DESIGN OF A ROBUST ADAPTIVE CONTROLLER FOR A CLASS OF UNCERTAIN NONLINER SYSTEMS WITH TIME-DELAY INPUT

Thi-Minh-Huong NGUYEN, Thi-Nga DINH
Center for Automation Technology – CAT
thiminhhuong_nguyen@yahoo.com

Thi-Ngoc NGUYEN, Van-Truong TRAN
Center for Automation Technology – CAT,
Hanoi, Vietnam

Abstract

This paper presents a systematic analysis and a simple design of a robust adaptive control law for a class of nonlinear systems with modeling errors and a time-delay input. The theory for designing a robust adaptive control law based on input-output feedback linearization of nonlinear systems with uncertainties and a time-delay in the manipulated input by the approach of parameterized state feedback control is presented. The main advantage of this method is that the parameterized state feedback control law can effectively suppress the effect of the most parts of nonlinearities, including system uncertainties and time-delay input in the pp-coupling perturbation form and the relative order of nonlinear systems is not limited.

Keywords: nonlinear, robust control, adaptive control, time-delay

1 Introduction

Recent developments in the theory of differential geometry provide useful methods for a class of nonlinear systems. The central concept of this approach is to algebraically transform the nonlinear system dynamics into an equivalent linear system, such that the conventional linear control techniques can also be applied [1, 2]. Generally, the feedback linearization techniques require the accurate mathematical model for the plant to achieve exact linearization of the close loop system. However, for many real processes, there exist inevitable uncertainties in their constructed structure models. Therefore, design of a robust controller for a nonlinear system is important subject.

In this paper, we present a systematic analysis and a simple design of an adaptive control law for a class of nonlinear systems with modeling errors and a time-delay input. The so-called matching conditions [3] for controlled nonlinear systems and the Smith predictor are not necessary. System uncertainty is considered as the non-vanishing case in the desired operating condition. Using the parameterized coordinate transformation, the original nonlinear system can be transformed to a class singular perturbation problem having distinct fast and slow dynamics. An adjustable parameter can be detuned to satisfy the desired control specification. When the lumped nonlinearity, including uncertainty and time-delay input, is constrained to a closed bounded set and satisfy the local Lepschitz condition [4], its effect on the output trajectory can be effectively suppressed using the proposed technique. More precisely, to treat the tracking problem, an ultimate bound of tracking error is investigated under the specific reference model.

2 Preliminaries and problem formulation

Consider the uncertain single input single output (SISO) nonlinear system with time-delay input:

$$x'(t)=f(x(t))+Df(x(t))+[g(x(t))+Dg(x(t))]u(t-d) \quad (1)$$
$$y(t)=h(x(t))$$

, where $x \in R^n$ is the state variable, $u \in R$ is the manipulated input, d>0 is the time delay in the manipulated input, $y \in R$ is the output .f(•), g(•), Df(•) and Dg(•) are smooth vector fields on R^n, and h(•) is a smooth function. The nominal system is then defined as follows:

$$x'(t)=f(x(t))+g(x(t))u(t) \quad (2)$$
$$y(t)=h(x(t))$$

, i.e. assume that Df(•) = 0, Dg(•) = 0, and d=0 in (1).

There has been a great deal of research in recent years over the development of a complete theory for explicitly linearizing the input-output map of the nominal system (2) using state feedback. Here we introduce some notations from differential geometry, namely the Lie derivative, which is frequently used in this paper. For more general treatment in this area, readers are advised to look in [1], [5]. Given a scalar function h(x(t)) and a vector field f(x(t)) on R^n, one can define a new scalar function $L_f h(x(t))$, called the Lie derivative of h(x(t)) with respect to f(x,t):

$$L_f h(x(t))=\frac{\P h(x(t))}{\P x(t)} f(x(t)) \quad (3)$$

Thus, the Lie derivative $L_f h(x(t))$ is the directional derivative of h(x(t)) along the direction of the vector f(x(t)). Higher order Lie derivative can be defined recursively as:

$$L_f^0 h(x(t))=h(x(t)) \quad (4)$$

$$L^k{}_f(x(t))=\sum_{j=1}^{n} \frac{\P}{\P x(t)} [L^{k-1}{}_f h(x(t))] f_j(x(t)),\ k = 1, 2, 3, \ldots \quad (5)$$

if g(x)is another smooth vector field on R^n, then one can define the Lie derivative of h(x(t)) with respect to two different vector fields:

$$L_g L_f^k h(x(t))= \sum_{j=1}^{n} \frac{\P}{\P x(t)} [L^k{}_f h(x(t))] g_j(x(t)),$$

$$k=1, 2, 3, \ldots \quad (6)$$

Furthermore, the minimum eigenvalue of a hermitian is denoted as $l_{min}(\bullet)$ and $l_{max}(\bullet)$, where as the transpose of the vector or of a matrix is written as $(\bullet)^T$ and $\|\bullet\|$ denotes the Euclidean norm.

