Identification of nonlinear mechatronic servo motor system having backlash

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Abstract- Volterra series representation is a useful method to describe a nonlinear system of single valued function, though the method to identify Volterra kernels are generally difficult. The authors have recently developed a new method for identification of Volterra kernels of nonlinear systems by use of pseudorandom M-sequence and correlation technique. Using this technique and the experimental setup consisiting of AC servo motor and the backlash, this paper makes the feasibilty study that Volterra kernel representation can describe not only single valued but also multi valued nonlinearity. The experimental results show that we can still use Volterra kernel representation for those backlash-type two-valued nonlinear systems.

I. INTRODUCTION

Backlash type nonlinear element exists in a mechanical transmission systems. An interval of transmission is a kind of nonlinear backlash characteristics. When the main tooth wheel makes a change of direction, the following wheel does not change until the interval is used up.

The nonlinear systems with backlash element which has multi-valued function between the input and output are generally thought difficult to be identified. Many researchers have tried to identify this kind of system by various methods. Simpson and Power[3)] have shown that a method of identification developed for a class of system containing a zero-memory nonlinearity is applicable to certain types of nonlinear elements with memory.

The extension of the Weierstrass theorem to the Volterra functionals was made in 1910 by Maurice Fréchet who showed that any continuous functional can be represented by a series of functionals of integer order whose convergence is uniform in all compact sets of continuous functions. Therefore, Volterra kernel expansion method can only be used for those nonlinear systems which are continuous for input-output relation, and single valued, theoretically. The nonlinear systems having backlash type element are multi-valued as far as input-output relationship is concerned, so these nonlinear systems are not suited to Volterra kernel representation.

However, Volterra kernel expression of nonlinear system is one of the most useful method for representing nonlinear systems. So we would like to know what would be the result if we apply Volterra kernel identification method to nonlinear system having backlash element.

In this paper, we apply the method of Volterra kernel identification by use of M-sequences to the identification of a nonlinear system consisting of backlash characteristics. In this method, M-sequences are applied to the nonlinear system and the crosscorrelation function between the input and the output is measured. ¿From the crosscorrelation function ,we can get not only the linear impulse response of the linear part of the system, but also cross-sections of high-order Volterra kernel up to 3rd order of the nonlinear system.[6)]

II. Identification of Volterra Kernels

One of the solutions of the identification ploblem of a nonliear system is based on the measurement of Volterra kernels. Consider the identification of a nonlinear system which can be described as follows,

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \cdots \tau_i) \times u(t-\tau_1) u(t-\tau_2) \cdots u(t-\tau_i) d\tau_1 d\tau_2 \cdots d\tau_i + n(t) \qquad (1)$$

where $u(t)$ is the input, and $y(t)$ is the output of the nonlinear system, and $g_i(\tau_1, \tau_2, ...)$ is called Volterra kernel of i-th order. $n(t)$ is noise. When $i = 1$, Eqn.(1) shows a linear systerm.

In order to get the Volterra kernels $g_i(\tau_1, \tau_2, ...)$, we use an M-sequence as an input to the nonlinear system. The crosscorrelation function $\phi_{uy}(\tau)$ between the input $u(t)$ and the output $y(t)$ can be written as,

$$\phi_{uy}(\tau) = \overline{u(t-\tau) y(t)}$$

$$= \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \cdots \tau_i) \times \overline{u(t-\tau)u(t-\tau_1)\cdots u(t-\tau_i)} d\tau_1 d\tau_2 \cdots d\tau_i \quad (2)$$

where $\overline{\quad\quad}$ denotes time average. Generally the n-th moment of $u(t)$ is difficult to obtain. But when we use M-sequence, we can get n-th moment of $u(t)$ easily. Namely, the $(i+1)$th moment of the input M-sequence $u(t)$ can be written as

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} = \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)} \end{cases} \quad (3)$$

where N is the period of the M-sequence. When we use the M-sequence with the degree greater than 16, $1/N$ is in the order of 10^{-5}. So Eqn.(3) can be approximated as a set of impulses which appear at certain τ's.

