Design of an Augmented Automatic Choosing Control with the Weighted Automatic Choosing Functions Using Hamiltonian and Genetic Algorithm

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Abstract

In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) for nonlinear systems with constrained inputs. It is designed by making use of the LQ controls and the weighted automatic choosing functions. Constant terms which arise from sectionwise linearization of a given nonlinear system are treated as coefficients of a stable zero dynamics. Parameters included in the control are suboptimally selected by minimizing the Hamiltonian with the aid of the genetic algorithm.

1 Introduction

A genetic algorithm (GA)[1] is one of evolutionary computing algorithms to carry out some designing problems in engineering. It has been used to solve such complicated tasks as nonlinear global optimization problems. The purpose of this paper is to present a nonlinear feedback control called AACC (Augmented automatic choosing control), which is designed by making good use of the GA.

Generally, it is easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[2]~[7]. One of most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion and the linear optimal control method to a given nonlinear system. This is only effective in a small region around the steady state point or in almost linear systems[2]~[4].

To overcome these weakness, the AACC is proposed for nonlinear systems with constrained inputs and its design procedure is as follows. Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls. These LQ controls are smoothly united by using the weighted automatic choosing functions of sigmoid type to synthesize a single nonlinear feedback controller.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, weights and gradients of the automatic choosing functions, and coefficients of the zero dynamics. These parameters must be selected optimally to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the GA suboptimally and successfully. The suboptimal values of these parameters are obtained by minimizing the Hamiltonian in this paper.

This approach is applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results show that the new controller using the GA can improve performance remarkably well.

2 Augmented Automatic Choosing Control

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbf{D} \tag{1}$$

subject to

$$u_{j,min} \le u[j] \le u_{j,max} \quad (j = 1, \cdots, r) \tag{2}$$

where $\cdot = d/dt$, $x = [x[1], \cdots, x[n]]^T$ is an n-dimensional state vector, $u = [u[1], \cdots, u[r]]^T$ is an r-dimensional control input vector, $f(x) : \mathbf{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x) : \mathbf{D} \to R^{n\times r}$ is a driving matrix with $g(0) \ne 0$ and is continuously differentiable, $u_{j,min}$: the minimum value of $u[j]$, $u_{j,max}$: the maximum value of $u[j]$, $\mathbf{D} \subset R^n$ is a domain , and T denotes transpose.

Considering the nonlinearity of the system (1), introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x[2]$ is the element which has the highest nonlinearity of (1), then

$$C(x) = x[2] \in D \subset R \quad (L = 1)$$

(see Section 4). The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \le i \le M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L}[a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.
Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \tag{3}$$

$$(x[n+1](0) \simeq 1, \quad 0 < \sigma_i < 1),$$

where the value of σ_i shall be selected so that $\sigma_i = -\dot{x}[n+1]/x[n+1] \le -\dot{x}[k]/x[k]$ holds for all $k(k = 1, \cdots, n)$. This tries to keep $x[n+1] \simeq 1$ for a good while when the system (1) is not on $C^{-1}(D_0)$.
Combine (1) with (3) to form an augmented system

$$\dot{\mathbf{X}} = \bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u \tag{4}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \quad \in \mathbf{D} \times R$$

$$\bar{f}(\mathbf{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \bar{g}(\mathbf{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

Let a cost function be

$$J = \frac{1}{2}\int_0^\infty \left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T\mathbf{R}u\right) dt \tag{5}$$

where

$$\mathbf{Q} \quad = \quad \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix}, \quad R \ni q > 0,$$

$Q = Q^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ which denote positive symmetrix matrices. Values of $\mathbf{Q}$ and $\mathbf{R}$ are properly determined based on engineering experience.
On each D_i, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$:

$$\begin{aligned} f(x) + g(x)u &\simeq A_i x + w_i + B_i u \\ &\simeq A_i x + w_i x[n+1] + B_i u \end{aligned} \tag{6}$$

where

$$\begin{aligned} A_i &= \partial f(x)/\partial x^T|_{x=\hat{X}_i}, \; B_i = g(\hat{X}_i), \\ w_0 &= 0, \; w_i = f(\hat{X}_i) - A_i\hat{X}_i. \end{aligned}$$

