Artificial Life-Based Search Technique on the Solution of Singular Configurations Concerning Screw Parameters in Helicoidal Robots

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ABSTRACT

The screw-based robot manipulators belonging to the *SBMF6* and *SBMF7* present closed-form equations describing joint variables as functions of end-effector coordinates; the mentioned closed-form equations depends on the so called screw parameters that, in some conditions, can not be determined because present singularities. It means that the inverse kinematics solved by this technique is not robust enough. The paper presents an auxiliary technique that will be helpful in finding the corresponding inverse kinematics. The mentioned technique is an artificial life-based search method that will be used only for those cases in which singularities are present and closed-form equations fail.

Key words: Artificial Life, Artificial Intelligence, Genetic Algorithms, Robotics, Inverse kinematics

1. INTRODUCTION

The *SBMF6* and *SBMF7* are sets containing manipulators having a particular mechanical architecture capable of transporting 3-dimensional work pieces by means of the helical or screw-based motions. This singular way of transferring work pieces from one pose (position and orientation) to another one is based on the well-known Chasles' Theorem and the Rodrigues' Formula [1], [2], [3], [4], [5], [6]. The *SBMF6* possesses the PPSP, RRSP, SPRP and RPSP arrangements, while the *SBMF7* contains the PPSPP, RRSPP, SPRPP and RPSPP manipulators; all these manipulators are named as *Helicoidal Manipulators*. The elements contained by the set *SBMF6* have 6 degrees of freedom, while elements belonging to the *SBMF7* posses 7 degrees of freedom. In all cases, *Helicoidal Manipulators* were designed according to the screw transformation that says that a piece described by three non-collinear points P, Q and R is brought from an initial to a final one by means of a screw displacement [1], [2], [3], [4], [5], [6]. In order to transfer a work piece from the initial pose to the final one, it is necessary to find the corresponding joint displacements; it is the duty of the inverse kinematics which is expressed by a set of closed-form equations depending on the screw parameters found by Rodrigues in 1840 [6]. These parameters can not be found for some initial and final positions and orientations of the work piece. This is a singular configuration dealing with the screw motion. In this case, it is necessary the aid of and auxiliary method to find the corresponding screw parameters. In this case, a search technique, based on the way species find the solution to the survival problem, was selected.

2. GOAL

To present an auxiliary technique employed to find the screw parameters when traditional or closed-form solutions fail. The mentioned screw parameters are necessary because the corresponding inverse kinematics is expressed in terms of them. This adjacent technique is an artificial life-based search method (ALBSM) because it uses the mechanisms employed by living species to be successful [5].

3. GENERAL SCREW DISPLACEMENT

Suppose that two poses of an object at time $t=0$ and at time $t=t_f$ are given and it is necessary to find the screw motion that interpolate them, then the description of computing pose interpolating screw parameters can be done as follows: it is a known fact that three non-collinear points $\mathbf{P_1}$, $\mathbf{Q_1}$ and $\mathbf{R_1}$, belonging to a work piece, are brought to positions $\mathbf{P_2}$, $\mathbf{Q_2}$ and $\mathbf{R_2}$ by a screw displacement (ϕ, d) about an axis with direction $\hat{e}$ passing through a point defined by $\bar{s}_0$ [1], [2], [3], [4], [5], [6]. They are called "screw parameters". (1), (2), and (3) present the mentioned functions and are known as the Rodrigues' formulae [6].

$$\bar{p}_2 - \bar{p}_1 = (\tan(\phi/2))\hat{e} \otimes (\bar{p}_2 + \bar{p}_1 - 2\bar{s}_0) + d\hat{e} \quad (1)$$

$$\bar{q}_2 - \bar{q}_1 = (\tan(\phi/2))\hat{e} \otimes (\bar{q}_2 + \bar{q}_1 - 2\bar{s}_0) + d\hat{e} \quad (2)$$

$$\bar{r}_2 - \bar{r}_1 = (\tan(\phi/2))\hat{e} \otimes (\bar{r}_2 + \bar{r}_1 - 2\bar{s}_0) + d\hat{e} \quad (3)$$

Where $\bar{p}_i, \bar{q}_i, \bar{r}_i$ are the position vectors describing $\mathbf{P_i}$, $\mathbf{Q_i}$ and $\mathbf{R_i}$ when i=1, 2. Symbol $\otimes$ represents the cross product.

