Modeling of Pneumatic Artificial Muscle Actuator

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Abstract: This paper mainly developed the static model of pneumatic artificial muscles in theoretically. From its physical model, the geometrical model was turned up, and the mathematical model was established. It discussed the contraction ratio, formulated the force output as a function of gas pressure and the structure parameters, and analyzed the stiffness.

Key word: Pneumatic artificial muscles, static model, contraction ratio, force output, stiffness

1. Introduction

In recent years, a new kind of artificial muscles-pneumatic artificial muscle (PAM) has been developed [1][2]. It consists of a rubber tube covered in tough braided fibre mesh, which shortens in length when inflated with compressed gas. Sometimes it is also called as air muscles, because usually the used gas is air. It can generate enough force and maintain proper compliance at the same time-a little like real muscles [3][4]. So, it can be used as the actuator in robots, so that the actuator will inherently behave a little like animals. Sometimes it is just called as pneumatic artificial muscle actuator. Fig. 1 is a photo of a pneumatic artificial muscle actuator made by Robot Shadow Company [4]. Fig. 2 is its enlarged part view.

Based on the possible applications in robots, it is necessary to establish its models. This paper is going to set up its static mathematics model.

Fig. 1 Pneumatic artificial muscle

Fig. 2 Physical structure

2. Geometric model

A piece of pneumatic artificial muscle can be geometrically modeled as a cylinder. Of course the cylinder can change its volume when gas pressure is applied. The non-cylindrical end effects are ignored, and the thickness of the braided mesh is assumed to be zero. The dimensions of this cylinder are the length L, diameter D, and the interweave angle θ. Neither of these dimensions remains constant, because all the three parameters will change when it contracts or extends. Assuming inextensibility of the mesh material, the geometric constants of the system are the mesh thread length b, and n-the number of turns for a single thread. The interweave angle θ is the angle between the thread and the long axis of the cylinder (look at Fig. 3). θ changes as the length of the actuator changes. The relationships between these parameters are shown in Fig. 4.

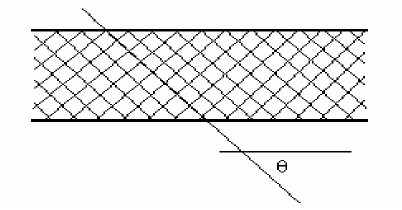

Fig. 3 Geometrical model

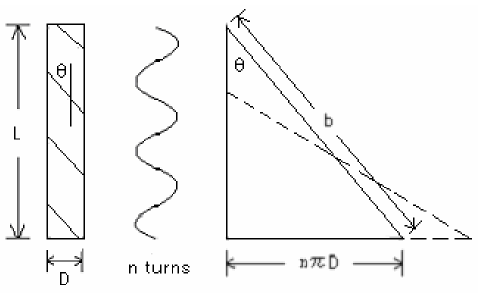

Fig. 4 Structure model

Now let's formulate the relationship between the length L and the angle θ.

$$L = b\cos\theta \qquad (1)$$

$$n\pi D = b\sin\theta \qquad (2)$$

The cylinder volume $V=\pi D^2 L/4$, combining (1) and (2),

$$V = b^3\sin^2\theta\cos\theta/(4\pi n^2) \qquad (3)$$

The minimum length (the maximum contracted length) should arise when the actuator's volume gets to its greatest, because this results in the equilibrium of the system. So, at the equilibrium point, there should be:

$$dV/d\theta = 0$$

Based on (3),

$$dV/d\theta = (2\sin\theta\cos^2\theta - \sin^3\theta)b^3/(4\pi n^2)$$
$$= 0$$

Then we get

$$\theta_{max} \approx 54.7°$$

We can say that the cylinder gets to the maximum of its volume when the interweave angle changes to 54.7°, or we say that the actuator gets to the greatest contraction by applied gas.

By now the geometrical model has been established.

Besides, the contraction ratio R_c is one of the most important characteristics. Now we can calculate it:

$$R_c = (L_0 - L)/L_0$$
$$= (L_0 - b\cos\theta_{max})/L_0 \qquad (4)$$

Here, L_o is the relaxed length, or called it free length.