Let us consider the case where we measure up to i-th Volterra kernel. Then for any integer $k_{i1}^{(j)}, k_{i2}^{(j)}, \cdots k_{i,i-1}^{(j)}$ (suppose $k_{i1}^{(j)} < k_{i2}^{(j)} < \cdots, k_{ii-1}^{(j)}$), there exists a unique $k_{ii}^{(j)} (mod\ N)$ such that

$$u(t)u(t+k_{i1}^{(j)})u(t+k_{i2}^{(j)})\cdots u(t+k_{i,i-1}^{(j)}) = u(t+k_{ii}^{(j)}) \quad (4)$$

where j is the number of a group $(k_{i1}, k_{i2}, \cdots, k_{i,i-1})$ for which Eqn.(4) holds. This property is called "shift and add property" of M-sequence. We assume that total number of those groups is m_i(that is, $j = 1, 2, \cdots, m_i$). Then Eqn.(3) becomes unity when

$$\tau_1 = \tau - k_{i1}^{(j)}, \tau_2 = \tau - k_{i2}^{(j)}, \cdots \tau_i = \tau - k_{ii}^{(j)} \quad (5)$$

Therefore Eqn.(2) becomes

$$\phi_{uy}(\tau) \simeq \sum_{i=1}^{\infty} \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)}) \quad (6)$$

Since $g_i(\tau_1, \tau_2, \cdots \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)})$ in Eqn.(6) appear in the crosscorrelation function $\phi_{uy}(\tau)$ when $\tau > k_{ii}^{(j)}$.

In order to obtain the Volterra kernels from Eqn.(6), $k_{ii}^{(j)}$'s in Eqn.(6) are sufficiently apart from each other. For this to be realized, we have to select suitable M-sequences, which make the appearance of each cross-section of Volterra kernels sufficiently apart from each other. Some of those usable M-sequences are shown in reference (7).

When we measure Volterra kernels up to 3rd order, the crosscorrelation function $\phi_{uy}(\tau)$ becomes,

$$\begin{aligned} \phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+ 2(\Delta t)^2 \sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}, \tau - k_{22}^{(j)}) \\ &+ 6(\Delta t)^3 \sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}, \tau - k_{32}^{(j)}, \tau - k_{33}^{(j)}) \end{aligned} \quad (7)$$

where

$$F(\tau) = (\Delta t)^3 g_3(\tau, \tau, \tau) + 3(\Delta t)^3 \sum_{q=1}^{m_1} g_3(\tau, q, q) \quad (8)$$

In general case, we have,

$$\begin{aligned} \phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+ \sum_{i=2}^{\infty} i!(\Delta t)^i \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)}) \end{aligned} \quad (9)$$

Here the function $F(\tau)$ is the function of τ and sum of the odd order Volterra kernels when some of its argument are equal. Since $F(\tau)$ appears together with $g_1(\tau)$ in a overlapped manner, $F(\tau)$ must be calculated from the odd order Volterra kernels and be subtracted from the measured $g_1(\tau)$ in order to obtain the accurate $g_1(\tau)$.

III. Simulation

We carried out Volterra kernel identification for the system having backlash type nonlinearity as is shown in **Figure 1**.

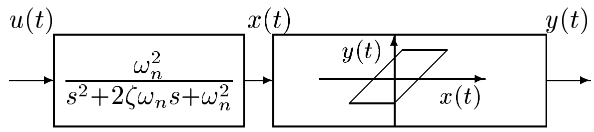

Figure 1: Nonlinear system consisting of backlash nonlinearity

As is well known, when the input to a backlash element is sinusoidal signal,

$$x(t) = X sin\omega t \quad (10)$$

the output of backlash element becomes as follows:

$$y(t) = \begin{cases} k_0(X sin\omega t - a) & 0 < \omega t < \frac{\pi}{2} \\ k_0(X - a) & \frac{\pi}{2} < \omega t < (x - \beta) \\ k_0(X sin\omega t + a) & (x - \beta) < \omega t < \pi \end{cases} \quad (11)$$

where $\beta = arcsin(1 - \frac{2a}{X})$, ρ is the slope of the inclined lines of the backlash characteristic curve and $k_0 = tg\rho$.

The parameters of the simulated system are as follows:

$\zeta = 0.4$
$\omega_n = 1.0$
Backlash width: $W = \mid 2a \mid = 0.8$
$\rho = \pi/4$
Sampling period: $\Delta t = 0.5s$

IV. Results of Simulation

As an input $u(t)$, 12 degree M-sequence with the characteristic polynomial ($f(x) = 15341$ in octal notation) is used. The first, second and third order Volterra kernels of the system are measured by calculating the crosscorrelation between the input and the output.

Figure 2 shows the first order Volterra kernel $g_1(\tau)$ thus measured. As is seen here, the response is not so smooth as the response of single-valued nonlinear system, because of the multi-valued characteristic of the backlash.

Figure 3 shows one of the crosssection of the third Volterra kernel $g_3(\tau_1, \tau_2, \tau_3)$, when $\tau_3 = 1\Delta t$.

