An Optimal Capturing Trajectory Planning for a Moving Object

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Abstract

An optimal capturing trajectory for a moving object is proposed in this paper based on the observation that a single-curvature path is more accurate than double- or triple-curvature paths. Traveling distance, tracking time, and trajectory error are major factors considered in deciding an optimal path for capturing the moving object. That is, the tracking time and distance are minimized while the trajectory error is maintained as small as possible. The three major factors are compared for the single and the double curvature trajectories to show superiority of the single curvature trajectory. Based upon the single curvature trajectory, a kinematics model of a mobile robot is proposed to follow and capture the moving object, in this paper.

Keywords Single curvature trajectory; Mobile robot; Capturing; Moving object

1 Introduction

From 1970's, 'Robot' has come close to our daily lives. More precise and fast control became possible with the development of integrated circuits, sensors, artificial intelligence, image processing, and computer technologies. As the result, intelligent robots which recognize changes of environment and decide their reactions automatically, appeared in daily lives[1][2].

There are two typical robots: a robotic manipulator and a mobile robot. The robotic manipulators that has a fixed base, have been utilized for precise assembly operations. However their workspace is limited within a small volume. On the contrary, workspace of a mobile robot is not limited to a certain area. However, in the control of the mobile robot, position control accuracy, velocity and acceleration limits, obstacle recognition, and trajectory planning must be considered carefully[3].

Therefore researches on a mobile robot can be globally classified into four categories: mobile robot navigation itself, trajectory planning, position estimation, and driving control. Trajectory planning of a mobile robot aims at providing an optimal path from an initial to a target position. Generally there are three major factors to be considered in the trajectory planning: the driving time, distance, and error. Note that these factors are not independent but closely related to each other. Therefore the trajectory planning aims at the selection of best trajectory in view of all the factors. There are not many researches on the trajectory planning, while many papers on the position estimation and trajectory control are published. The importance of driving trajectory planning which reduces driving distance and time has not been studied closely[4].

This paper proposes a single curvature trajectory for a precise motion of a mobile robot and it is applied for the trajectory of a mobile robot to capture a moving object. Based on the kinematics modeling of a mobile robot, a theoretical driving model is proposed; the driving characteristics of the single curvature trajectory, that is, driving time, distance, and error have been measure and analyzed through the simulations and real experiments. In chapter 2, driving characteristics of a mobile robot have been analyzed through kinematics analysis. The single curvature planning is described in chapter 3 concretely. In chapter 4, the capturing algorithm following the single curvature trajectory is introduced with theoretical formulars. The comparison between a single curvature trajectory and a double curvature trajectory in terms of tracking error, time, and distance has been shown in chapter 5. The real capturing experiments are also demonstrated in the chapter. Chapter 6 concludes this paper.

2 Kinematics Modeling and Moving Characteristics of a Mobile Robot

To form a trajectory for a mobile robot to follow the desired path, kinematics analysis which shows the relation between robot control variables and robot position and velocity needs to be done previously.

2.1 Kinematics analysis of a mobile robot

As shown in Fig. 1-(a), a mobile robot with differential driving mechanism has two wheels on the same axis, and each wheel is controlled by an independent motor. Let us define v_L as the velocity of left wheel, v_R as that of the right wheel, and l as the distance between the two wheels. The robot motion can be determined by the two wheel velocities, v_L and v_R, and the linear and angular velocities of the mobile robot can be described in terms of v_L and v_R as follows[5]:

$$v_1 = \frac{v_R + v_L}{2} \qquad (1)$$

$$v_2 = \frac{2(v_R - v_L)}{l} \qquad (2)$$

Kinematics model of a mobile robot with differential driving mechanism can be described as Fig. 1-(b). On the two dimensional $X-Y$ Cartesian coordinates, position of the mobile robot is described by $x_R(t)$ and $y_R(t)$ while the orientation is represented as $\theta_R(t)$.