That is, an approximation of (4) is

$$\dot{\mathbf{X}} = \bar{A}_i\mathbf{X} + \bar{B}_i u \quad \text{on} \quad C^{-1}(D_i) \times R \tag{7}$$

where

$$\bar{A}_i \quad = \quad \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[3] to (5) and (7) yields

$$u_i(\mathbf{X}) = -\mathbf{R}^{-1}\bar{B}_i^T\mathbf{P}_i\mathbf{X} \tag{8}$$

where the $(n+1) \times (n+1)$ matrix $\mathbf{P}_i$ satisfies the Riccati equation :

$$\mathbf{P}_i\bar{A}_i + \bar{A}_i^T\mathbf{P}_i + \mathbf{Q} - \mathbf{P}_i\bar{B}_i\mathbf{R}^{-1}\bar{B}_i^T\mathbf{P}_i = 0. \tag{9}$$

Introduce an automatic choosing function of sigmoid type with weight d_i:

$$\begin{aligned} I_i(x) \quad &= \quad d_i \prod_{j=1}^{L} \left\{ 1 - \frac{1}{1 + \exp\left(2N\left(C_j(x) - a_{ij}\right)\right)} \right. \\ &\left. - \frac{1}{1 + \exp\left(-2N\left(C_j(x) - b_{ij}\right)\right)} \right\} \end{aligned} \tag{10}$$

where d_i and N are positive real values, $-\infty \le a_{ij}$ and $b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Figure 1).

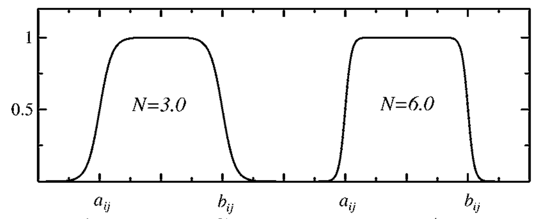

Figure 1: Automatic Choosing Function(N=3.0, 6.0) when $d_i = 1.0$

Uniting $\{u_i(\mathbf{X})\}$ of (8) with $\{I_i(x)\}$ of (10) yields

$$\begin{aligned} \hat{u}(\mathbf{X}) \quad &= \quad [\hat{u}(\mathbf{X})[1], \cdots, \hat{u}(\mathbf{X})[j], \cdots, \hat{u}(\mathbf{X})[r]]^T \\ &= \quad \sum_{i=0}^{M} u_i(\mathbf{X}) I_i(x). \end{aligned}$$

We finally have an augmented automatic choosing control which is satisfied with the constraint condition (2) by

$$u(\mathbf{X}) = [u(\mathbf{X})[1], \cdots, u(\mathbf{X})[j], \cdots, u(\mathbf{X})[r]]^T \tag{11}$$

where

$$u(\mathbf{X})[j] = \begin{cases} u_{j,max} & \text{if} \quad \hat{u}(\mathbf{X})[j] \ge u_{j,max} \\ u_{j,min} & \text{if} \quad \hat{u}(\mathbf{X})[j] \le u_{j,min} \\ \hat{u}(\mathbf{X})[j] & \text{otherwise} \end{cases}$$

$$(1 \le j \le r).$$

3 Parameter Selection by GA

The Hamiltonian for Eqs.(4) and (5) is given by

$$H(\mathbf{X},u,\lambda) = \frac{1}{2}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X}+u^T\mathbf{R}u\right) + \lambda^T\left(\bar{f}(\mathbf{X})+\bar{g}(\mathbf{X})u\right). \quad (12)$$

Assume that the adjoint vector $\lambda(\mathbf{X}) \in R^{n+1}$ is defined by

$$\lambda(\mathbf{X}) = [\lambda^I(\mathbf{X})^T, \lambda^{II}(\mathbf{X})^T]^T \quad (13)$$

where $\lambda^I(\mathbf{X}) = [\lambda[1],\cdots,\lambda[r]]^T = -(G^T(x))^{-1}\mathbf{R}u(\mathbf{X})$, $\lambda^{II}(\mathbf{X}) = [\lambda[r+1],\cdots,\lambda[n+1]]^T = [\mathbf{0},E]\hat{\lambda}$,

$$\hat{\lambda} = \sum_{i=0}^{M}\{(\bar{B}_i - \bar{g}(\mathbf{X}))\bar{g}(\mathbf{X})^\dagger + E\}^T\mathbf{P}_i\mathbf{X}I_i(x) \quad \in R^{n+1},$$

$\bar{g}(\mathbf{X})^\dagger\bar{g}(\mathbf{X}) = E$, E is an appropriate-dimentional unit matrix, and † denotes pseudo inverse.
The necessary condition of the optimality is $\partial H/\partial u = 0$ or $u = -\mathbf{R}^{-1}\bar{g}(\mathbf{X})^T\lambda = -\mathbf{R}^{-1}G^T(x)\lambda^I(\mathbf{X})$, which is satisfied with Eq.(11) from Eq.(13). By it, Eq.(12) becomes

$$H(\mathbf{X},u,\lambda) = \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u^T\mathbf{R}u + \bar{f}^T(\mathbf{X})\lambda. \quad (14)$$

Thus we can define a performance

$$PI = \int_{\mathbf{D}} |H(\mathbf{X},u,\lambda)|/\mathbf{X}^T\mathbf{X}d\mathbf{X}. \quad (15)$$

A set of parameters included in the control (11):

$$\bar{\Omega} = \left\{M, N, d_i, a_{ij}, b_{ij}, \hat{X}_i\right\}$$

is suboptimally selected by minimizing PI with the aid of GA[1] as follows.