Figure 4 shows the result of comparison of the actual output (solid line) and the output (x) calculated by use of first Volterra kernel and the output(+) calculated from up to third order Volterra kernels. From these results of simulation we can say that the use of Volterra kernel indentification method is effective even for backlash type nonlinear system.

V. Experiment for AC servo moter with backlash

An experiment is carried out for AC servo motor system having backlash element. The block diagram of the motor is shown in **Figure 1**. Driven by this motor, a mechanical backlash element is attached, and the rotary encoder of high resolution measures the output. Here the torque constant K_T and the moment of inertia J are given as follows.

$$K_T = 0.03606(Nm/A) \quad (12)$$

$$J = 0.00025(kgm^2) \quad (13)$$

The transfer function of the motor system is as follows,

$$G(s) = \frac{2\zeta\omega_n s + {\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \quad (14)$$

where

$$\omega_n = \sqrt{\frac{K_I K_T}{Jm}} \quad (15)$$

$$\zeta = \frac{K_P}{2}\sqrt{\frac{K_T}{J_m K_I}} \quad (16)$$

The parameters of the experiment are as follows;
Characteristic polynomial of the M-sequence: $f(x) = 40473$ (in octal notation),
Sampling period: $\Delta t = 8ms$,
Width of backlash: $W = 0.72 \quad or \quad 1.5(degrees)$

Figure 5 shows the schematic diagram of experimental setup. The PC commands the velocity command of M-sequence, and the pulse width modulated signal becomes the input voltage on the motor driver. The motor drives the rotary encoder connected through the backlash coupling at opposite side in order to measure the output. The cross-correlation function between the velocity command and output is calculated for extracting the volterra kernels. **Figure 6** shows the control block diagram. **Figure 7** shows the first Volterra kernel extracted from the crosscorrelation function. **Figure 8** shows the second Volterra kernel showing that the second Volterra kernel is very small in this case. **Figure 9** shows a crossection of the third Volterra kernel when $\tau_3 = 1$. **Figure 10** shows the comparison between the actual output of the AC servo motor system having backlash element and the output calculated from the measured Volterra kernels. The dotted line shows the actual output, (+)line shows the comparison when we use up to the third order Volterra kernles. It is seen that when we use up to the third order Volterra kernel, the actual output can be very closely estimated from the measured Volterra kernels.

VI. Conclusion

A method for identification of Voterra kernels using M-sequence was applied to identify a kind of multi valued function, which appeares frequently at the mechatronics field as the backlash. M-sequences are applied to the two-valued nonlinear system and the crosscorrelation function between the input and the output are measured. From the crosscorrelation function, we obtain not only the linear impulse response, but also cross-sections of 3rd order Volterra kernels of backlash element nonlinear system.

After verifying the effect by simple simulation, this method was applied to the actual AC servo motor system having backlash element. The calculated result using up to third order Volterra kernels showed good agreement with the actual output which was driven by M-sequence velocity command through the backlash coupling. ¿From these results , we can say that the method of Volterra kernels is practically useful for indentification of backlash type nonlinear system.

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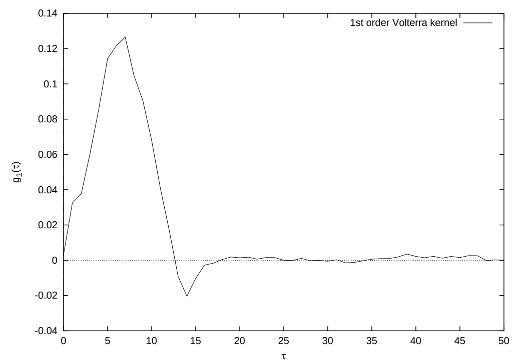

Figure 2: Obtained first Volterra kernel $g_1(\tau_1)$ in the simulation.

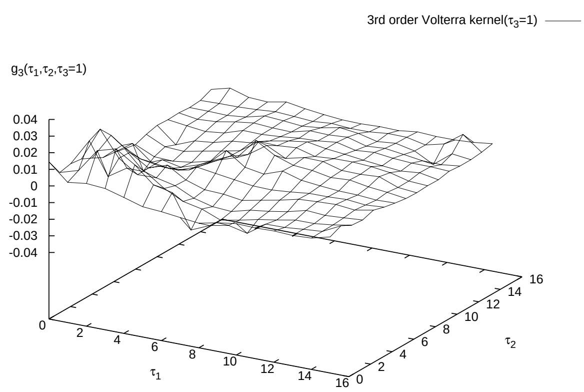

Figure 3: Obtained third Volterra kernel $g_3(\tau_1, \tau_2, \tau_3 = 1)$ in the simulation.