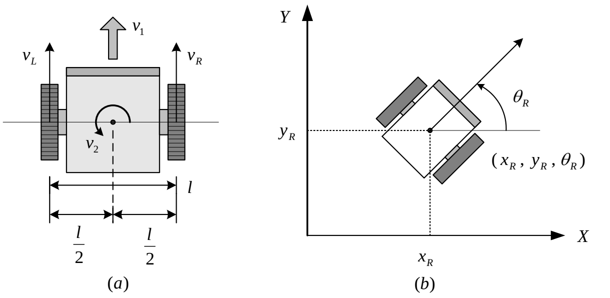


Fig. 1. Kinematics model of a mobile robot.

Then $\dot{x}_R(t)$ and $\dot{y}_R(t)$ represent the linear velocity of a mobile robot while $\dot{\theta}_R(t)$ represents the angular velocity. The velocity vector of the mobile robot is defined as

$$\dot{P} = [\dot{x}_R \quad \dot{y}_R \quad \dot{\theta}_R]^T \tag{3}$$

where $P = [x_R \quad y_R \quad \theta_R]^T$.

Now the kinematics model of the mobile robot can be represented as[6]

$$\dot{P} = \begin{bmatrix} \cos\theta_R & 0 \\ \sin\theta_R & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{4}$$

Kinematics analysis aims at the proper velocity assignment to each wheel to drive the mobile robot to a desired position and orientation[7].

2.2 Driving Principle of a Mobile Robot

Motion states of the mobile robot with differential driving mechanism are changing according to the two wheel velocities. A robot with multiple wheels rotates along a rotation center instantaneously, and this rotation center is defined as ICC (Instantaneous Center of Curvature). As it is shown in Fig. 2-(a), ICC locates on the cross-section point of the extension-lines of the wheel centers. For a mobile robot with differential driving mechanism, as shown in Fig. 2-(b), ICC can be located any point on the wheel axis since the two wheel axes are on the same line[5].

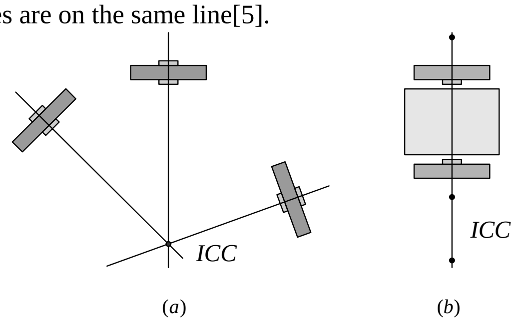


Fig. 2. Center of instantaneous rotation.

In this case, ICC will be determined by the velocity ratio between the two wheels'. Fig. 3 illustrates ICC along with the robot's velocity and position. There exists proportional relationship between the wheel velocity and the distance from the wheel to the ICC, which is represented as

$$v_L : v_R = R - \frac{l}{2} : R + \frac{l}{2}. \tag{5}$$

Equation (5) can be simply represented as

$$R = \frac{l}{2}\left(\frac{v_R + v_L}{v_R - v_L}\right). \tag{6}$$

Note that the rotation radius of the mobile robot is determined by the values of left and right wheel velocities. When the robot is following a straight line, $R = \infty$ and $v_R = v_L$. When $v_R \neq v_L$, the robot follows a circular trajectory with a certain rotation radius.

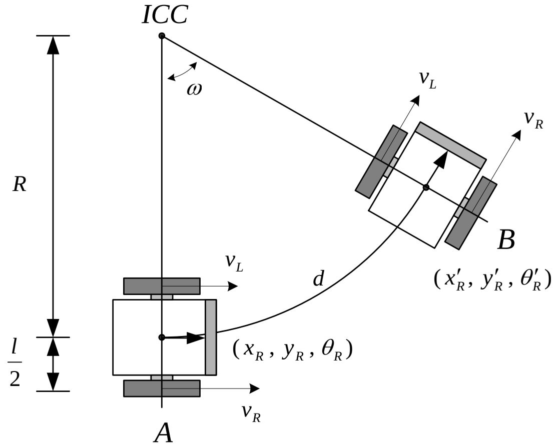


Fig. 3. ICC of a mobile robot

In Fig. 3, when the mobile robot is moving from A where the robot is located on (x_R, y_R, θ_R) at time $= t$ to B where the position is on (x'_R, y'_R, θ'_R) at time = $t + \delta t$, the coordinates of ICC can be determined as

