Rule ecology dynamics for studying dynamical and interactional nature of social institutions

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Abstract

In this paper, we address a new concept to view a social system as consisting of diversified institutions interacting with each other and dynamically changing through actions of individuals in a society. A mathematical framework for the concept is formalized. The framework, called Rule Ecology Dynamics (RED), is an extension of multi game dynamics, in which players play plural games simultaneously, by introducing time dependent weights for the plural games and a "meta-rule". The meta-rule is a rule to determine the change of rules. We show simulation results of two kind of meta-rules , average payoff type and inverse standard deviation type. We discuss that this framework is a realization of rule dynamics and it has certain relevance to describe real phenomena of institutional dynamics.

Keywords: Rule Ecology Dynamics, Formation and Change of Institutions, Replicator Equation, Multi-Game Dynamics, Evolutionary Game Theory, Rule Dynamics

1 Introduction

Social institutions are rules for individuals' behavior and cognition broadly accepted and used in a society. In our society, there exist various institutions/rules ubiquitously. Seiyama[1] describes such situation as "We are living with institutions, such as family, commuter passes, trains, universities, telephones, postal services, E-mails, meetings, lectures, laws, norms, conventions and so on. There is no one that is not institutional. We engage in our daily life postulating such institutions. They are omnipresent. It is very difficult to pick out non-institutional aspect, even only one, from our daily life." As we can easily imagine, each institution has different importance or influence for individuals' behavior.

Aoki and Okuno-Fujiwara[2] discusses that diversity of institutions exists in different social systems. But, as quoted above, there are great diversity of institutions in a society. Aoki and Okuno-Fujiwara[2] point out the important feature of institutions, that is an institutional complementarity. When an institution provides a reason to exist another institution, these two institutions are refereed to as in the relationship of institutional complementarity. Since only one of such institutions cannot change independently, Aoki and Okuno-Fujiwara insist that they are stable. However, social institutions often changes and many institutions interact with each other. Accordingly, a change of an institution induces changes of other institutions. The chain of change may go on to all institutions.

This chain of change through interaction among institutions is like an ecological system of biological species. In this paper, we conceptualize such dynamic and interactional nature of institutions as an ecology of rules. Here we use a term 'rule' as a more abstract version of institutions. We propose a new framework to formalize the ecology of rules and its dynamics as an extension of evolutionary game theory. This framework is named "Rule Ecology Dynamics (RED)[1]". This framework can integrate two game theoretical treatment of institutions, one treats institutions as game rules, and the other as equilibria of games[3].

[1] The previous version of the framework was called "Meta-Evolutionary Game Dynamics" since it is an extension of game dynamics (replicator dynamics) introducing a meta-rule[3].

2 Rule Ecology Dynamics

In our framework, rules and individual behaviors are modeled by games and strategies of players in the games, respectively. We start the formalization of the RED from the replicator equation that governs the dynamics of population of each strategy[4]:

$$\dot{x}_i = x_i(u_i - \bar{u}) \qquad (i = 1 \cdots N) \ , \tag{1}$$

where x_i is a population share of the i-th strategy and satisfies $\sum_{i=1}^{N} x_i = 1$, u_i is a payoff of the i-th strategy, $\bar{u} = \sum_{i=1}^{N} x_i u_i$ is the average of the payoffs, and N is the number of strategies.

To represent various compositional rules, multiple games that all players play are brought in, as proposed in Multi Game Dynamics[5]:

$$\begin{aligned} \dot{x}_{i_1 \cdots i_M} &= x_{i_1 \cdots i_M} \sum_{g=1}^{M} (u^g_{i_1 \cdots i_M} - \bar{u}^g) \\ & \qquad (g = 1 \cdots M) \ , \end{aligned} \tag{2}$$

where $x_{i_1 \cdots i_M}$ is a population share of a strategy $(i_1, i_2, \cdots, i_M)$, which means that a player plays the strategy i_1 at the game 1, the strategy i_2 at the game 2 and so on, satisfying $\sum_{i_1=1}^{r^1} \sum_{i_2=1}^{r^2} \cdots \sum_{i_M=1}^{r^M} x_{i_1 \cdots i_M} = 1$, where r^g is the number of options at the game g, $u^g_{i_1 \cdots i_M}$ is a payoff of the strategy $(i_1, i_2, \cdots, i_M)$ at the game g, $\bar{u}^g$ is the average payoff at the game g, and M is the number of games.