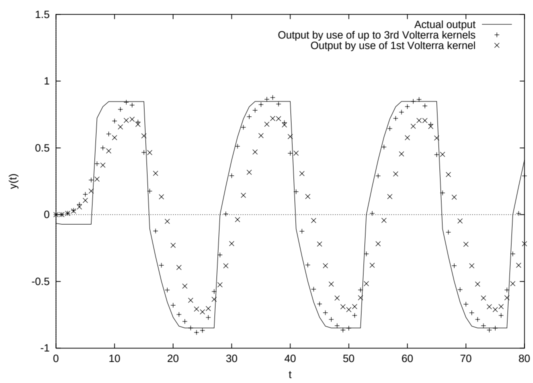

Figure 4: Comparison between actual output with calculated output from Volterra kernels in the simulation.

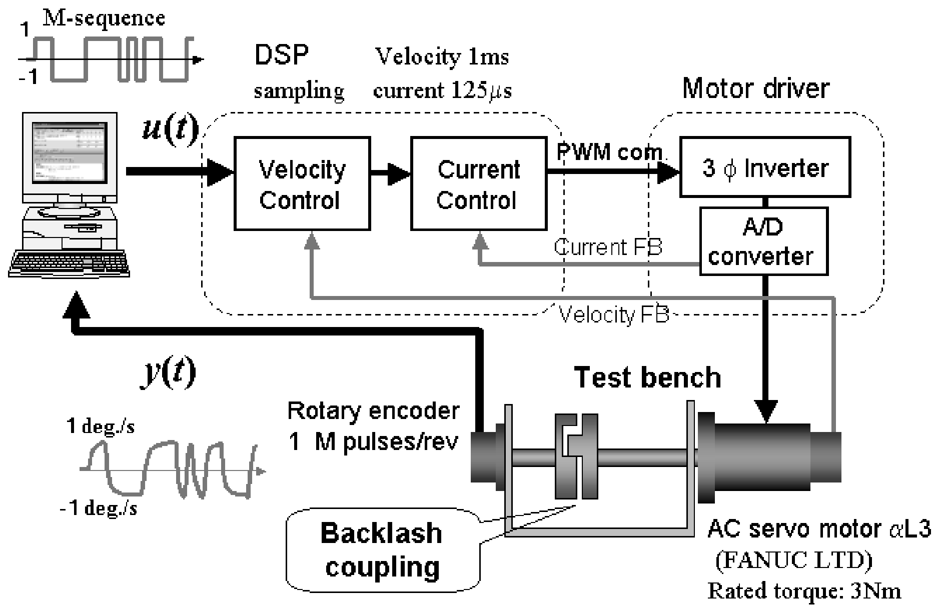

Figure 5: Experiment for AC servo motor with backlash.

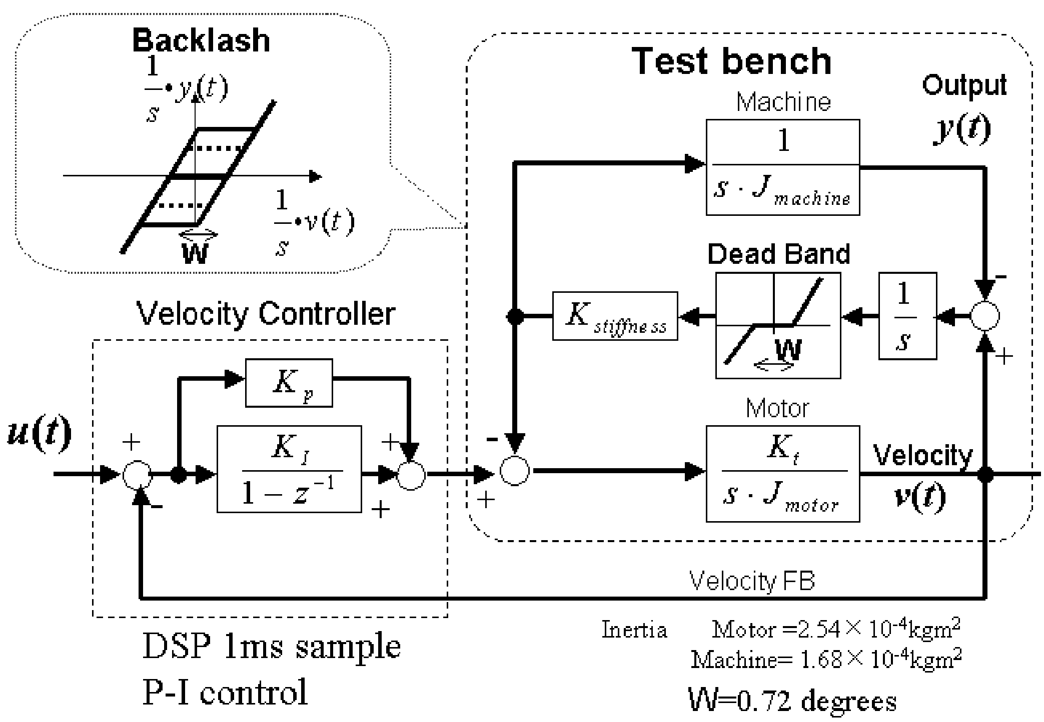

Figure 6: Control system block.