$$ICC = [x_R - R\sin(\theta_R),\ \ y_R + R\cos(\theta_R)] \tag{7}$$

Also the mobile robot position, (x'_R, y'_R, θ'_R), at time= $t + \delta t$, can be expressed in terms of position of ICC and angular velocity, ω, as follows:

$$\begin{bmatrix} x'_R \\ y'_R \\ \theta'_R \end{bmatrix} = \begin{bmatrix} \cos(\omega\,\delta t) & -\sin(\omega\,\delta t) & 0 \\ \sin(\omega\,\delta t) & \cos(\omega\,\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R - ICC_X \\ y_R - ICC_Y \\ \theta_R \end{bmatrix} + \begin{bmatrix} ICC_X \\ ICC_Y \\ \omega\,\delta t \end{bmatrix}. \tag{8}$$

Now, the total distance, d, and the rotation angle, φ, of the mobile robot movement from location A to B can be obtained as follows:

$$d = \int_t^{t+\delta t} v_1\,dt = \int_t^{t+\delta t} \frac{v_L + v_R}{2}\,dt \tag{9}$$

$$\varphi = \frac{d}{R} = \frac{\int_t^{t+\delta t} (v_L + v_R)\,dt}{l(v_L + v_R)}(v_R - v_L) \tag{10}$$

Using these equations, when the rotation radius, distance of movement, and rotation angle of the mobile robot are planned previously, the desired linear and angular velocities, v_L, v_R, and ω can be obtained when it makes a curved motion[5].

3 Single-Curvature Trajectory

3.1 Curvature

Curvature, k, is defined as ratio of $\Delta\theta$ to Δs, when a mobile robot is rotating from a point, P, to Q as it is illustrated in Fig. 4. The curvature is defined as[8]

$$k = \lim_{\Delta s \to 0} \left| \frac{\Delta\theta}{\Delta s} \right| = \left| \frac{d\theta}{ds} \right| \quad (11)$$

Rotation radius can be defined as the inverse of the curvature as

If $k > 0$, then $\rho = 1/k$. (12)

As it can be clearly recognized from Eq. (11), the total travel distance along the curve, Δs, is the length of the arc and is proportional to the curve radius. Since curvature, k, is inverse proportional to Δs, curvature k is inverse proportional to the radius of the curve. If $k = 0$, then the radius of the curve, in other words, rotation radius becomes infinite. That is, $k = 0$ implies a straight line which is a circle with the infinite radius [5].

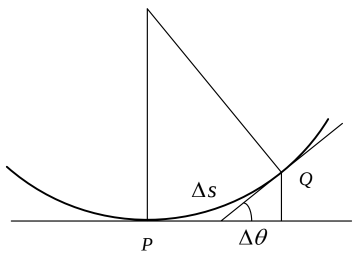


Fig. 4. Curvature.

When the mobile robot is moving along the curved trajectory, the rotation radius affects severely on the driving error. Generally the mobile robot has smaller error along a straight line path $(k = 0)$, than the curved $(k \neq 0)$. Theoretically, the curved trajectory can be calculated with the Eq's (5)~(8) assuming the pure rolling and non-slipping conditions. Practical driving may have some differences from the theoretical values. When the mobile robot is following a curved path, there exist centrifugal and centripetal forces all together. The friction force between the surface and the wheels acts as centripetal force to ICC and keeps the curved motion of the mobile robot. Under the ideal conditions, the driving error becomes zero without the slippage. However, practically, there always exists the driving error caused by the slippage. From the following equation, it can be formulated that the centrifugal force is a function of rotation radius and velocity:

$$F = \frac{mv^2}{r} \quad (13)$$

Fig. 5 illustrates the driving situation of a mobile robot, starting from A and following a circular curved path. In the ideal condition, the estimated position of robot becomes B1. However, practically the robot arrives at B2 by the driving error. So far there are lots of researches to reduce the driving error caused by the rotation radius and driving speed.