An important property of a nonlinear system is its relative degree. For a linear system defined in transfer function form, the relative degree is usually defined as the order of the denominator minus the order of a numerator. Amore general definition is used for nonlinear systems:

Definition 1: The system (eqn.2) is said to have a constant relative degree r [6], if there exists a positive integer $1<=r<=\infty$,such that

$L_gL_f^kh(x(t))=0, k< r-1$ (7)

$LgLfr\text{-}1h(x(t))!=0$ for all $x\in R^n$ and $t\in [0,\infty)$ (8)

Throughout this paper, we assume that the nominal system (2) possesses a relative degree r. Based on the assumption, it has been shown that [1] there exits a neighborhood U of the operating point x_s such that the mapping:

$p: U\text{-}> R^n$ (9)

defined as

$p_i(x(t))=E_i(t)=L_f^{i-1}h(x(t)), i=1,2,3,...,r$ (10)

$p_k(x(t))=N_k(t), \quad k=r+1,r+2,...,n$ (11)

, and satisfying:

$L_gp_k(x(t))=0, k=r+1,r+2,...,n$ (12)

, is a diffeomorphism onto image. To obtain a linear input-output relation of (2), start with the external dynamics:

$$E'_1(t)=\frac{\partial p_1}{\partial x}\frac{dx}{dt}=\frac{\partial h}{\partial x}\frac{dx}{dt}=L_fh(x(t))=p_2(x(t))=E_2(t) \quad (13)$$

…

$$E'_{r+1}(t)=\frac{\partial p_{r-1}}{\partial x}\frac{dx}{dt}=\frac{\partial L_f^{r-1}}{\partial x}\frac{dx}{dt}=L_f^{r-1}h(x(t))$$
$$=p_2(x(t))=E_r(t) \quad (14)$$

$$E'(t)=\frac{\partial p_r}{\partial x}\frac{dx}{dt}=\frac{\partial L_f^{r-2}}{\partial x}\frac{dx}{dt}$$
$$=L_f^rh(x(t)) + L_gL_f^{r-1}h(x(t))u(t) \quad (15)$$

Set

$$a(E(t)), N(t)=L_gL_f^{r-1}h(x(t))u(t)\,|_{x=p}^{-1}\,(E(t),N(t)) \quad (16)$$

$$b(E(t), N(t))=L_gL_f^{r}\,h(x(t))u(t)\,|_{x=p}^{-1}\,(E(t),N(t)) \quad (17)$$

(15) can be written as:

$E'_r(t) = b(E(t),N(t))+a(E(t),N(t))u$ (18)

Next, under eqn.12 the integral dynamics is considered as:

$$N'_k(t) = \frac{\partial p_k}{\partial x}(f(x(t))+g(x(t))u(t) = L_fp_k(x(t))+$$
$$L_gp_k(x(t))u(t) = L_fp_k(x(t)), k=r+1,r+2,r+3, \ldots \quad (19)$$

Thus, in short, the state space description of the system in the new co-ordinates as follows:

$E'_i(t)=E_{i+1}(t), i=1,2,3, \ldots$ (20)

$E'_r =b(E(t),E(t))+a(E(t),N(t))u(t)$ (21)

$N'(t)=q(E(t),N(t))$ (22)

$y(t)=E_1(t)$ (23)

, where $q_i(E(t),N(t)) = L_fp_k(x(t))\,|_{x=p}^{-1}\,(E(t),N(t))$, $k = r+1,r+2, \ldots, n$

Generally, (20), (21) are called the external dynamics of the system, and (22) is called the internal dynamics of the system. The proper choice of a linearizing control law is now apparent from (21). As b(E(t),N(t)) is bounded away from zero, its inverse is well defined. Thus the following linearising feedback law can be derived from (16), (17) and (21):

$$u(t)= f(t),v(t)) = (L_gL_f^{r-1}h(x(t))^{-1}(-L_f^rh(x(t))+v(t))$$
$$=a^{-1}(E(t),N(t))[-b(E(t),N(t))+v(t)] \quad (24)$$

, where v(t) is a new external control to be designed for the purpose of tracking signals. Note that the control law of (24) makes the state vector N(t) completely unobservable at the output. Since we are interested in achieving stable state tracking, it is required that N(t) remain bounded for the bounded E(t). However, we observe that E(t) can be thought as an external input vector with respect to the dynamics of N(t). Since E(t) is expected to track arbitrary time functions, it is clear that the boundedness of N(t) is entirely depend on the vector field q(E(t),N(t)). It can be easily verified that the equation:

$E'(t)=q(0,N(t))$ (25)

, is referred to as the zero dynamics [1]. The system in which the zero dynamics is asymptotically stable referred to as the minimum phase system. Stable tracking requires a stronger stability criterion for the dynamics

$E'(t)=q(E(t),N(t))$ (26)

be bounded input bounded state (BIBS) stable. In addition to the relative degree assumption, a further property of the zero dynamics required. This is illustrated in the following assumptions:

Assumption 1: The zero dynamics of eqn.25 is exponentially stable. Moreover, the function q(E(t),N(t)) is Lepchitz uniformally in N(t).

Remarks:

(i)Since the zero dynamics is exponentially stable by assumption therefore by a converse theorem of Lyapunov [7], there exists a Lyapunov Vo(N(t))which satisfies the following properties:

$K1\|N(t)\|^2<= Vo(N(t))<=K2\|N(t)\|^2$ (27)

$$\frac{\partial Vo(N(t))}{\partial N(t)}q(0,N(t)) <=-K3\|N(t)\|^2 \quad (28)$$

$$\left\|\frac{\partial vo(N(t))}{\partial N(t)}\right\|<=K4\|N(t)\| \quad (29)$$

,, where K1, K2, K3 and K4 are some appropriate positive constants.

(ii) Due to the fact that q(E(t),N(t)) is Lepschitz in E(t) there exists a positive constant L such that

$\|q(E(t),N(t))\text{-} q(0,N(t))\|<=L\|E(t)\|, N(t)\in R^{n-r}$ (30)

, and Lis called a Lepschitz constant of q(E(t),N(t)) [1].

Now, we are ready to design the tracking controller for the system (1) to minimize the trajectory error and to stabilize the close loop control system in the presence of system uncertainties and time-delay input.

3 Results of research

In this part, the robust control objective is to design a parameterized feedback linearizing control law such that the desired output trajectory of the close loop system is achieved and the effects of system uncertainty attenuated while maintaining the boundedness of all signals inside the control loops. For simplicity, all uncertain components can be lumped and the uncertain nonlinear system can be reduced to

$x'(t)=f(x(t))+g(x(t))u(t-d)+\sum(x(t),u(t-d))$

$y(t)=h(x(t))$ (31)

, where $\sum(x(t),u(t-d))=Df(x(t))+Dg(x(t))u(t-d)$.

Applying the nominal change of co-ordinates of equations (10)-(12) and the nonlinear state feedback of (24) to the system (1) yields

$$E'_1(t)=E_2+\frac{\P h(x(t))}{\P x(t)}(\Sigma(x(t),u(t-d))) \quad (32)$$

. . .

$$E'_{r-1}(t)=E_r(t)+\frac{\partial L_f^{r-2}h(x(t))}{\partial x(t)}\sum(x(t),u(t-d)) \quad (33)$$

$$E'_r(t)=v(t)+\frac{\partial L_f^{r-1}}{\partial x(t)}\sum(x(t),u(t-d))+g(x(t)(u(t-d)-u(t)) \quad (34)$$

$$N_1'(t)=q_1(E(t),N(t))+\frac{\partial p_n}{\partial x(t)}\sum(x(t),u(t-d)) \quad (35)$$

$$N'_{n-r}(t)=q_{n-r}(E(t),N(t))+\frac{\partial p_{r+1}}{\partial x(t)}\sum(x(t),u(t-d)) \quad (36)$$

Taking advantage of the identities in the nominal transformations and by some derivations, it can be easily verified the equations (32)-(36) can be transformed into:

$E'(t)=AE(t)+Bv(t)+DA(x(t),u(t), u(t-d))$ (37)

$N'(t)=q(E(t),N(t))+D\Phi(x(t),u(t-d))$ (38)

$y(t)=CE(t)$ (39)

, where $E=[E_1,E_2,...,E_n]^T$, $N=[N_1,N_2,...,N_{n-r}]^T$,

$DA=[DA_1, DA_2, \ldots, DA_r]^T=$

$$\begin{bmatrix} \frac{\P h(x(t))}{\P k(t)}\Sigma(x(t),u(t-d)) \\ \ldots \\ \frac{\P L_f^{r-2}h(x(t))}{\P x(t)}\Sigma(x(t),u(t-d)) \\ \frac{\P L_f^{r-1}h(x(t))}{\P x(t)}\Sigma(x(t),u(t-d)+g(x(t))u(t-d)-u(t) \end{bmatrix}$$

