

Decision Method of Reference Input Time Interval and Sampling Time Interval that Considered Contour Control Performance in Software Servo System

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Abstract

In a software servo system, reference input time interval and sampling time interval are two important control parameters. In this paper, these two control parameters are adjusted to achieve high performance of contour control for circle drawing. The performance is analyzed through mathematical models concerning two critical control performance criteria, namely decrease in radius and surface roughness. Further, validity of the analytical equations is confirmed through experiments in an actual software servo system.

Keywords: contour control, linear interpolation, sampling time interval, reference input time interval, decrease in radius, surface roughness, average radius, padé approximation, software servo system

1 Introduction

From 1980's, mechatronics servo system has been changed from hardware servo system to software servo system. The change causes some problems in the contour control especially the following 2 points.

1. Input command became changed from the pulse allocation method of the hardware circuit to linear interpolation by the microcomputer computation. Therefore, small fluctuation appears in input command.
2. The reference input time interval and the sampling time interval of the control system become remarkably shorten by high performance of the microprocessor in recent years. Therefore, an input command wave changes largely according to the ratio of the reference input time interval and sampling time interval.

The control performance, i.e., the decrease in radius and surface roughness in circular have not been investigated. The above control performance is analyzed

mathematically in this paper. The appropriateness is also shown by a real machine experiment.

2 Mathematical Model of Software Servo System

Software servo system is modelled by the first order system including dead time, where the essence of the sampling control is considered[1]. The block diagram of software servo is shown in Fig. 1.

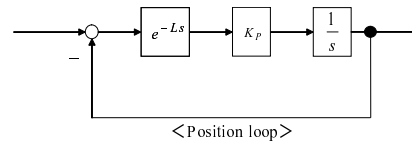


Figure 1: Block diagram of software servo system.

In Fig. 1, $K_p(s^{-1})$ shows the position loop gain and $L(s)$ shows the dead time. The total dead time is expressed by computation time delay and the delay of 0th order hold as $L = 1.5\Delta t_p$ [1].

where $\Delta t_p(s)$ is the sampling time interval of the servo system.

3 Relationships among Reference Input Time Interval, Sampling Time Interval and Shape Error

3.1 Relationship between time intervals and decrease in radius

3.1.1 Average radius

In case of the circular tracking approximated by the linear interpolation, command locus becomes the

polygon shape. Namely the circle becomes a polygon. Therefore the radius of an actual circle drawn by the software servo system must be considered. Then, the radius of the circle approximated by the polygon is investigated. An example of the linear interpolation is shown in Fig. 2. From the reference input time interval T (s) and the input angular frequency ω_i (rad/s) of circle drawing, division several N of the circle is calculated by eq.(1).

$$N = 2\pi / (\omega_i T) \quad (1)$$

Next, area S (m²) of a triangle inside the polygon is calculated by eq.(2).

$$S = r^2 \cos(\theta/2) \sin(\theta/2) \quad (2)$$

where θ is set as $360/N$. Therefore all the area S' (m²) of the polygon can be calculated by NS. By using the ratio between the area of the real circle and all the area of the polygon, the equivalent circular radius of the approximated polygon is obtained by eq.(3).

$$\bar{r} = r \sqrt{(1/\omega_i T) \sin \omega_i T} \quad (3)$$

Here, this radius \bar{r} (m) is defined as "average radius". From eq.(3), the radius of the circle which is approximated the polygon linear interpolation, is smaller than the radius of the real circle. The radius is fluctuated for the approximated polygon.

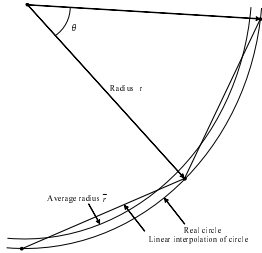


Figure 2: Concept of average radius.

3.1.2 Derivation of decrease in radius

The circular drawing by the interpolated straight line is considered for the control system of Fig. 1. In the circular drawing in radius r (m) and tangential speed v (m/s), the response of the servo system is the syntheses of 2 axis (X , Y axis) motion. The decrease in radius is obtained by the gain corresponding to the circular drawing speed in continuous system. The same idea is used for sampling control system.

