn(x)$ is a function satisfying,

$$sgn(x) = \begin{cases} 1\ (x > 0) \\ -1\ (x < 0) \end{cases} \quad (12)$$

This method is called soft-thresholding. If the standard deviation σ of the Gaussian noise is already known, the threshold level λ can be determined by

$$\lambda = \sigma\sqrt{2\ log\ L} \quad (13)$$

Noise reduction is completed by reconstructing the signal by using the coefficient $W_x^{'(j)}(k)$ processed by Eq.(11).

4 Impulsive noise reduction method

The noise reduction method for both impulsive noise and white noise in AFM signal by using M-transform and wavelet shrinkage is as follows[4].

Let $x(i)$ be a time signal which includes both impulsive noise $p(i)$ and white noise $e(i)$.

$$x(i) = u(i) + e(i) + p(i) \quad (14)$$

$$X_n = (x(0), x(1), \cdots, x(N-1))^T \quad (15)$$

Then, M-transform A of the noisy signal X_n is calculated by Eq.(5).

$$A = A_u + A_e + A_p \quad (16)$$

Here, A_u, A_e and A_p are the M-transform of the original signal, white noise and impulsive noise, respectively. As mentioned above, both impulsive noise

$p(i)$ and white noise $e(i)$ are converted into small-amplitude random signals through M-transform. So, if we apply the wavelet shrinkage method to the M-transform $\alpha(i)$ intead of the time signal $x(i)$, it is possible to remove both impulsive noise and white noise.

The method for noise reduction proposed in this paper is as follows. First, M-transform A of the AFM signal $x(i)$ is calculated. Then, the wavelet shrinkage method is applied to the M-transform A. After the wavelet shrinkage, the filtered signal is transformed into time domain through the inverse M-transform and the noise reduction procedure is completed.

Fig.2 shows the procedure of the proposed noise reduction method.

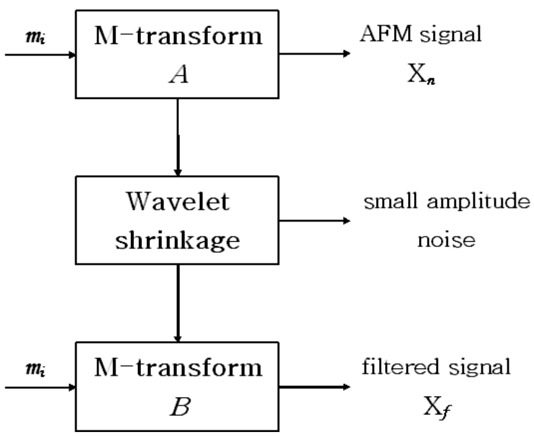

Figure 2: Procedure of the proposed noise reduction method

5 Application of the proposed method to atomic force microscope signal

Atomic force microscope(AFM) can measure the surface profile very precisely by using the atomic force operating between a probe and the sample surface. A schematic configuration of AFM is shown in Fig. 3. AFM mainly consists of three parts, a cantilever, a displacement sensor and scanning element. The cantilever converts the atomic force that is received by a probe into displacement. The displacement sensor detects the deflection of the cantilever. The scanning element is used to move the sample in high accuracy in three-dimensional space. AFM has the following features.

1. AFM possesses the spatial resolution at an atomic level.

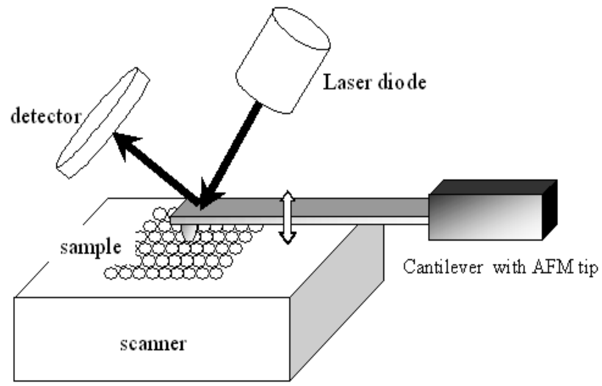

Figure 3: Structure of AFM

2. AFM does not need a special operating environment, and be able to measure the surface profile in atmosphere, in liquid, and in the vacuum.

3. Unlike the scanning electron microscope, AFM can measure surfaces of the conductive material, ceramics, the polymeric material, and the biological material.