, $DA\in R^{rx1}$, $D\Phi=\begin{bmatrix} \frac{\P p_{r+1}}{\P x(t)}(\Sigma(x(t),u(t-d))) \\ \ldots \\ \frac{\P p_n}{\P x(t)}(\Sigma(x(t),u(t-d))) \end{bmatrix}\in R^{(n-r)\times 1}$

$$A=\begin{bmatrix} 010.0 \\ 001.0 \\ \ldots\ldots \\ 000.1 \\ 00000 \end{bmatrix}\in R^{r\times r},$$

$B=[0\ \ 0\ \ldots\ 0]^T\in R^{r\times 1}$, $C=[0\ \ 0\ \ldots\ 0]^T\in R^{1\times r}$

Consider that the output y(t) will track the output $y_d(t)$ of the reference model. The desired model reference is

$z'(t)=A_0z(t)+B_0y_{sp}(t)$ (40)

where z(t) is the state variable, A_0,B_0,C_0 are matrices and vectors with appropriate dimensions. $Y_{sp}(t)$is the external input, and $y_y(t)$is the desired output trajectory. The following assumptions are needed to achieve the desired output trajectory:

Assumption 2:

The desired trajectory output and its first r derivatives are all uniformly bounded, and satisfy:

$º(y_d(t)\ y_d^{(1)}(t)\ \ldots.\ \ y_d^{®}(t))º <=B_d$ (41)

, where B_d is the some positive constant.

Remark:

(iii) The fact that the model reference must be of relative order of r, to avoid the use of differentiators in the controller. Furthermore, if the desired trajectory and its derivatives are uniformly bounded, one can make the output tracking error, $y(t)-y_d(t)$, as minimum as possible and the whole system states locally stable. This is further verified in the subsequent theorem:

Define: $e_i(t)=E_i(t)-y_d^{(i-1)}(t)$, $i=1,2,...,r$ (42)

Then equations (24) and (37)-(39) become:

$e'(t)=Ae(t)+B(v(t)-y_d^r(t))+DA(x(t),u(t),u(t-d))$ (43)

$N'(t)=q(E(t),N(t))+D\Phi(x(t),u(t-d))$ (44)

$e_1(t)=Ce(t)$ (45)

Define the tracking error with parameterization

$e_i^*(t)=pp^{i-1}e_i(t)$, $i=1,2,...,r$ (46)

, for a positive constant pp<=1. Then we obtain the following equations:

$ppe'^*(t)=Ae^*(t)+pp^rB(v-y_d^{®}+ppDA^*(x(t),u(t),u(t-d)$ (47)

$N'(t)=q(E(t),N(t))+D\Phi(x(t),u(t-d))$ (48)

$e_1(t)=Ce^*(t)$ (49)

, where $DA^*=[DA_1\quad ppDA_2\ \ldots\ \ pp^{r-1}A_r]^T$

Now, we propose the control law v(t) of the following form:

$$v(t)=\Phi_0(e^*(t),y_d^{(r)}(t))=y_d^{(r)}(t)-pp^{-r}\sum_{t=1}^{r}a_ie^*_i(t)$$
$$=y_d^{(r)}(t)+\Gamma(a,pp)e(t) \quad (50)$$

, where $\Gamma(a,pp)=[-pp^{-r}a_1,...,-pp^{-1}a_r]$

Note that $a_1,a_2,...,a_r$ are chosen such that

$s^r+a_rs^{r-1}+\ldots+a_1$ (51)

is a Hurwitz polynomial and s is the Laplace operator. By some derivations we obtain a standard singularly perturbed system of the form:

$ppe'^*(t)=A_ce^*(t)+ppDA^*(x(t),u(t),u(t-d))$ (52)

$N'(t)=q(E(t),N(t))+D\Psi(x(t),u(t-d))$ (53)

, where $A_c=\begin{bmatrix} 0\ \ 1\ 0\ 0\ 0 \\ 0\ \ 0\ 1\ 0\ 0 \\ .\ \ .\ .\ .\ . \\ 0\ \ 0\ 0\ 0\ 1 \\ -a_1-a_2\ .\ \ .-a_r \end{bmatrix}$ is companion matrix.

Moreover, we define P as a symmetric positive define matrix satisfying the Lyapunov equation:

$A_c^TP+Pa_c=-I$ (55)

, where I is an identity matrix.