The transfer function of the software servo system in Fig. 1 is given in eq.(4).

$$G(s) = \frac{K_p e^{-Ls}}{s + K_p e^{-Ls}} \quad (4)$$

It is not easy to be treated just as the eq.(4) because of the nonlinear term e^{-Ls} of the dead time. Therefore, e^{-Ls} is transformed into the (1,1) order padé approximation[2]. From the Padé approximation, the software servo system eq.(4) is approximated by eq.(5).

$$G(s) = \frac{K_p 2/L - s}{s^2 + (2/L - K_p)s + 2K_p/L} \quad (5)$$

From eq.(5), the gain $|G(j\omega_i)|$ of the frequency response function is eq.(6).

$$|G(j\omega_i)| = \sqrt{\frac{\omega_i^2 + (2K_p/L)^2}{(2K_p/L - \omega_i^2)^2 + ((2/L - K_p)\omega_i)^2}} \quad (6)$$

From the gain of eq.(6) and the average radius \bar{r} , the decrease in radius by delay of the servo system is obtained. Actual radius is reduced in the average radius from the radius of a real circle with that approximated the polygon. Therefore decrease in radius Δr of the circle by the interpolated straight line is derived by eq.(7).

$$\Delta r = r - \bar{r}|G(j\omega_i)| \quad (7)$$

From eq.(7), decrease in radius depends on the position loop gain K_p , reference input time interval T , sampling time interval Δt_p and input angular velocity ω_i .

3.2 Relationship between time intervals and the surface roughness

3.2.1 Consideration of input in linear interpolation

When input command is interpolated by the straight line in the case of circular drawing, the circle becomes a polygon approximation, and command causes by some fluctuation. Furthermore the reference input time interval is divided by the sampling time interval of the servo system. Shape of the input wave differs from the difference of the ratio of the sampling time interval and the reference input time interval, as shown in Fig. 3. Here, the ratio of the reference input time interval and the sampling time interval is set as $q(= T/\Delta t_p)$. Generally q equals to or greater than 1.

From Fig. 3, one piece of command of small fluctuation becomes the stairs shape with the sampling

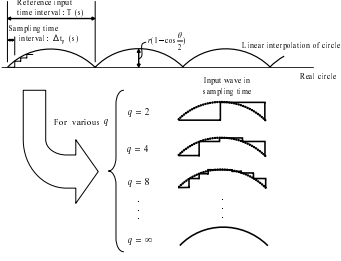


Figure 3: Concept of input wave by difference of q (=reference input time interval T /sampling time interval Δt_p).

time interval with the difference of q . DFT (Discrete Fourier Transform) is used for analyzing the wave of the stairs form. By using DFT, the main frequency component is obtained and the surface roughness is derived. DFT is expressed by eq.(8).

$$F(n) = \sum_{k=0}^{N-1} f(k)e^{-j\frac{2n\pi k}{N}} \quad (8)$$

Here $f(k)$ is the function of DFT derivation. Table 1 shows the DFT results for the input wave for various q from the eq.(8).

Table 1: DFT results of input wave for various q (q = (reference input time interval T /sampling time interval Δt_p), k = ordinary).

q \ k	1st	2nd	3rd	4th	5th
2	1.270	0	0.419	0	0.247
3	1.01	0.454	0	0.271	0
4	0.943	0.313	0.250	0	0
5	0.898	0.209	0.169	0.177	0
10	0.860	0.223	0	0	0
20	0.850	0.213	0	0	0
50	0.846	0.209	0	0	0

Table 1 shows the normalized amplitude between the real circle and string for each order of the frequency component where $r(1 - \cos(\theta/2))=1$. From Table 1, in the case that q is small, the higher order frequency components needs to be considered to represent the input wave. Zeros in Table 1 means the amplitude is small compared with 1 order and is able to be neglected.

3.2.2 Derivation of surface roughness

From the consideration of the previous paragraph, the input wave of the interpolated straight line is made by the base frequency as N times of input angular velocity ω_i . The input wave of higher harmonics is ex-

pressed with the basic wave. In the case that q is big, the basic wave and the 2nd order harmonics wave components must be considered. However, in the case that q is small ($q=5$ or less), the higher harmonics wave components to the 2 ~ 5th order must be considered. The conceptual figure of the derivation of surface roughness, is shown in Fig. 4.

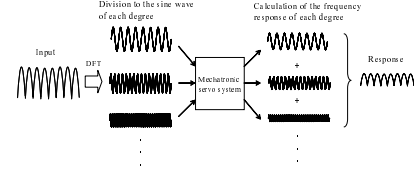


Figure 4: Consideration of surface roughness.

As shown in Fig. 4, the surface roughness is derived from principle of superposition. Surface roughness is calculated by the summation of gain for each frequency. However, the above study is considering only the gain. Therefore, the value corresponds to the maximum value of the quantity.