It is possible to find the corresponding four screw parameters in terms of the three mentioned points by using (4), (5) and (6).

$$\tan\left(\frac{\phi}{2}\right)\hat{e} =$$

$$= \frac{\{[(\bar{r}_2 - \bar{q}_2) - (\bar{r}_1 - \bar{q}_1)] \otimes [(\bar{p}_2 - \bar{q}_2) - (\bar{p}_1 - \bar{q}_1)]\}}{\{[(\bar{r}_2 - \bar{q}_2) - (\bar{r}_1 - \bar{q}_1)]^T [(\bar{p}_2 - \bar{q}_2) + (\bar{p}_1 - \bar{q}_1)]\}} \tag{4}$$

$$\bar{s}_0 = \tfrac{1}{2}\begin{bmatrix} \bar{p}_1 + \bar{p}_2 + (\hat{e} \otimes (\bar{p}_2 - \bar{p}_1)/\tan(\phi/2)) - \\ -(\hat{e}^T(\bar{p}_2 + \bar{p}_1))\hat{e} \end{bmatrix} \tag{5}$$

Where $\hat{e}^T \bar{s}_0 = 0$

$$d = \hat{e}^T(\bar{p}_2 - \bar{p}_1) \tag{6}$$

4. ANALYSIS OF MATHEMATICAL SINGULARITIES

Consider (7) which is the denominator of (4). It is not possible to find the screw axis and the rotation angle when (7) becomes null. As a result, the other two screw parameters, (5) and (6), can not be found because they depend on the screw axis and the rotation angle.

$$den = [(\bar{r}_2 - \bar{q}_2) - (\bar{r}_1 - \bar{q}_1)]^T [(\bar{p}_2 - \bar{q}_2) + (\bar{p}_1 - \bar{q}_1)] \tag{7}$$

The mentioned denominator becomes zero when position vectors $[(\bar{r}_2 - \bar{q}_2) - (\bar{r}_1 - \bar{q}_1)]$ and $[(\bar{p}_2 - \bar{q}_2) + (\bar{p}_1 - \bar{q}_1)]$ rest perpendicular. In some cases it is desirable for the pick and place operation to have particular initial and final position and orientations, but they could make (4) singular, so then, inverse kinematic equations, depending on screw parameters, can not be found. This is the main reason to use a method whose robustness ignores limitations dealing with this singularity [5].

5. THE ARTIFICIAL LIFE-BASED SEARCH METHOD (ALBSM)

5.1 GENOTYPE

The member *n*, belonging to the species of sets of screw parameters and representing a particular solution, must contain the most basic information, responsible for the determination and transmission of hereditary characteristics corresponding to the four screw parameters (8).

$$solution(n) = \langle\langle \hat{e}_g \rangle\langle \phi_g \rangle\langle \bar{s}_0 \rangle\langle d_{\hat{e}} \rangle\rangle \tag{8}$$

Where $\langle \hat{e}_g \rangle$ represents the screw axis which contains three elements due to the fact that it is a Cartesian vector (9).

$$\langle \hat{e}_g \rangle = \langle\langle a_1 \rangle\langle a_2 \rangle\langle a_3 \rangle\rangle \tag{9}$$

So then, (10) must be used to obtain the screw axis.

$$\hat{e}_g = \frac{1}{\sqrt{\sum_{i=1}^{3} a_i^2}} \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix}^T \tag{10}$$

The second part of (8), $\langle \phi_g \rangle$, representing the rotation angle, is constituted by one genetic structure, (11).

$$\langle \phi_g \rangle = \langle b_1 \rangle \tag{11}$$

The third part of (8), $\langle \bar{s}_0 \rangle$, contains four genetic structures, (12).

$$\langle \bar{s}_0 \rangle = \langle\langle c_1 \rangle\langle c_2 \rangle\langle c_3 \rangle\langle c_4 \rangle\rangle \tag{12}$$

These four elements help to find the position vector describing the point where the screw axis passes, (13).

$$\bar{s}_0 = \left(\frac{1}{\sqrt{\sum_{i=1}^{3} c_i^2}} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \right) c_4 \tag{13}$$

The last part of (8), $\langle d_{\hat{e}} \rangle$, is a scalar parameter related to the linear displacement along the screw axis, (14).

$$d_{\hat{e}} = d_1 \tag{14}$$

So then, equation (8) can be expressed by (15).

$$solution(n) = \langle\langle a_1 \rangle\langle a_2 \rangle\langle a_3 \rangle\langle b_1 \rangle\langle c_1 \rangle\langle c_2 \rangle\langle c_3 \rangle\langle c_4 \rangle\langle d_1 \rangle\rangle \tag{15}$$