Intuitively, to reduce the effects of the nonlinearity including state and input perturbations to the tracking error e(t) while maintaining the boundedness of N(t), we can choose a large gain in the control law v(t) eqn.50. However, the input error between the delay in put u(t-d) and the desired control law u(t) will be large due to inappropriate high gain feedback control. Therefore how to choose a suitable tuning parameter pp in the present control law, which guarantees the overall system stability and good performance, is the main key issue of in this paper.

Owing to state feedback control the resulting closed loop system of (52) and (53) may include state delay and input delay. Hence, the bound information of the delayed state x(t-d) and current state x(t) is a priori. This is addressed in the following assumption.

Assumption 3:

Consider that $W(e^*(t))=ppe^*(t)^T pe^*(t)$ is a positive define function, where $e^*(t)$ is the parameterized variable as shown in eqn.46, and P satisfies eqn.55. If there exists

$W(e^*(t-d))<=\varepsilon^2 c^*$º$e^*(t)$º (56)

where c^* is the ratio between maximum and minimum eigenvalues of matrix P.

Remarks:

(iv) This assumption is based on the stability theorem of Razumikhin [7]. Under the continuous inversion of parameterized co-ordinate transformation (equations (10), (11), (42), (46)), i.e.:

$x_l(t) = p_k^{-1}(E_i(t),N_j(t)) = p_k^{-1}(e_i(t)+y_d^{i-1}(t),N_j(t))$
$= p_k^{-1}(pp^{1-i}e_i^*(t)+ y_d^{(i-1)}(t)N_j(t)),\ i=1, 2, \dots$

The result of (56) represents that if the current state x(t) is bounded, the delayed state x(t-d)is uniformly bonded. This assumption had been used in the literature for state-delayed systems [8].

From the assumptions above, the error can be expressed further as follows:

³u(t-d)-u(t)³= ³Φ*(x(t-d),v(t-d)* - Φ(x(t),u(t)))
³=³-a^{-1}(E(t-d),N(t-d))
b(E(t-d),N(t-d))+$y_d^{®}$(t-d)a^{-1}(E(t-d), N(t-d)) –
$y_d^{®}$(t)a^{-1}(E(t),N(t))+Γ(a,pp)
a^{-1}(E(t-d),N(t-d))e(t-d)-a^{-1}(E(t),N(t))e(t))³ (57)

After taking derivation of Lepschitz Jacobian near the origin, one can show that:

³u(t-d)-u(t) ³<=pp^{-r}(c_1ºE(t-d)-E(t) º+ c_2ºN(t-d)-N(t) º+
c_3<=k^*/pp^r (58)

, where c_1,c_2,c_3 and $k^*$are positive constants.

Based on the ongoing analysis, a simple design procedure for better output tracking in the presence of plant uncertainties and time delay is proposed. The basic principle is to design the parameterized state feedback control law for getting a good system response and to tune the parameter pp for achieving robust stability and performance specification.

The control structure is shown in Figure 1, and the design procedure consists of five steps:

Step 1: Select the controlled output and calculate the co-ordinate transformation based on the nominal system as shown in equations (10)-(12).

Step 2: Transform the nonlinear system into an equivalent linear system as shown in equations (20)-(23).

Step 4: Calculate the desired reference model based on the specific specification as shown in equation (40).

Step 5: Adjust the tuning parameter pp to satisfy the performance specification.

4 Conclusions

The theory for designing a robust adaptive control law based on input-output feedback linearization of nonlinear systems with uncertainties and a time-delay in the manipulated input by the approach of parameterized state feedback control has been presented. The main advantage of this method is that the parameterized state feedback control law can effectively suppress the effect of the most part of nonlinearities, including system uncertainties and time-delay input in the pp-coupling perturbation form and the relative order of nonlinear systems is not limited.

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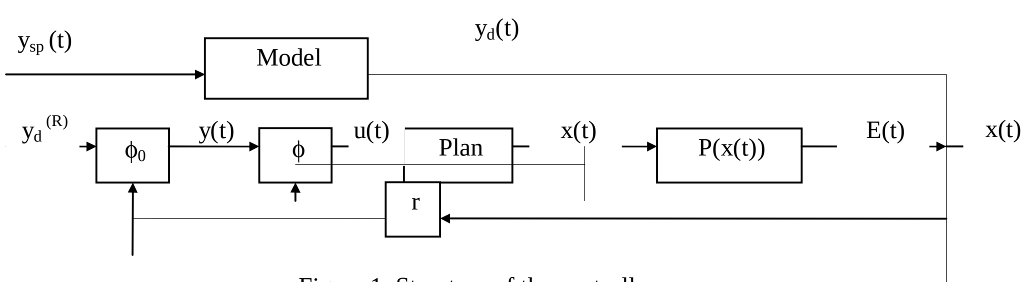

Figure 1: Structure of the controller