Gain $|G(j\omega_i N)|$ of the division number N of the circle is obtained by eq.(9).

$$|G(j\omega_i N)| = \sqrt{\frac{(\omega_i N)^2 + (2K_p/L)^2}{(2K_p/L - (\omega_i N)^2)^2 + ((2/L - K_p)\omega_i N)^2}} \quad (9)$$

When q changes, the higher harmonics order number of the input wave must be considered as shown in Table 1. Therefore, surface roughness changes $\omega_i N$ of the eq.(9) with 1,2... times and obtains the gain. Therefore, the surface roughness Δm in particular q is obtained the eq.(10).

$$\Delta m = r \sum_{k=1}^5 (1 - \cos(\omega_i T/2)) l_{q,k} |G(jk(\omega_i N))| \quad (10)$$

Here, $l_{q,k}$ is the coefficient in Table 1. Also, q shows the vertical axis and k is the horizontal axis of the Table 1. Also from the eq.(10), surface roughness changes by K_p , T , Δt_p and input angular velocity ω_i .

4 Inspection by Experiment

4.1 Setting of experimental device constitution and experimental conditions

Surface roughness is confirmed by a real machine experiment. The experiment system is the numerical

control unit consisted of SPX-8000 (techno Co., Ltd. Mfg.). As shown in Fig. 5 the experiment system is composed of ball screw systems of 2 axes (X-axis, Y axis) that used 2 units of AC servo motor and the servo controller and also the motion controller board (SPX-8000). The motion controller board gives command to the servo controller and PC (personal computer).

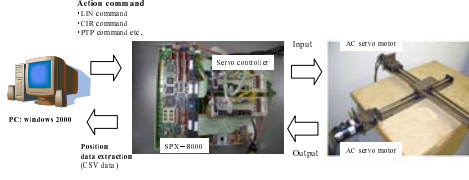


Figure 5: Schematics of experimental.

The experimental conditions are as follows. The condition of big fluctuation is chosen to check the influence (polygon approximation) by linear interpolation.

1. $\omega_i=1\text{rad/s}$, $r=50\text{mm}$, $\Delta t_p=2\text{ms}$, $T=260\text{ms}$, $q=130$, $N=24$
2. $\omega_i=1\text{rad/s}$, $r=50\text{mm}$, $\Delta t_p=2\text{ms}$, $T=130\text{ms}$, $q=65$, $N=48$

Furthermore, the position loop gains of the servo controllers of 2 axis are adjusted with $K_p=40\text{s}^{-1}$. Also AC servo motor and servo controller for the experiment is 100W (Yasukawa Electric Mfg).

4.2 Experimental results of decrease in radius and surface roughness

The size of decrease in radius and surface roughness are confirmed by a actual machine experiment. The experimental results are shown in Fig. 6. Here the left side of Fig. 6 is the input wave in the experiment. The wave on the right side of Fig. 6 is the error between the reduced synthesis locus of the position detection device of 2 motors from the real circle. In Fig. 6(a), the division number N of the circle is 24, the output wave is smooth and the same as an input wave. However, in Fig. 6(b) the division number N is 48 is distorted. This causes the low resolution of the position detection device of 2048pulse/rev.

The experimental results and the analyzed results of the decrease in radius and surface roughness are shown in Table 2. In the case of the division number of the circle 24, as for decrease in radius and surface roughness one of the analysis solution is bigger than the experimental results. This is caused that eq.(10) of

surface roughness are not considered the phase. Also the size of decrease in radius and surface roughness almost become the same, in the case of division several N of the circle 48. As for this the influence of the phase is almost disappearing, because the size of the input of originally is small with that the reference input time interval becomes short. Appropriateness of decrease in radius and surface roughness is confirmed from the results of Table 2.

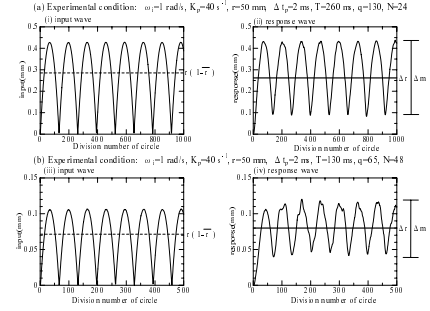


Figure 6: Experimental results.

Table 2: Comparison between analytical solutions and experimental results.

	Condition (a)		Condition (b)	
	Analysis	Experiment	Analysis	Experiment
Decrease in radius Δr (μm)	297	262	83	80
Surface roughness Δm (μm)	383	343	73	70

5 Conclusion

The analysis equations of the decrease in radius and surface roughness were derived, that becomes the control performance guideline of the circular drawing that was represented to the contour control. Appropriateness was confirmed by the experimental results.

References

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