MQPSO Algorithm Based Fuzzy PID Control for a Pendubot System

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Abstract

In this study, a modified quantum-behaved particle swarm optimization (MQPSO) approach is proposed to design an optimal fuzzy PID controller for asymptotical stabilization of a pendubot system. In the fuzzy PID controller, parameters are determined by using MQPSO algorithm. The MQPSO method and other PSO methods are then applied to design an optimal fuzzy PID controller in a pendubot system. Comparing the simulation results, the feasibility of the MQPSO is verified.

Keywords: modified quantum-behaved particle swarm optimization, fuzzy PID controller, pendubot system

1. Introduction

Most industrial processes nowadays are still controlled by PID controllers. However, a conventional PID controller may have poor control performance for nonlinear and/or complex systems that have no precise mathematical models. In Keel et al[1] and Cervantes et al[2], how to tune the PID controller based on mathematical models is proposed, but complex mathematical computation is generally required in tuning procedures. For the methods in Whidborne and Istepanian[3] and Lin et al,[4] since the PID gains are fixed, the main disadvantage is that they usually lack in flexibility and capability.

Fuzzy controllers provide reasonable and effective alternatives for conventional controllers. Many researchers attempted to combine conventional PID controllers with fuzzy logic.[5] Despite the significant improvement of these fuzzy PID controllers over their classical counterparts, it should be noted that they still have disadvantages.

The PSO algorithm possesses the ability of high convergent speed, easily falling in some local optima is its fatal defect. Many researchers have presented revised PSO algorithms and obtained good results.[6] Another improvement on traditional PSO algorithm is quantum-behaved particle swarm optimization (QPSO).[7] However, in QPSO, particles fall into local optimal state in multimode optimization problems and cannot find any better state. To overcome the premature phenomenon in QPSO, a modified quantum-behaved particle swarm optimization (MQPSO) is proposed to find an optimal PID fuzzy controller. To show the flexibility and capability of the proposed method, an underactuated pendubot system is adopted as an illustrative example.

2. Fuzzy PID Controllers

In the proposed fuzzy PID controller, the input variables of the fuzzy rules are the error signals and their derivatives, while the output variables are the PID gains. Asymmetric Gaussian functions are used as the antecedent fuzzy sets in this paper. This means that input membership functions are represented as

$$X_k^{m_k}(x_k)=\begin{cases}\exp\left[-\left(\dfrac{x_k-\rho_k^{m_k}}{\sigma_{kl}^{m_k}}\right)^2\right] & if \quad x_k \le \rho_k^{m_k}\\ \exp\left[-\left(\dfrac{x_k-\rho_k^{m_k}}{\sigma_{kr}^{m_k}}\right)^2\right] & if \quad x_k > \rho_k^{m_k}\end{cases}$$

for $k=1,2,\ldots,4$,
$1\le m_1 \le n_1, 1\le m_2\le n_2, 1\le m_3\le n_3, 1\le m_4\le n_4$ (1)

where x_k represents the input linguistic variables, $\rho_k^{m_k}$, $\sigma_{kl}^{m_k}$, and $\sigma_{kr}^{m_k}$ denote the values of the centers, the left widths, and the right widths of the input membership functions, respectively. For the output membership functions, singleton sets are adopted. In the defuzzification process, Wang used the center of gravity method to determine the output crisp values.[8] Then, if the PID control law is used, the control signal is determined as

$$\begin{aligned}u(t) &= k_{p1}e_1(t)+k_{I1}\int e_1(t)dt + k_{D1}\dot{e}_1(t)\\ &+ k_{p2}e_2(t)+k_{I2}\int e_2(t)dt + k_{D2}\dot{e}_2(t)\end{aligned} \quad (2)$$

From the above description, one can find that the gains of the fuzzy PID controller are adaptive such that the controller should have more flexibility and capability than the conventional ones. The MQPSO method is proposed to search for the optimal values of these parameters simultaneously.

