# Improving Odometry Accuracy for a Car Using Tire Radii Measurements 

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#### Abstract

Odometry is the most widely used method for determining the momentary position of a mobile robot. In most practical applications, odometry provides easily accessible real time positioning information in-between periodic absolute position measurements. Odometry errors are caused by two dominant error sources in vehicles: Systematic errors and nonsystematic errors. Systematic errors stay almost constant over prolonged periods of time and can be calibrated. In most case of mobile robot, unequal wheel diameters are systematic errors. But, they are not systematic errors in case of flexible tires like passenger car case. Radii of flexible tires are always varied by road conditions, rolling of vehicle and etc. So, it is important to measure the variations of tire radii for accurate positioning in odometry navigation of car-like vehicles. The method for measurement of tire radii is described and experimental results are presented in this paper.


## 1 Introduction

In most positioning system, relative and absolute positioning methods are employed together. Absolute positioning methods usually rely on satellite-based navigation signals (GPS), landmarks or beacons, and map matching. GPS can be used only outdoors and it has poor resolution in a local range (Its errors are about $10 \mathrm{~m}[2]$ ). With a radio station as a compensative reference, differential GPS (DGPS) method has been developed to reduce the errors. GPS suffers from satellite mask occurring in urban environments, under bridges, tunnels or in forests. Moreover, radio frequency-based systems are very expensive. Landmarks or beacons usually require costly installations and maintenance. Map matching methods provide the position and pose of the vehicle. If there were several areas with similar feature, the method would obtain mistake result. In general, absolute positioning methods have the errors that do not accumulate with the movement of the vehicle. Dead reckoning is the representative of relative positioning methods. It has the advantage of cheapness, simplicity, good performance in short term and working in real-time. But its positioning
errors accumulated with the traveled distance, and grow without bound. Many cases of relative positioning methods use inertial navigation with accelerometers and gyros. Accelerometer data must be integrated twice to yield position thereby making these sensors exceedingly sensitive to drift. Gyros provide information only on the rate of rotation of vehicle so their data must be integrated once to provide the heading. Besides the deterministic errors contained in accelerometers and gyros measurements, they have also stochastic errors which call for the use of estimation and optimal filtering to correct them. It is common to combine relative positioning with other absolute positioning methods[2][3][4].
Odometry errors are caused by two error sources in vehicles: Systematic errors and nonsystematic errors[1]. Systematic errors (uncertainty of wheelbase, unequal wheel diameters, etc.) stay almost constant over prolonged periods of time and can be calibrated. In most case of mobile robot, unequal wheel diameters are systematic errors. But, they are not systematic errors in case of flexible tires like passenger car. Radii of flexible tires are always varied by road conditions, rolling of vehicle and etc. So, it is important to measure the variations of tire radii for accurate positioning in odometry navigation of car-like vehicles.
Although odometry has several disadvantages, it is important positioning method. Improved odometry can reduce the cost for installations of vehicle systems because it simplifies the fundamental problem of position determination, and the improvement in accuracy of odometry could make high positioning accuracy and robustness by fusing other absolute positioning methods. In modern cars, breaking system is assisted with ABS systems that utilize angular encoders attached to the wheels. In this case, the sensors basically measure the wheel speeds and this measure can be use to estimate travel distances. So, extra encoders to measure wheel rotations are not needed.
This paper reduces such odometry problems with calibration of systematic errors and tire radii measurement. Experiments show the efficiency of consideration of tire radii variation.

## 2 Odometry Model for a Car

Consider a car-like vehicle. The mobile frame is chosen with its origin $P$ attached to the center of the rear axle. The $X$-axis is aligned with the longitudinal axis of the car. At time $t_{k}$, the vehicle position is represented by the $\left(x_{k}, y_{k}\right)$ Cartesian coordinates of $P$ in a world frame. The heading angle is denoted $\theta_{k}$.


Fig. 1 Elementary displacement between two samples
Let $P_{k}$ and $P_{k+1}$ be two successive positions. Supposing the road is perfectly planar and horizontal, as the motion is locally circular. (Fig.1)

$$
\begin{equation*}
\Delta P=R \cdot \Delta \theta \tag{1}
\end{equation*}
$$

where $\Delta P$ is the length of the circular arc followed by $P, \theta, R$ (the radius of curvature), I (the instantaneous center of rotation)

Supposing the car is moving forward, the variation on the position is expressed as:

$$
\begin{align*}
& \Delta x=\left|P_{k} P_{k+1}\right| \cdot \cos \left(\theta_{k}+\Delta \theta_{k} / 2\right) \\
& \Delta y=\left|P_{k} P_{k+1}\right| \cdot \sin \left(\theta_{k}+\Delta \theta_{k} / 2\right) \tag{2}
\end{align*}
$$