We introduce a weight for each game to treat the degree of influence or importance of each rule. The weight of each game change with time through individual behavior. The change of the game weights is governed by a meta-rule. A meta-rule is a more basic and stabler rule than focal rules. It is for determine which rules are relatively desirable or relatively strong in influence for individual behavior. Introducing a meta-rule is a representation of the hierarchical structure of interaction and stability of rules. Thus, the meta-rule corresponds to the constitution or social values and norms, which are relatively not easy to change. In our framework, it is modeled as an evaluation function of games. The RED is defined by the following three equations:

$$\dot{x}_{i_1 \cdots i_M} = x_{i_1 \cdots i_M} \sum_{g=1}^{M} w^g (u^g_{i_1 \cdots i_M} - \bar{u}^g) \ , \tag{3}$$

$$\tau \dot{w}^g = w^g (\lambda^g - \bar{\lambda}) \ , \tag{4}$$

$$\lambda^g = \lambda^g(\boldsymbol{x}, \boldsymbol{u}^g) \ , \tag{5}$$

where w^g is the weight of the g-th game, satisfying $\sum_{g=1}^{M} w^g = 1$, λ^g is an evaluation of the g-th game, and $\bar{\lambda} = \sum_{g=1}^{M} w^g \lambda^g$ is the weighted average of the evaluations of games, τ is a time coefficient for the changing velocity of the games relative to that of the strategy populations, $\boldsymbol{x} = (\{x_{i_1 \cdots i_M}\})$ $(i_g = 1 \cdots r^g)$ is the strategy profile that is a vector of population share of each strategy, and $\boldsymbol{u}^g(\boldsymbol{x}) = (\{u^g_{i_1 \cdots i_M}(\boldsymbol{x})\})$ is the combined payoff or the payoff profile that is a vector of the payoff of each strategy.

The equation (5) is a meta-rule to give evaluation of each game. The variables of the evaluation function is the strategy profile $\boldsymbol{x}$ and the combined payoff $\boldsymbol{u}^g(\boldsymbol{x})$. This setting makes an interaction loop among individual behavior, the rules and the meta-rule closed. Namely, the change of rules depends on the consequences of the behavior under the rules.

3 Simulation Result

We show simulation results of the system for tow settings of the meta-rule. The meta-rule should be considered appropriately for the objective system to be modeled.

The first example is the average type meta-rule

$$\lambda^g_{\mathrm{A}}(\boldsymbol{x}, \boldsymbol{u}) = \sum_{i_1, \cdots, i_M} x_{i_1 \cdots i_M} u^g_{i_1 \cdots i_M} \ . \tag{6}$$

This is a model of a market economics and each rule is thought of as describing a market. We suppose in the market economics that the more profit a market or a rule gives for individuals/organizations, the more important the market/rule is, as in stock markets.

The simulation result is depicted in Fig. 1[2]. In this case, a phenomenon like globalization is observed. Namely, the system is monopolized by a strategy and then games in which the monopoly strategy wins develop their weights.

The second example of the meta-rule is to describe the egalitarianism that is the doctrine of the equality of mankind and the desirability of political, economic and social equality. This meta-rule is formalized as the inverse of standard deviation of agents' payoffs:

$$\lambda^g_{\mathrm{IV}}(\boldsymbol{x}, \boldsymbol{u}) = \sum_{i_1, \cdots, i_M} \left\{ x_{i_1 \cdots i_M} \left(u^g_{i_1 \cdots i_M} - \bar{u}^g \right)^2 \right\}^{-1} \ . \tag{7}$$

[2] In the following simulations, we simplify the RED (Eq.(3)-(5)) as follows: The number of options of all games are the same, $r^g = N \ \ (g = 1 \sim M)$ and each strategy is limited to take the same one at the all games and thus the strategy share is indicated as $x_{i_1 \cdots i_M} \equiv x_i$.

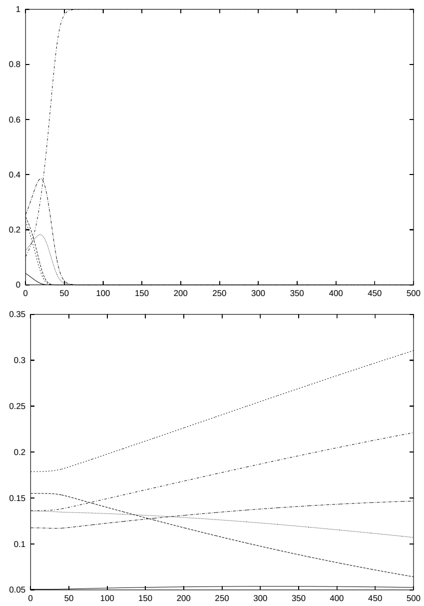

Figure 1: An example of dynamics of RED with the average type meta-rule (Eq.(6)). The y axes are the strategy distribution for the top graph and the weights of games for the bottom, respectively. The x axis is time steps. The system size is $N = 6, M = 7$.

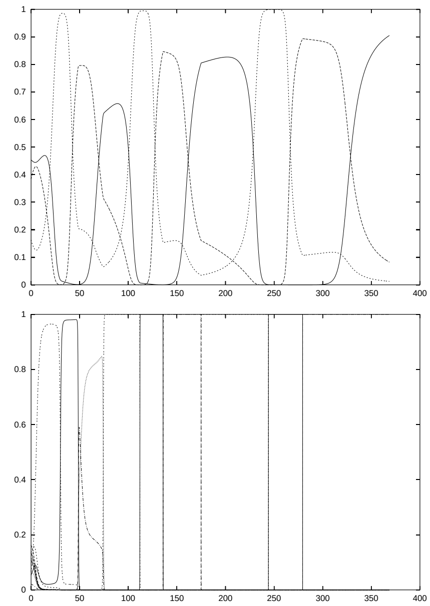

Figure 2: An example of dynamics of RED with the the inverse standard deviation type type meta-rule (Eq.(7)). The y axes are the strategy distribution for the top graph and the weights of games for the bottom, respectively. The x axis is time steps. The system size is $N = 3, M = 10$.