To two pieces of the same kind of pneumatic artificial muscles in different lengths, their interweave angle should be same. Therefore,

$$L1_0 = b1\cos\theta$$

$$L2_0 = b2\cos\theta$$

$$R_{c1} = (L1_0 - b1\cos\theta_{max})/L1_0$$
$$= 1 - L1_0/b1\cos\theta_{max}$$
$$= 1 - \cos\theta_{max}\cos\theta_0$$

$$R_{c2} = (L2_0 - b2\cos\theta_{max})/L2_0$$
$$= 1 - L2_0/b2\cos\theta_{max}$$
$$= 1 - \cos\theta_{max}\cos\theta_0$$

Where, θ_0 is the interweave angle corresponding to the relaxed length. So,

$$R_{c1} = R_{c2} = \text{constant}$$

This means that their contraction ratios are same to two actuators with the same structure in different lengths.

3. Static model

Because it is a kind of motion-generating device, we should try to find the force expression as a function of the related factors. Probably the force F has relationship with gas pressure, because the actuator acts when gas pressure is applied. Besides the relationships between the force and the length, interweave angle and so on will be developed too.

Here we use a simple energy analysis. It is assumed that the actuator is a conservative system in which the work in (W_i) is equal to the work out (W_o). The losses will be accounted for later. Work is input to the actuator when gas pressure is applied to the inner bladder surface.

$$dW_i = \int_{S_i}(P_{abs} - P_{atm})dl_i * ds_i$$
$$= (P_{abs} - P_{atm})\int_{S_i} dl_i * ds_i$$
$$= P_r dV \qquad (5)$$

Where,

P_{abs} = Absolute internal gas pressure

P_{atm} = Atmosphere pressure (a little more than 1 bar)

P_r = relative gas pressure

s_i = Total inner surface

ds_i = Area vector

dl_i = Inner surface displacement

dV = Volume change

The output work arises when the actuator shortens due to the change in volume.

$$dW_o = -FdL \quad (6)$$

Now the ideal system assumption can be applied. The work input to the system should be equal to the work done by the actuator.

$$dW_i = dW_o$$

Combining (5) and (6),

$$P_r dV = -FdL$$

So,

$$F = -P_r dV/dL$$
$$= -P_r (dV/d\theta)/(dL/d\theta)$$
$$= P_r b^2(2\cos^2\theta - \sin^2\theta)/(4\pi n^2)$$
$$= P_r b^2(3\cos^2\theta - 1)/(4\pi n^2) \quad (7)$$

When θ=54.7°, F= $P_r b^2(3\cos^2\theta-1)/(4\pi n^2)$ =0. So please note that at the maximum interweave angle 54.7° the force output of the actuator is zero. In another word, the force output will be zero at the greatest contraction. Besides, under the condition of P_r is constant, when θ is equal to its minimum cosθ gets to its maximum, so F will get to its maximum. We denote the maximum of F as F_{max}, therefore,

$$F_{max} = P_r b^2(3\cos^2\theta_0-1)/(4\pi n^2),$$

θ_0 is the interweave angle when the actuator is relaxed.

Besides, in (7), based on Fig. 4, we can imagine the diameter D will get to its maximum D_{max} when θ=90°. Then

$$b = n\pi D_{max}$$

Take this into (7), it will become:

$$F = \pi D_{max}^2 P_r(3\cos^2\theta-1)/4 \quad (7')$$

The geometric variables used above provide a straightforward calculation, but to use the resulting equations in practice, they need to be modified, because the parameters b and θ are not easy or convenient to be measured. Now let us discuss this. If the cylindrical mesh is opened and laid flat, the weave geometry is easily observed (Fig. 5). The shape of the weave quadrilateral is governed by the interweave angle θ and the quadrilateral side length l.