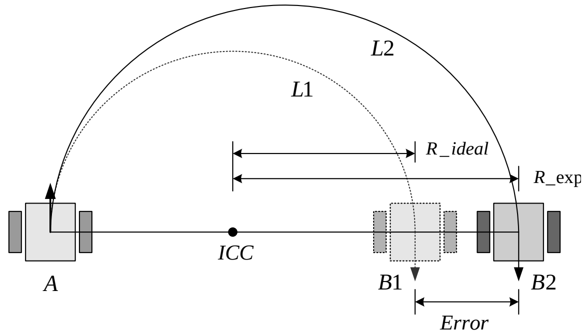


Fig. 5. Driving error of a mobile robot on a curved path.

Fig. 6 shows the error characteristics according to the driving radius and speed with a real mobile robot. The speed of right wheel is kept constant, while that of left wheel was changed to make a curved motion of the mobile robot. With the increase of the left wheel velocity, the driving error is increased as well. Also note that the driving error becomes large with the small rotation radius even though the speed is same. From the analysis of Fig. 6, it can be concluded that the driving error of a mobile robot becomes larger with the smaller rotation radius and with the higher speed while it is driving along the curved path[4].

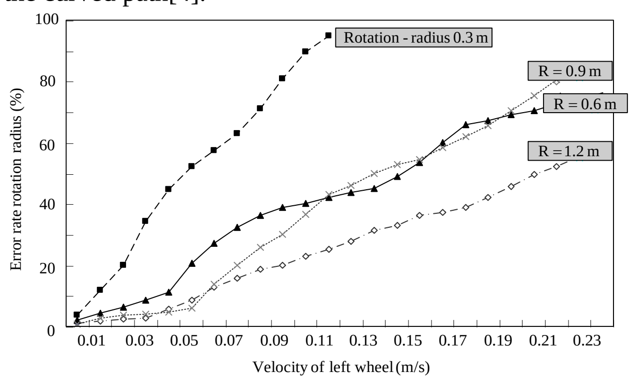


Fig. 6. Driving errors in terms of speed and rotation radius

3.2 Single curvature trajectory

The single curvature trajectory keeps the same curvature while the double curvature trajectory changes the driving direction and curvature at the point of inflection.. That is, the points of inflection exist at several locations. While a mobile robot is moving along the trajectory , the wheel velocities of the mobile robot need to be changed whenever the rotation radius and driving direction are changed according to Eq. (6).

For the same travel distance, the single curvature trajectory has the biggest rotation radius, while the others have varying radius with smaller values. Therefore it can be predicted that when the mobile robot is traveling along the single-curvature trajectory, it may have the least tracking error. To verify the fact, the tracking errors are going to be recorded and analyzed through the real experiments. Before the real experiments, pre-simulations are performed to show the rotation angles, traveling distance, trajectory, etc.

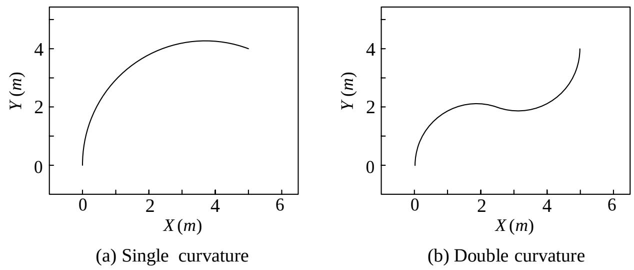


Fig. 7. Single-curvature and double-curvature trajectories.