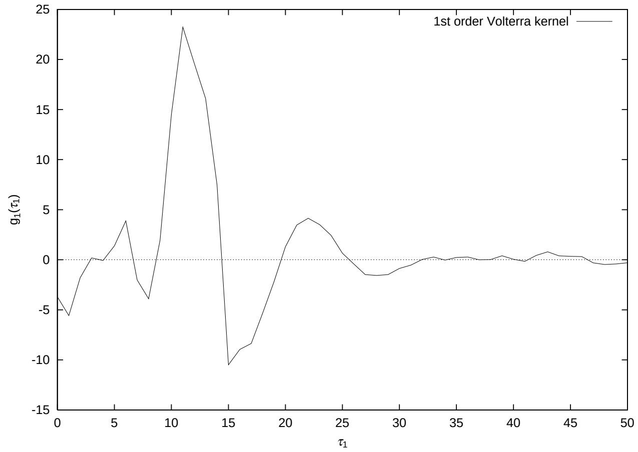

Figure 7: First Volterra kernel $g_1(\tau_1)$ in case of AC servo motor system.

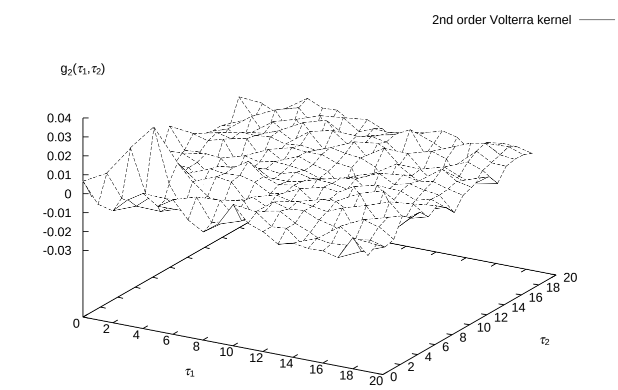

Figure 8: Second Volterra kernel $g_2(\tau_1,\tau_2)$ in case of AC servo motor system.

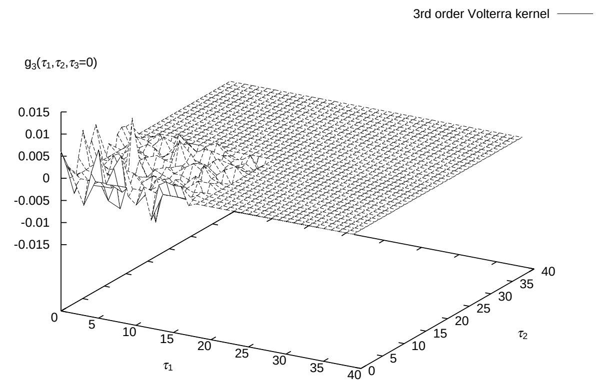

Figure 9: Third Volterra kernel $g_3(\tau_1,\tau_2,\tau_3),\tau_3=1)$ in case of AC servo motor system.

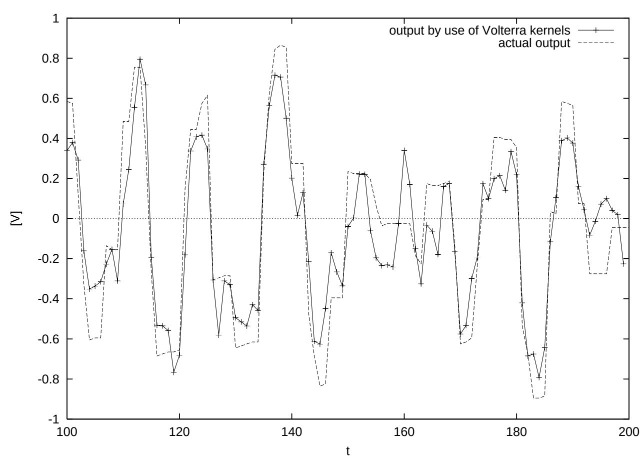

Figure 10: Comparison between the actual output with the output calculated from Volterra kernels in case of AC servo motor system.