5.2 LENGTH OF BASIC GENETIC STRINGS

The length of the genetic vectors or strings depends on the length of the domain of the search space and the required precision [5]. The chromosomes corresponding to $\langle a_i \rangle$; $i = 1,2,3$, $\langle b_1 \rangle$, $\langle c_i \rangle$; $i = 1,2,3$, $\langle c_4 \rangle$ y $\langle d_1 \rangle$ by means of binary strings are given by (16)-(20), taking in account that $b_{*,*} = 0, 1$.

$$\langle a_i \rangle = \langle b_{K1-1,ai} \ldots b_{2,ai} b_{1,ai} b_{0,ai} \rangle \tag{16}$$

$$\langle b_1 \rangle = \langle b_{K2-1,b1} \ldots b_{2,b1} b_{1,b1} b_{0,b1} \rangle \tag{17}$$

$$\langle c_i \rangle = \langle b_{K3-1,ci} \ldots b_{2,ci} b_{1,ci} b_{0,ci} \rangle \tag{18}$$

$$\langle c_4 \rangle = \langle b_{K4-1,c4} \ldots b_{2,c4} b_{1,c4} b_{0,c4} \rangle \tag{19}$$

$$\langle d_1 \rangle = \langle b_{K5-1,d1} \ldots b_{2,d1} b_{1,d1} b_{0,d1} \rangle \tag{20}$$

The problem consists in finding K1, K2, K3, K4 and K5 in (16)-(20) satisfying the search space and the precision requirements [5].

5.3 PHENOTYPES

The mapping from binary strings into real numbers can be done by (21)-(25).

$$a_j = -1 + \left(\sum_{i=0}^{K1-1} b_{i,aj} 2^i\right)\left(\frac{2}{2^{K1}-1}\right); \quad j = 1,2,3 \tag{21}$$

$$b_1 = \left(\sum_{i=0}^{K2-1} b_{i,b1} 2^i\right)\left(\frac{2\pi}{2^{K2}-1}\right) \tag{22}$$

$$c_j = -1 + \left(\sum_{i=0}^{K3-1} b_{i,cj} 2^i\right)\left(\frac{2}{2^{K3}-1}\right); \quad j = 1,2,3 \tag{23}$$

$$c_4 = \left(\sum_{i=0}^{K4-1} b_{i,c4} 2^i\right)\left(\frac{c_{MAX}}{2^{K4}-1}\right) \tag{24}$$

$$d_1 = \left(\sum_{i=0}^{K5-1} b_{i,d1} 2^i\right)\left(\frac{d_{MAX}}{2^{K5}-1}\right) \tag{25}$$

Where c_{MAX} and d_{MAX} are the upper limits of the search spaces.

5.4 EVALUATION FUNCTION

The evaluation function takes in account the final pose acquired by the work piece. If the object is close to the required final pose, then the corresponding solution is evaluated with a high score, otherwise, it will have a poor effectiveness. Each solution, generated by the ALBSM, provides a final pose described by $\mathbf{P_2}(\boldsymbol{n})$, $\mathbf{Q_2}(\boldsymbol{n})$, $\mathbf{R_2}(\boldsymbol{n})$. The comparison between the required final pose ($\mathbf{P_2}$, $\mathbf{Q_2}$, $\mathbf{R_2}$) and ($\mathbf{P_2}(\boldsymbol{n})$, $\mathbf{Q_2}(\boldsymbol{n})$, $\mathbf{R_2}(\boldsymbol{n})$) will permit evaluate solution *n*. It is important to take in account that whatever points ***S*** and ***S(n)*** are represented by sets of three coordinates:

$$S([w_1, s], [w_2, s], [w_3, s]) \tag{26}$$

And,

$$S(n)([w_1, s](n), [w_2, s](n), [w_3, s](n)) \tag{27}$$

With this in mind, it is possible to define the scalar $g_{S,wi}$ as follows,

- If $\|[w_i, s](n)\| \geq \|[w_i, s]\|$ then $g_{S,wi} = \dfrac{[w_i, s]}{[w_i, s](n)}$.
- If $\|[w_i, s](n)\| < \|[w_i, s]\|$ then $g_{S,wi} = \dfrac{[w_i, s](n)}{[w_i, s]}$.

Then, the proximity of the work piece corresponding to solution *n* is expressed by $sum(n) \in [-9, 9]$, (28).

$$sum(n) = \sum_{S=P,Q,R} \left(\sum_{i=1,2,3} g_{S,wi} \right) \tag{28}$$

The final evaluation function, $eval(n) \in [0, 1]$, is defined by (29).

$$eval(n) = \frac{9 + sum(n)}{18} \tag{29}$$

Obviously, the exact solution corresponds to $eval(n) = 1$, belonging to the member *n*, Fig 1.