In general, the sampling rate of state is very small compared to their rate of change, so we can approximate $\Delta P \approx\left|P_{k} P_{k+1}\right|$. The integration process is then:

$$
\begin{align*}
& x_{k+1}=x_{k}+\Delta P \cdot \cos \left(\theta_{k}+\Delta \theta / 2\right) \\
& y_{k+1}=y_{k}+\Delta P \cdot \sin \left(\theta_{k}+\Delta \theta / 2\right) \\
& \theta_{k+1}=\theta_{k}+\Delta \theta \tag{3}
\end{align*}
$$

In Fig.1, the distance traveled, $\Delta P$, and the angle changed, $\Delta \theta$, resulting from the movement $P_{k+1}$ form $P_{k}$ can be calculated in terms of the incremental changes of the odometric measurements of the right and left wheel motions.

Let us $\mathrm{T}, \Delta P_{R R}$, and $\Delta P_{R L}$ denote the
wheelbase, the covered distances of the right and left rear tires respectively, and we assume that, between two samples, the wheels do not slip and that the distance $T$ is known and constant. Then

$$
\begin{align*}
& \Delta P_{R R}=(R+T / 2) \Delta \theta \\
& \Delta P_{R L}=(R-T / 2) \Delta \theta \\
& \Delta P=R \cdot \Delta \theta \tag{4}
\end{align*}
$$

Thus, we have

$$
\begin{align*}
& \Delta P=\left(\Delta P_{R R}+\Delta P_{R L}\right) / 2 \\
& \Delta \theta=\left(\Delta P_{R R}-\Delta P_{R L}\right) / T \tag{5}
\end{align*}
$$

Equation (5) shows that computation of odometry use left and right traveled distance of tires and wheelbase. That is, $\Delta P$ is the average of the right and left traveled distance of rear tires and $\Delta \theta$ is proportional to the difference of the right and left traveled distance of rear tires. Let $R_{R R}$ and $R_{R L}$ be right and left radius of rear tire of a car respectively, $\Delta P_{R R}$ and $\Delta P_{R L}$ can be expressed as:

$$
\begin{gather*}
\Delta P_{R R / R L}=C \cdot R_{R R / R L} \cdot N_{R R / R L}  \tag{6}\\
C=2 \pi / N_{E} \tag{7}
\end{gather*}
$$

where $C_{\text {RR/RL }}$ are conversion factor that translate encoder pulses into linear displacement of right and left rear tires. $N_{E}, N_{R R}$ and $N_{R L}$ denote encoder resolution, right and left incremental pulses of rear tires respectively.

## 3 Tire Radius Measurement Sensor

Tires are attached to axle and they have rotational mechanism, so it is more difficult problem to measure their radius. We suppose that distance between center of tire rotation and the ground which is contacted with tire is approximately same with actual tire radius. From this assumption, the Sharp GP2D12 is used in this paper. It is a short range infrared (IR) proximity sensor. Its output voltage is proportional to the distance between it and an object directly in front of it. It works well in a variety of lighting conditions.

### 3.1 GP2D12

GP2D12 use triangulation and a small linear CCD array to compute the distance and/or presence of objects in the field of view. The angles vary based on the distance to the object. This method of ranging is almost immune to interference from ambient light and offers amazing indifference to the color of object being detected. Detecting a black wall in full sunlight is possible. Characteristics of
the GP2D12 are listed below:

| - Output Type | Analog value $(0 \mathrm{~V}$ to $\sim 3 \mathrm{~V})$ based on distance <br> measured |
| :--- | :--- |
| - Range | $: 10 \mathrm{~cm}-80 \mathrm{~cm}$ |
| - Enable Method $:$ Continuous readings $\sim 38 \mathrm{~ms}$ per reading |  |



Fig. 2 Analog output voltage vs. distance to reflective object (GP2D12)

### 3.2 Non-linear Outputs and data fitting

Because of some basic trigonometry within the triangle from the emitter to reflection spot to receiver, the output of these detectors is non-linear with respect to the distance being measured. The Fig. 2 shows typical output from these detectors. The output of the detectors within the stated range (10 $\mathrm{cm}-80 \mathrm{~cm}$ ) is not linear but rather somewhat logarithmic. This curve will vary slightly from detector to detector so it is a good idea to fit the sensor outputs. In this way, we calibrate each detector and end up with polynomial data that is consistent from detector to detector. Moreover, amplifiers used because of small variation of sensor outputs near nominal tire radius (about 32 cm ).