The dynamics of the strategy share and the games' weights are shown in Fig. 2. This case gives revolutionary changes of predominant rules. Namely, no one game and no one strategy can dominate the system stably and temporarily prevalent games and strategies change continually. It is a rather paradoxical situation that egalitarianism induces destabilization of established rules and revolutions.

4 Discussion – RED as Framework for Rule Dynamics

When we try to understand some object, it is often described by a set of states and a system of static functions. The functions are rules to govern the change of states. In other words, the rules are operators and the states are operands. Describing with static functions implies that the decomposability between rules and states is presupposed. The decomposability may occasionally not be able to be presupposed. This undecomposability between rules and states, or between operators and operands is one of the main features of complex systems[6]. In complex systems, rules are often not static but dynamically changing. The typical phenomena are found in evolution, development, learning, adaptation and emergence. We call such dynamic phenomena rule dynamics.

The social rule is also an example having undecomposability between operators and operands. Social structures such as institutions and norms are formed, maintained and changed with time. The remarkable dynamic nature in such systems are self-modification. Individuals behave under some institutions and change the institutions.

Some efforts to study and to describe rule dynamics have been launched [7, 8, 9, 10]. The RED proposed here is also a framework to describe the rule dynamics, especially observed in social systems. Actually, RED (Eqs.(3)-(5)) can be written in a matrix form as

$$\dot{\boldsymbol{x}} = \left(G_{\mathrm{T}}(t)\boldsymbol{x} - \boldsymbol{x} \cdot G_{\mathrm{T}}(t)\boldsymbol{x}\right)\boldsymbol{x} \ , \qquad (8)$$

where $G_{\mathrm{T}} = \sum_{g=1}^{M} w^g A^g$ is a total game that is the weighted some of all games and A^g $(g = 1, \cdots, M)$ is the payoff matrix of the g-th component game. It is clearly seen that RED is a replicator equation with time dependent interaction matrix $G_{\mathrm{T}}(t)$.

The RED can be resolved into a multi-population replicator system[11] of the individual strategies and the rules:

$$\dot{x}_{i_1 \cdots i_M} = \left[(A\boldsymbol{w})_{i_1 \cdots i_M} - \boldsymbol{x} \cdot A\boldsymbol{w} \right] \dot{x}_{i_1 \cdots i_M} \;, \quad (9)$$

$$\tau \dot{w}_g = \left[\lambda^g - \bar{\lambda} \right] w^g \quad (10)$$

where the element of a matrix A is $(A)_{i_1 \cdots i_M g} = u^g_{i_1 \cdots i_M}$. This representation clearly show that RED is interactions between the individual strategies and the rules.

5 Conclusion

We have proposed a new model for studying the dynamics of social institutions. The model called Rule Ecology Dynamics (RED). The key concept of RED is that rules in a society is diverge and ubiquitous, interact with each other and change through behavior of individuals acting under the rules. The ecological interaction of rules is described by a modification of the replicator equation. The notable point is to incorporate not only the interactions among individuals but also those of rules. In RED, an interaction loop between individual behavior and rules is realized by introducing a meta-rule. Accordingly, RED is a mathematical model of the micro-macro loop proposed by Shiozawa[12] to understand economic dynamics.

The formation and the change of social institutions is a representative of rule dynamics phenomena, which is a key feature of complex systems. We show that our model is a framework to describe rule dynamics. We also show that RED describes the interaction (game) between individual behavior and rules.

By setting two kind of meta-rules, the average type and the inverse standard deviation type, we show the simulation results that show actually the dynamics of rules and support certain effectiveness of RED to study the dynamics of institutions.

We need to promote the relevance of the RED for describing the real phenomena of rule dynamics. We can suggest a concrete example of the rule dynamics to be described by RED. It is changes of monetary credit systems in Argentina where a local currency, GRT, has been used. Since the crash of national currency, Peso, caused by the governmental default in 2002, the credit of the national and the local currencies, mechanisms to establish and support the currencies, relationship between them, users' impression about them have changed. All of these factors constitute institutions and interact with each other ecologically. This phenomena may be able to be modeled with RED. Through such effort, the framework of RED come to be a tool of designing institutions from the viewpoint of evolutionary economics.

Acknowledgements

This research is partly supported by Canon Foundation Europe; and a Grant-in-Aid for Scientific Research (No.15700183) from the Ministry of Education, Culture, Sports, Science and Technology of Japan and by the Japan Society for the Promotion of Science.

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