<ALGORITHM>

step1:A-priori: Set values $\bar{\Omega}_{apriori}$ appropriately.
step2:Parameter: Choose a subset $\Omega \subset \bar{\Omega}$ to be improved and rewrite it by $\Omega = \{M, N, d_i \cdots\} = \{\alpha_k : k = 1, \cdot\cdot, K\}$.
step3:Coding: Represent each α_k with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.
step4:Initialization: Randomly generate an initial population of $\widetilde{q}$ strings $\{\Omega_p : p = 1, \cdot\cdot, \widetilde{q}\}$.
step5:Decoding: Decode each element α_k of Ω_p by $\alpha_k = (\alpha_{k,max} - \alpha_{k,min})\, A_k/\left(2^{\widetilde{L}} - 1\right) + \alpha_{k,min}$ where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and A_k:decimal value of α_k.
step6:Control: Design $u = u(\mathbf{X})_p \quad (p = 1, \cdots, q)$ for Ω_p by using Eq.(11).
step7:Adjoint: Make $\lambda = \lambda(\mathbf{X})_p \quad (p = 1, \cdots, q)$ for Ω_p by using Eq.(13).
step8:Fitness value calculation: Calculate

$$PI_p = \int_{\mathbf{D}}\Big|\frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u(\mathbf{X})_p^T Ru(\mathbf{X})_p + \bar{f}^T(\mathbf{X})\lambda(\mathbf{X})_p\Big|/\mathbf{X}^T\mathbf{X}d\mathbf{X} \quad (16)$$

by Eq.(15), or fitness $F_p = -PI_p$.
Integration of (16) is approximated by a finite sum.
step9:Reproduction: Reproduce each of individual strings with the probability of $F_p/\sum_{j=1}^{\widetilde{q}} F_j$.
step10:Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability P_c.
step11:Mutation: Alter a bit of string (0 or 1) by a mutation probability P_m.
step12:Repetition: Repeat step5∼step11 until prespecified G-th generation. If unsatisfied, go to step2.

As a result, we have a suboptimal control $u(\mathbf{X})$ for the string with the best performance over all the past generations.

4 Numerical Example

Consider a field excitation control problem of power system which is described[6][7] by

$$\widetilde{M}\frac{d^2\delta}{dt^2} + \widetilde{D}(\delta)\frac{d\delta}{dt} + P_e(\delta) = P_{in}$$
$$P_e(\delta) = E_I^2 Y_{11}\cos\theta_{11} + E_I\widetilde{V}Y_{12}\cos(\theta_{12}-\delta)$$
$$E_I + T'_{d0}\frac{dE'_q}{dt} = E_{fd}$$
$$E_I = E'_q + (X_d - X'_d)I_d(\delta)$$
$$I_d(\delta) = -E_I Y_{11}\sin\theta_{11} - \widetilde{V}Y_{12}\sin(\theta_{12}-\delta)$$
$$\widetilde{D}(\delta) = \widetilde{V}^2\left\{\frac{T''_{d0}(X'_d - X''_d)}{(X'_d + X_e)^2}\sin^2\delta + \frac{T''_{q0}(X_q - X''_q)}{(X_q + X_e)^2}\cos^2\delta\right\},$$

where δ: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, P_{in}: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, E_I: open circuit voltage, and E_{fd}: field excitation voltage. Put $x=[x[1],x[2],x[3]]^T=[E_I - \hat{E}_I, \delta - \hat{\delta}_0, \dot{\delta}]^T$ and $u = E_{fd} - \hat{E}_{fd}$, so that

$$\begin{bmatrix}\dot{x}[1]\\ \dot{x}[2]\\ \dot{x}[3]\end{bmatrix} = \begin{bmatrix}f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix} + \begin{bmatrix}g_1(x)\\ 0\\ 0\end{bmatrix}u \quad (17)$$

where

Table 1: Performances $\widetilde{J}$

Method	$x^T(0)$				
	[0, 0.4, 0]	[0, 0.6, 0]	[0, 1.2, 0]	[0, 1.35, 0]	[0, 1.39, 0]
LOC	0.954	×	×	×	×
AACC(d_i:fix)	1.121	2.065	3.155	2.769	×
AACC(d_i:GA)	1.364	2.478	2.723	2.165	2.496