Table 1. Computed single-curvature and double-curvature trajectories

	Single Curvature	Double Curvature
Initial Position	Pi=(0, 0, 90^o)	Pi=(0, 0, 90^o)
Final Position	Pf=(5, 4, 109.6^o)	Pf=(5, 4, 90^o)
Path Distance(m)	7.18 m	7.18 m
Curve Radius(m)	3.75 m	1.88 m
Rotation Angle(deg)	109.65^o	109.65^o, 109.65^o
ICC Coordinates	(3.75, 0)	(1.87, 0) (3.13, 4)

Simulation results the total travel distance is same for the single-curvature and double-curvature trajectories. However, the rotation radius is double for the single-curvature trajectory. In more detail, the rotation radius for the single-curvature trajectory is 3.75m, while that of double-curvature is 1.875m, which has the point of inflection at (2.5, 2). As the result of close observation of Fig. 6, it can be concluded that the single curvature trajectory has less tracking error than the double-curvature.

4 Capturing Algorithm

When a mobile robot is tracking to capture a moving object, the decision of tracking trajectory is a very important factor for a successful operation. To minimize the tracking error, a single curvature trajectory is selected and the mobile robot is planned to follow the trajectory. Fig 8 shows the mobile robot path to capture a moving object along the single-curvature trajectory. The total path is composed of two single-curvature and a straight-line trajectories. The left-wheel velocity and acceleration of the robot are defined as v_L and a_L, respectively. At the first part of the path, the robot needs to rotate clock-wise along the circular trajectory, the left-wheel velocity is higher than the right-wheel velocity, $v_L > v_R$. The velocities can be related to the accelerations as follows:

$$v_L = \int_0^t a_L(\tau)d\tau = a_L t \qquad (14\text{-a})$$

$$v_R = \int_0^t a_R(\tau)d\tau = a_R t \qquad (14\text{-b})$$

As it can be noted from the relations between the velocities and accelerations, the velocity is proportional to the acceleration, that is, $a_L > a_R$. The second segment is a straight line along which the robot follows from (x_{line1}, y_{line1}) to (x_{line2}, y_{line2}). At the third part of the path, the mobile robot estimates the position, (x_{obj}, y_{obj}), and velocity of the moving object using a CCD camera attached on its body, and follows the single-curvature trajectory to capture the moving object precisely.

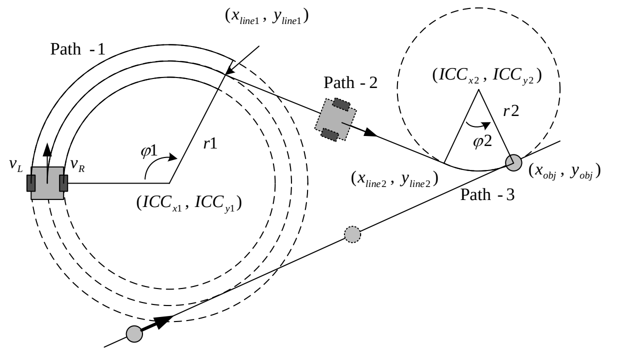


Fig. 8. Capturing trajectory of a moving object

5 Experiments

5.1 Experimental system

Experiments were performed in the research laboratory building. The floor was flat and slippery.

A. Moving object

The moving object does not need any specific characteristics. Therefore it is implemented as a simple and small mobile robot. To estimate the position of the moving object easily, a red color patch is attached on the object. The color information of the patch is obtained by the CCD camera on the mobile robot and is utilized to estimate the position of the moving object.

B. Mobile Robot

The mobile robot utilized for the experiments has a two DOF active vision camera to recognize the objects and surrounding environments. To enable the autonomous navigation, several sensors including rate-gyroscope sensors are used for the mobile robot. The hardware specifications of the mobile robot are summarized in Table 2.