3. Modified Quantum Particle Swarm Optimization

From the view of classical dynamics, to avoid explosion and guarantee convergence, particles must be bounded and fly in an attractive potential field. Clerc and Kennedy[6] have proved that if these coefficients are properly defined, the particle's position p_i will converge to the center of potential field, $pf^c=[pf_1^c, pf_2^c,\cdots,pf_n^c]$, and is defined as:

$$pf_i^c=\frac{(c_1\cdot r_1\cdot p_i^l + c_2\cdot r_2\cdot p^g)}{(c_1\cdot r_1 + c_2\cdot r_2)},\ i=1,2,\cdots,n. \quad (3)$$

where p_i^l and p^g are the best position of the ith particle and the global best position; c_1 and c_2 are cognitive and social constriction coefficients, respectively; r_1 and r_2 are random numbers between 0 and 1.

Inspired by the behavior that particles move in a bounded state and preserve the global search ability, Sun et al.[9] proposed the QPSO algorithm. In the QPSO model, the solution of time-independent Schrödinger equation for this system in one dimensional space can be expressed as:[10]

$$p_i = pf_i^c \pm \frac{L}{2}\cdot \ln\left(\frac{1}{\lambda}\right), \quad (4)$$

where λ is a random number uniformly distributed on [0, 1] and L is the characteristic length of delta potential well (called "Creativity" of particles) which specifies the search scope of a particle. The mainstream thought point and can be expressed as the following forms:[8]

$$mbest=\left[\sum_{i=1}^{n}\frac{p_{i,1}}{n},\sum_{i=1}^{n}\frac{p_{i,2}}{n},\cdots,\sum_{i=1}^{n}\frac{p_{i,n}}{n}\right],\ i=1,2,\cdots,n, \quad (5)$$

$$L=2\cdot\beta\left|mbest-p_i\right| , \quad (6)$$

The creative coefficient β with adaptive annealing learning mechanism according to the change rate of optimal estimation has the form:

$$\beta=\beta_{\max}-\Delta\beta\cdot(\Delta fit)^{\gamma} , \quad (7)$$

$$\Delta fit=\left|p^g-p_i^l\right|, \quad (8)$$

where $\Delta\beta$ is step length of β, Δfit is the change rate of optimal estimation so far. The mechanism of adaptive annealing learning can overcome the stagnation problem to accelerate the convergent speed.

4. An Simulation Example

4.1. *The Pendubot System*

The general dynamic model of underactuated mechanisms with m actuated joints from a total of n joint can be expressed as follows (Spong and Vidyasagar) :[11]

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\ \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})=\boldsymbol{\tau} \quad (9)$$

where $\boldsymbol{q}\in R^n$ is the position vector indicating link angles, $\boldsymbol{M}(\boldsymbol{q})$ denotes the inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\ \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\in R^n$ is the vector of damping, coriolis, and centrifugal torques, $\boldsymbol{G}(\boldsymbol{q})\in R^n$ represents the gravitational term and $\boldsymbol{\tau}\in R^n$ is the vector of control torque which has $(n-m)$ zero components.

For the pendubot system in Fig. 1, let m_1 and m_2 denote the distributed mass of the actuated link (called link 1) and the unactuated link (called link 2), respectively. Meanwhile, let q_1 and q_2 denote the angles of the two links, l_1 and l_2 denote the lengths of the two links, and l_{1c} and l_{2c} denote the distances to the center of masses, respectively.

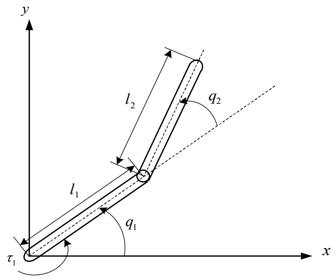

Fig. 1. Dynamics of the pendubot system

When the configuration is at equilibrium state; that is, pendubot balances at a state $\dot{q} = 0$ and $\ddot{q} = 0$, the following can be derived from (10).