### 3.3 Sensor implementation

Getting the best results of tire radius measurement with the GP2D12 will require some adjustment. It works best to mount the sensor vertically about 4 cm offset from the plane of the tire. Although the sensor has a very narrow beam-width, if the sensor is mounted too close to tire surface it may detect tire itself. Fig. 3 shows an implementation of wheel encoder and infrared range finder sensor (GP2D12). Wheel encoder is attached to the center of each tire rotation and a rod is used to prevent case for encoder from rotation. Vertical movement of the rod is supported for suspension system of a car. Infrared range finder is attached to encoder case and it detects the distance between rotation center of a wheel and the ground near a point of the tire contact.


Fig. 3 Implementation of wheel encoder and infrared range finder sensor

## 4 Calibration of Systematic Enor

Equation (5) and (6) show that computation of odometry use wheelbase, tire radii and counted pulse of each wheels. In this section, installation uncertainties of infrared range finders and effective wheelbase are considered to calibrate systematic errors.

In case of linear translation, odometry use only information of tires. Therefore, linear translation of specified distance can provide offset of infrared range finder from wheel center.

$$
\begin{equation*}
D=\sum \Delta P_{R R / R L}=\sum C \cdot R_{R R / R L} \cdot N_{R R / R L} \tag{8}
\end{equation*}
$$

where $D$ is specified distance of linear translation for a test car .
If we suppose slow translation, $R_{R R / R L}$ are constants.

$$
\begin{align*}
& D=\left(R_{\text {RR.OFF } / \text { RL.OFF }}+\operatorname{average}\left(R_{R R . S / R L . S}\right)\right) \cdot C \sum N_{R R / R L} \\
& R_{\text {RR.OFF/RL.OFF }}=\frac{D}{C \sum N_{R R / R L}}-\operatorname{average}\left(R_{\text {RR.S/RL.S }}\right) \tag{9}
\end{align*}
$$

where $R_{\text {Rr.off/RL.off }}$ and $R_{\text {RR.S/RL.S }}$ are offset of Infrared range finder from wheel center and measurements of tire radii respectively.

We drive the test car along a 10 m straight lane for 5 times. From equation (9), the average of $R_{\text {Rr.OFF / RL.OFF }}$ are calculated. If offsets of range finder sensors are calculated, effective wheelbase $T$ can be calculated form equation (5). The test car is driven along circular path (CW direction and CCW direction) and accumulated heading angle $\theta$ is compared with data of electrical compass.

$$
\begin{equation*}
T=\left(\sum\left(\Delta R_{R R}-\Delta R_{R L}\right)\right) / \theta_{\text {COMPASS }} \tag{10}
\end{equation*}
$$

where $\theta_{\text {COMPASS }}$ denotes heading angle of electrical compass. Initial heading angle of electrical compass is treated as zero degree.

Average of $T$ (for five CW/CCW direction test) is selected as an effective wheelbase.

## 5 Experimental Results



Fig. 4 A test Area
Fig. 4 shows a test area. The tested paths are shape of "8" which can test CW direction and CCW direction together (Fig.5). The car is run twice time for an each test. And other tested paths are also considered for a long distance (Fig.6). Tested road is asphaltic, but it is not flat.
Infrared range finder offset and the effective wheelbase are calculated from the results of section 4. Two rear wheels of the test car are used only. Two cases of results are compared. One case is that the effective wheelbase and initial tire radii (fixed tire radii) are used for odometry computation (dotted line of experimental results). Another case is that the effective wheelbase and unfixed tire radii are used for computation (solid line of experimental results).


Fig. 5 An experimental result (Case 1)
Form Fig. 5 and Fig.6, we can see that odometry computation using fixed tire radii has a weak point for roll motion of the car or uneven road surface. Experimental results using information of tire radii show more robust to those situations. The results are meaningful because the positions of the car using odometry are closed for closed reference path. Accumulative error can not be eliminated for odometry computation, but improving odometry accuracy is important for a long navigation which is not available for absolute positioning methods. In a
city (especially in a tunnel) or in the forest, many of GPS system can not be available.


Fig. 6 An experimental result (Case 2)

## 6 Conclusions

This paper presents improving accuracy of odometry for car-like vehicles. Systematic odometry errors are calibrated by straight and circular path navigation. Tire radii are measured by infrared range finders and they are used to reduce nonsystematic odometry errors. Experimental results show that calibration of systematic errors is not sufficient for vehicles which use flexible tires. In case of odometry navigation using information of tire radii, results show good performance without any other sensor fusion.

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