Table 2. Specifications of the mobile robot

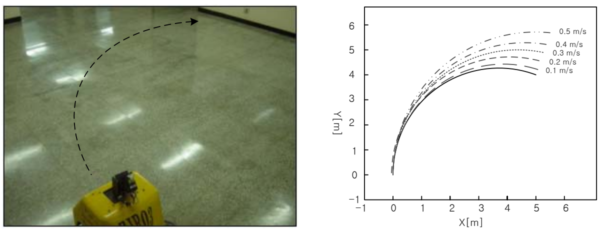
Item	Specifications
Size (mm)	660 * 430 * 465 (L*W*H)
Weight (kg)	19.5 kg
Distance between Wheels (mm)	400 mm
Diameter of Wheel	210 mm

5.2 Experimental results and discussions

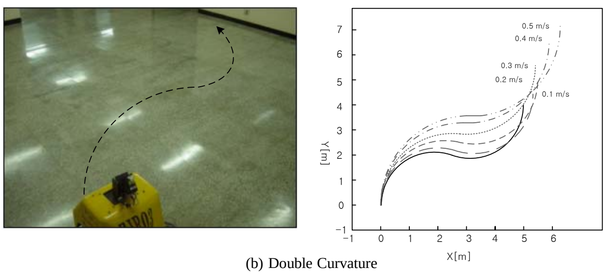
In the first experiment, the single-curvature and double-curvature trajectories are compared in terms of tracking error, traveling distance, and tracking time with the same mobile robot for a moving object. Figure 9 shows the real experimental environment. The experimental results are summarized in Table 4 and 5. Figure 10 shows tracking error, traveling distance, and tracking time as a graph according to the real experimental values.

Table 3. Computed velocity of the mobile robot for each trajectory

Lists		Single-curvature Trajectory (r=3.75m)		Double-curvature Trajectory (r=1.875*2m)	
v_1 (m/s)	Time(s)	v_L (m/s)	v_R (m/s)	v_L (m/s)	v_R (m/s)
0.1	72.8	0.105	0.095	0.089	0.111
0.2	37.9	0.211	0.189	0.179	0.221
0.3	26.9	0.316	0.284	0.268	0.332
0.4	21.9	0.421	0.379	0.357	0.443
0.5	19.3	0.527	0.473	0.447	0.553



(a) Single Curvature



(b) Double Curvature

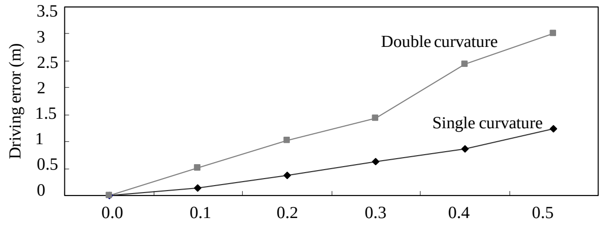
Fig. 9. Real experiments of the mobile robot Driving

Table 4. Rotation radius, driving error, time, and distance along the single-curvature trajectory

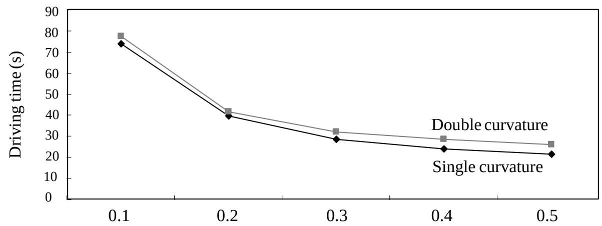
v_1 (m/s)	R_real (m)	Error (m)	Time (s)	Distance (m)
0.1	3.88	0.13	74.0	7.31
0.2	4.13	0.38	39.5	7.54
0.3	4.37	0.62	28.8	7.75
0.4	4.61	0.86	24.0	7.98
0.5	4.98	1.23	21.5	8.32

Table 5. Rotation radius, driving error, time, and distance along the double-curvature trajectory