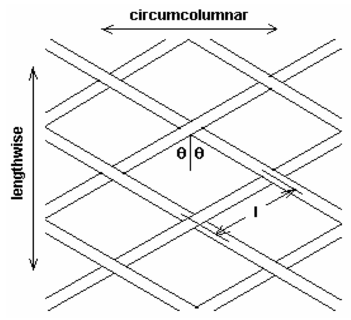

Fig. 5 The weave mesh structure

The cylinder length L and circumference C of the actuator is easy to be formulated by the following formulas:

$$L = 2Al\cos\theta \quad (8)$$
$$C = 2Bl\sin\theta \quad (9)$$

Where:

A = number of lengthwise quadrilaterals

B = number of circumferential quadrilaterals

Since the circumference can also be expressed as πD, so there is:

$$\pi D = 2Bl\sin\theta$$
$$D = 2Bl\sin\theta/\pi \quad (10)$$

Recall equations (1) and (2),

$$L = b\cos\theta$$
$$D = b\sin\theta/n\pi$$

Comparing (1) and (8), (2) and (10) results in the following:

$$b = 2Al \quad (11)$$
$$n = A/B \quad (12)$$

Therefore, to practically characterize an actuator only the quadrilateral size and count are necessary. Now let us try to remove θ from the equations. It is difficult to sense the interweave angle during operation of the actuator. But it is much easier to measure the length. If the equations can be rewritten in terms of force, pressure, and length, then they will be more useful, because these variables can be measured most easily. Recall the triangle from Fig. 4, and note that the length of side opposite to angle θ has been rewritten in terms of b and L.

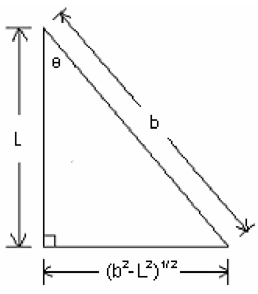

Fig. 6 The triangle relationship

The relationships between θ, L, and b can be expressed as:

$$\cos\theta = L/b \qquad (13)$$

$$\sin\theta = (b^2-L^2)^{1/2}/b \qquad (14)$$

Substituting these relationships into the volume (3) and force (7) equations,

$$V = L(b^2-L^2)/(4\pi n^2) \qquad (15)$$

$$F = P_r b^2(3L^2/b^2-1)/(4\pi n^2) \qquad (16)$$

It is easy to imagine that pneumatic artificial muscles act somewhat like springs. We know that the spring has an important parameter-the constant elasticity coefficient. Does the actuator exhibit the same property – the constant stiffness as the spring? According to the concept of stiffness, it is simply a derivative of force with respect to length.

$$K = dF/dL$$

Differentiating (16) with respect to L,

$$K = P_r b^2(3L^2/b^2-1)/(4\pi n^2) dP_r/dL + 3P_r L/(2\pi n^2) \qquad (17)$$

The first term (dP_r/dL) is the most difficult to formulate. When the valves are closed, the pressure changes proportionally with volume according to gas laws. As a result, the relationship dV/dL would need to be developed. However, when the gas valves are opened, this relationship (dP_r/dL) is even more difficult to model. But we think that the pressure change as a function of length is small, and can be neglected. Besides, there is some space in the inlet hose, and it can help to cause the pressure change to remain minimal through the actuators' range of motion. In this case we can assume:

$$dP_r/dL \approx 0 \qquad (18)$$

So, actuator stiffness is now given by

$$K = 3P_r L/(2\pi n^2) \qquad (19)$$

Or solving (16) for P_r and substituting the result into (19),

$$K = 6F/(3L-b^2/L) \qquad (20)$$

4. Conclusion

From the above theoretical analysis, we can conclude:

- Pneumatic artificial muscle is a linear motion engine, because the force output is proportional to the stimulating gas pressure (see (7));
- To a constant gas pressure, force output is related with the length. And it will become zero when it gets to the shortest length (see (7));
- The stiffness is not constant. It changes with both the gas pressure and the length;
- To determine actuator state, only two of the parameters are needed: gas pressure, length, force, or stiffness. The other two can be calculated from the above formulas.

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