× : very large value

$$
\begin{aligned}
f_1(x) &= -\frac{1}{kT'_{d0}}\left(x[1]+\hat{E}_I\right) \\
&\quad +\frac{(X_d-X'_d)\widetilde{V}Y_{12}}{k}x[3]\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right) \\
f_2(x) &= x[3] \\
f_3(x) &= -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right) \\
&\quad -\frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)^2-\frac{\widetilde{D}(x)}{\widetilde{M}}x[3]+\frac{P_{in}}{\widetilde{M}} \\
\widetilde{D}(x) &= \widetilde{V}^2\left\{\frac{T''_{d0}(X'_d-X''_d)}{(X'_d+X_e)^2}\sin^2\left(x[2]+\hat{\delta}_0\right)\right. \\
&\quad \left.+\frac{T''_{q0}(X_q-X''_q)}{(X_q+X_e)^2}\cos^2\left(x[2]+\hat{\delta}_0\right)\right\} \\
g_1(x) &= \frac{1}{kT'_{d0}},\quad k=1+\left(X_d-X'_d\right)Y_{11}\sin\theta_{11}.
\end{aligned}
$$

Assume that the constrained input is subject to

$$u_{min}+\hat{E}_{fd}\le E_{fd}\le u_{max}+\hat{E}_{fd}.$$

Parameters are $\widetilde{M}=0.016095[pu]$, $T'_{d0}=5.09907[sec]$, $\widetilde{V}=1.0[pu]$, $P_{in}=1.2[pu]$, $X_d=0.875[pu]$, $X'_d=0.422[pu]$, $Y_{11}=1.04276[pu]$, $Y_{12}=1.03084[pu]$, $\theta_{11}=-1.56495[pu]$, $\theta_{12}=1.56189[pu]$, $X_e=1.15[pu]$, $X''_d=0.238[pu]$, $X_q=0.6[pu]$, $X''_q=0.3[pu]$, $T''_{d0}=0.0299[pu]$, $T''_{q0}=0.02616[pu]$.

Steady state values are $\hat{E}_I=1.52243[pu]$, $\hat{\delta}_0=48.57°$, $\hat{\dot{\delta}}_0=0.0[deg/sec]$, $\hat{E}_{fd}=1.52243[pu]$. Set $\mathbf{X}=[x^T,x[4]]^T=[x[1],x[2],x[3],x[4]]^T$, $n=3$, $\hat{X}_0=\hat{\delta}_0=48.57°$, $C(x)$=$x[2]$, $L=1$, $\mathbf{Q}$=diag(1,1,1,1), $\mathbf{R}$=1, $d_0=1$ and $x[4](0)=1$. Experiments are carried out for the new control(AACC) and the ordinary linear optimal control(LOC)[2][3].

1) AACC(d_i:GA):

The parameters are suboptimally selected along the algorithm of section 3. $u_{max}=-u_{min}=0.5$. Ω=$\{M,N,d_i,a_{ij},b_{ij},\hat{X}_i\}$. $\widetilde{G}$=100, $\widetilde{q}$=100, $\widetilde{L}$=8, P_c=0.8, P_m=0.03, $\mathbf{D}$=[-1,1]×[-1,1.5]×[-5,5]×[0,1.5]. It results that $M=1$, $N=8.34$, $d_1=0.20$, $a=49.0°$, $\hat{X}_1=75°$.

2) AACC(d_i:fix):

The parameters are suboptimally selected by using a similar way of the **AACC**(d_i:GA) when the weight is fixed at $d_i=1(i=1,\cdots,M)$. Ω=$\{M,N,a_{ij},b_{ij},\hat{X}_i\}$. It results that $M=1$, $N=6.65$, $a=74.9°$, $\hat{X}_1=75°$.

Table1 shows performances by the AACC and the LOC. The cost function of Table1 is $\widetilde{J}=\frac{1}{2}\int_0^{20}\left(\mathbf{X}^T\mathbf{QX}+u^T\mathbf{R}u\right)dt$. These results indicate that the AACC is better than the LOC.

5 Conclusions

We have studied an augmented automatic choosing control using the weighted automatic choosing functions for nonlinear systems with constrained inputs. This approach have been applied to a field excitation control problem of power system. Simulation results have shown that this controller using the GA can improve performance remarkably well.

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