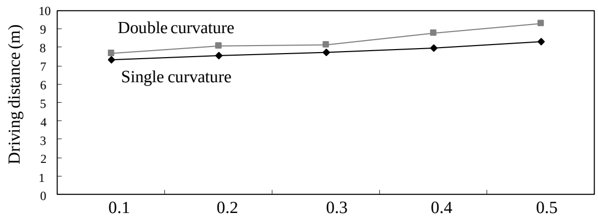
v_1 (m/s)	R1_real (m)	R2_real (m)	Error (m)	Time (s)	Distance (m)
0.1	2.02	2.06	0.51	77.2	7.67
0.2	2.25	2.25	1.02	41.6	8.08
0.3	2.47	2.38	1.44	32.1	8.12
0.4	2.85	2.89	2.43	28.8	8.80
0.5	3.12	3.15	3.01	26.2	9.33



(a) Comparison of driving- error



(b) Comparison of driving- time



(c) Comparison of driving- distance

Fig. 10. Comparison of single- and double-curvature driving characteristics

The second experiment is performed to show the tracking and capturing of a moving object by the mobile robot. As it is shown in Fig. 8, the mobile robot follows two curved paths and a straight path to capture a moving object. Figures 12 and 13 show the real trajectories followed by the mobile robot. In this experiment, the mobile robot's velocity is 0.1m/s, while the moving object has the velocity of 0.05m/s. To minimize the tracking error of the mobile robot, the maximum allowable acceleration was kept as $0.05m/s^2$. The initial position of the mobile robot was (1, 2, 90^o), and the moving object was initially located at (0, 2, 25.5^o) before the experiment. The first curved path has rotation radius of 1.5m. When the mobile robot followed this path, it had tracking error of 0.6m. Along the second straight line path, the mobile robot moved 2.8m, and by following the third curved path with rotation radius of 1m, the mobile robot hits the moving object to capture. The total traveling distance of the mobile robot was 6.91m, the the tracking error was 1.4m. The total time to capture the moving object was 71.2seconds.

Fig. 11. Capturing operation along the single-curvature trajectory

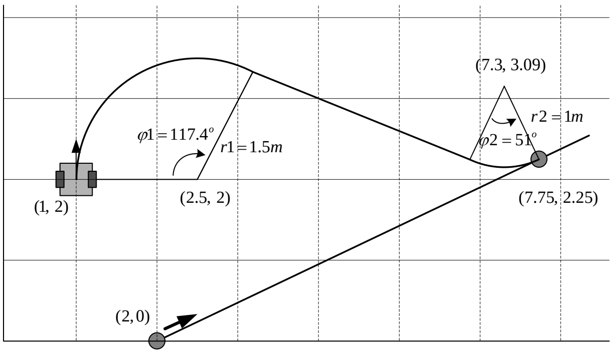


Fig. 12. Tracking and Capturing Trajectory

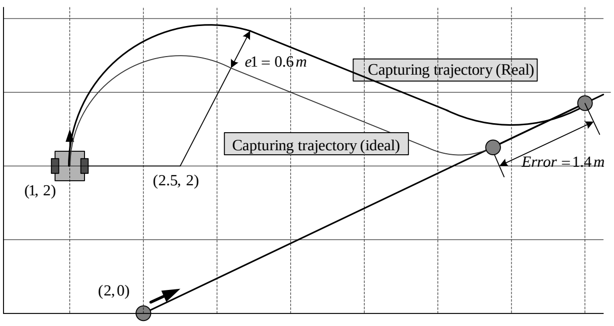


Fig. 13. Tracking error

6 Conclusions

This paper demonstrated optimality of a single curvature trajectory in capturing a moving object by a mobile robot. For the trajectory planning of a mobile robot in a curved path, superiority of the single curvature trajectory is verified based on the experiences and theoretical backgrounds. To practically show the effectiveness of the proposed algorithm, several experiments are performed with other trajectories, for an example, double-curved trajectory. Since the mobile robot using only dead-reckoning sensors aggregates the position estimation error while it is navigating, an error correction algorithm is necessary to support a precise and prompt control performance. Frequent error correction processes degrade the driving performance critically and may cause unstable operations. Providing a trajectory which causes less error for the mobile robot is very effective for the precise and fast curved motion. To improve the driving accuracy of the mobile robot, intelligent position estimation schemes and driving control algorithms are necessary to develop further as future research works.

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