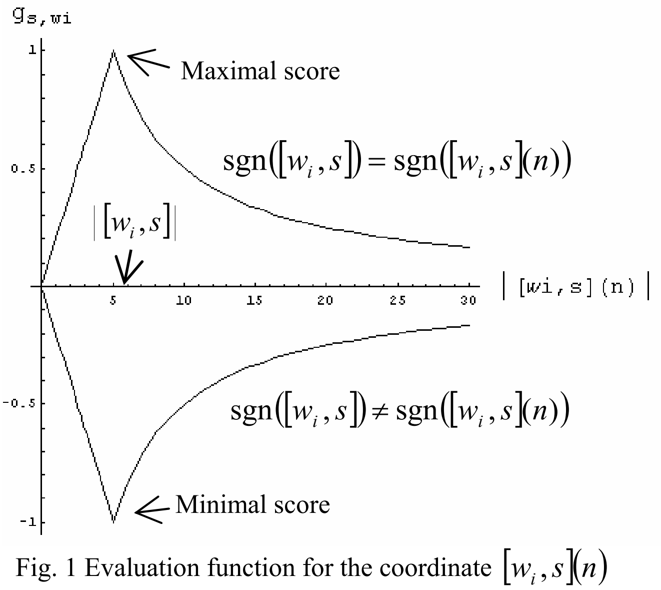

Fig. 1 Evaluation function for the coordinate $[w_i, s](n)$ belonging to the member *n*

5.5 THE ALBSM FLOWCHART

Fig. 2 presents the flowchart representing the different stages followed by the ALBSM.

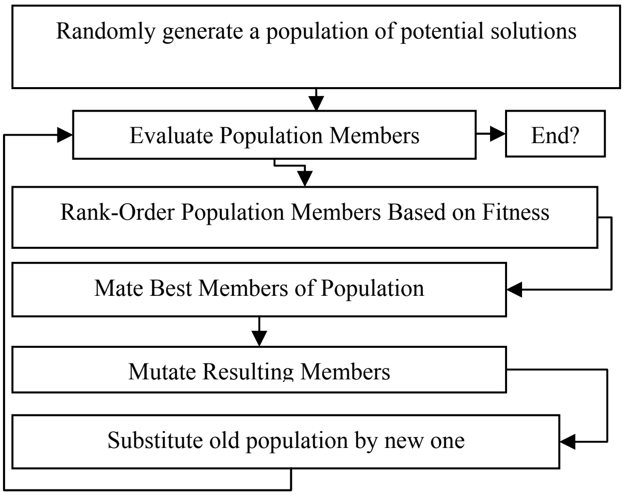

Fig. 2 Flowchart representing the different stages of the ALBSM

6. EXPERIMENTATION

It is necessary to transfer a work piece from the initial and final positions described in table 1.

Object	coordinates
P_1	(2, 2, 0)
Q_1	(3, 2, 0)
R_1	(2, 3, 0)
P_2	(2, 2, 0)
Q_2	(1, 2, 0)
R_2	(2, 1, 0)

Table 1. Initial and final poses

For the mentioned initial and final poses presented in table 1, (4) becomes singular. The screw parameters, shown in table 2, were found with the aid of the ALBSM.

Screw Parameters	
$\hat{e}$	$(0 \quad 0 \quad 0.995)^T$
ϕ	179.898º
$\bar{s}_0$	$(1.997 \quad 2.001 \quad 0.003)^T$
d	0

Table 2. Screw parameters resulting from the ALBSM

The exact values of the screw parameters are shown in table 3. The comparison of both tables, It is concluded that the ALBSM provides a suitable result.

Screw Parameters	
$\hat{e}$	$(0 \quad 0 \quad 1)^T$
ϕ	180º
$\bar{s}_0$	$(2 \quad 2 \quad 0)^T$
d	0

Table 3. The exact screw parameters

7. CONCLUSIONS

The ALBSM has demonstrated its effectiveness in finding the solution of singular configurations. However, this method should be only used for those cases in which it is not possible to find the screw parameters by other means. It is necessary to take in account that the ALBSM is an iterative one; therefore, it spends more time to find the solution in comparison to the closed-form equations. The main advantage resides in its robustness because this method does not use the equations that could become singular. All the manipulators, belonging to the *SBMF6* and *SBMF7*, have their own inverse kinematics, but these equations depends on the screw parameters, so then, the ALBSM can be applied to all the screw-based manipulators.

8. REFERENCES

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