4. AFM can measure various kinds of force such as van der Waals force, repulsive force and adhesive force.

In order to shorten the measuring time, it is necessary to increase the scanning speed of a probe that traces the surface of the test piece. However, when the scanning speed becomes faster, impulsive noise is likely to occur, and it becomes impossible to measure the precise surface profile. Those impulsive noises cannot be removed by using ordinary low-pass filter or a nonlinear median filter.

The AFM used in this paper is SPI3700 manufactured by SEIKO Instruments Inc., Japan, and the measuring range of the AFM is from $1\mu m \times 1\mu m$ to $150\mu m \times 150\mu m$. The maximum resolution of displacement is 0.2nm in th x-y place and 0.01nm in z axis direction.

An example AFM signal which includes impulsive noise is shown in Fig. 4. Here, the sample used for the measurement was a silicon plate which was polished like a mirror finished surface. In this sample, the scanning speed is 1Hz and the measure range is $25.57\mu m \times 25.57\mu m$. In Fig. 4, 127×127 pixels of a measured image of 256×256 pixels are displayed. From this figure it is clear that a lot of impulse noises are included in the AFM signal, and it is not possible to measure accurate surface profile of the sample.

The proposed noise reduction method is applied to the AFM signal. Since the size of the image shown in Fig. 4 is 127×127, the degree of M-sequence is chosen to be 7 and the characteristic polynomial f(x) is

$$f(x) = x^7 + x^4 + x^3 + x^2 + 1 \tag{17}$$

The level j of the Wavelet transform is j=3.

The result of the noise reduction is shown in Fig. 5. Although, there remain some impulsive noises, most the impulsive noises were removed from the original AFM signal.

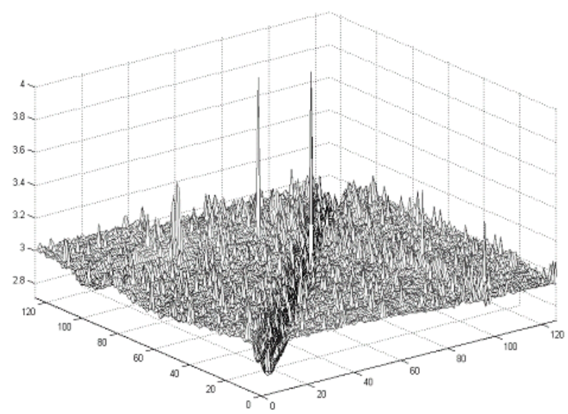

Figure 4: AFM signal which includes both impulsive noise and white Gaussian noise

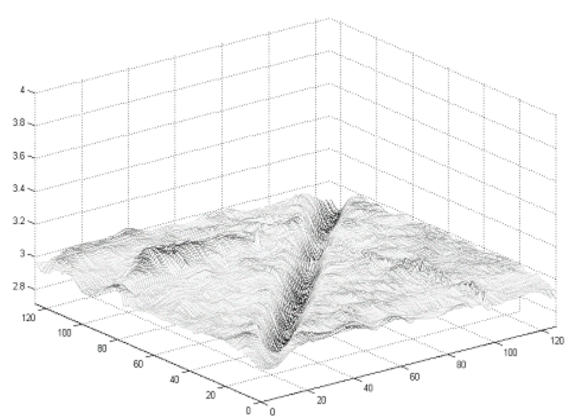

Figure 5: AFM signal after impulsive noise and white noise reduction by using M-transform and wavelet shrinkage

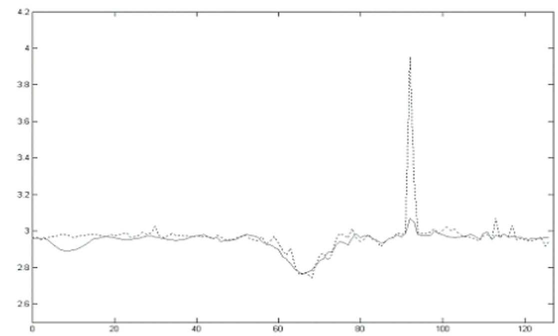

Figure 6: Original signal and noise reduced signal

An example of a column of AFM signal containing original signal and noise reduced signal is shown in Fig. 6. The dotted line is the original signal, and the solid line is noise reduced signal by using M-transform and wavelet shrinkage. From this figure, we see the proposed method is very effective for impulsive noise reduction.

6 Conclusion

In this paper, the authors propose a new method for impulsive noise reduction by using M-transform and wavelet shrinkage. From the results of computer simulation, it is shown that the proposed method is very efficient to remove both impulsive noise and white noise in AFM signals.

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