$$(m_1 g l_{c1} + m_2 g l_1)\cos q_1 + m_2 g l_{c2} \cos(q_1 + q_2) = \tau_1 \qquad (10)$$

$$m_2 g l_{c2} \cos(q_1 + q_2) = 0 \qquad (11)$$

4.2. MQPSO tuning Fuzzy PID Controller

In the pendubot system, the desired value of $q_1(t)$ and $q_2(t)$ are denoted by q_{1d} and q_{2d}. If the PID control law is employed, then the input-output relation of the pendubot system is expressed as

$$\begin{aligned}\tau(t) &= k_{P1}e_1(t) + k_{I1}\int e_1(t)dt + k_{D1}\dot{e}_1(t) \\ &+ k_{P2}e_2(t) + k_{I2}\int e_2(t)dt + k_{D2}\dot{e}_2(t)\end{aligned} \qquad (12)$$

where $e_1(t) = q_{1d} - q_1(t)$, $e_2(t) = q_{2d} - q_2(t)$, $\dot{e}_1(t) = \dot{q}_{1d} - \dot{q}_1(t)$, and $\dot{e}_2(t) = \dot{q}_{2d} - \dot{q}_2(t)$.

4.3. *Fitness*

In designing the fuzzy PID controller, the primary goal is to drive a pendubot system from the given initial state to the desired final state. The performance criteria can be included in the fitness as follows:

$$f = \int t\left[e_1^2(t) + e_2^2(t)\right]dt \qquad (13)$$

From the definition (14), the fitness value can be calculated to evaluate the performance of the fuzzy PID controller and a lower fitness value denotes a better performance.

5. Simulation Results

The parameters of the pendubot system shown in Fig. 1 are chosen as $m_1 = 2.0$kg, $m_2 = 1.5$kg, $l_1 = 0.3m$, $l_2 = 0.5m$, $l_{1c} = 0.15m$, $l_{2c} = 0.25m$, and $g = 9.8$ m/s^2. The initial state and the desired final state of the pendubot system are $[q_1, \dot{q}_1, q_2, \dot{q}_2] = [-\pi/2,\ 0,\ 0,\ 0]$ and $[q_1, \dot{q}_1, q_2, \dot{q}_2] = [\pi/2,\ 0,\ 0,\ 0]$. Meanwhile, the input torque $\tau(t)$ of the motor is assumed to be within the range $[-10$ Nm, 10 Nm$]$.

In the proposed algorithm, the population size, the maximal iteration number, the crossover rate, and mutation rate are chosen to be 20, 10000, 0.8, and 0.2, respectively. Moreover, it is assumed that the values of the singletons of the output linguistic variables are all chosen as real numbers in the range $[-10, 10]$.

In PID tuning techniques, we proposed MQPSO method to design a fuzzy PID controller to asymptotically stabilize the pendubot and maintain the equilibrium state over all control processes. From Figs. 2 and 3, the illustration results demonstrate the proposed MQPSO fuzzy PID controller has good performance for asymptotical stabilization of the pendubot system.

6. Conclusions

In PID tuning techniques, the PID gains are difficult to obtain the optimal values for stabilizing a pendubot system. In this paper, we proposed MQPSO to design a fuzzy PID controller to asymptotically stabilize the pendubot and maintain the equilibrium state over all control processes. From the simulation results, one demonstrates the proposed fuzzy PID controller has good performance for asymptotical stabilization of the pendubot system.

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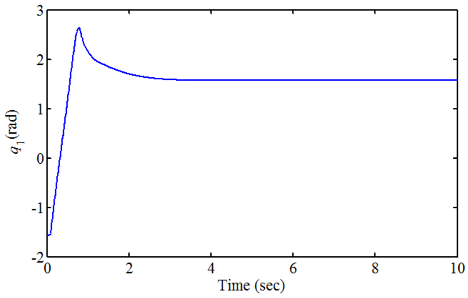

Fig. 2. Plot of angle $q_1(t)$ of the pendubot system.

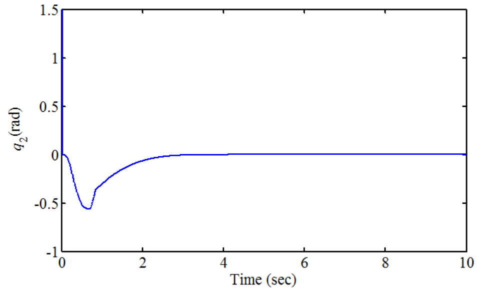

Fig. 3. Plot of angle $q_2(t)$ of the pendubot system.