Histogram Matching Based on Gaussian Distribution Using Regression Analysis
Variance Estimation

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Abstract

This paper describes an improvement method for variance estimation which is used in Histogram Matching based on Gaussian Distribution (HMGD). In the previous papers, based on curvature computation, we have described that how to estimate the variance of reference histogram, which is used in HMGD processing. However, we have considered that the histogram of original image is not always ideal shape. And the variance estimation method based on curvature computation might not have high reliability. In this paper, we propose improvement variance estimation method using regression analysis. As for the method, first, we detect the histogram peak of original image by using curvature computation; next, we perform regression analysis using approximation formula of curvature. Then, we illustrate processing results through some experimentation.

Keywords: Image processing, Curvature, Variance estimation, Histogram matching, HMGD

1. Introduction

These days, automated image processing for enhancement of color images has been more familiar to us, for example, Digital Signage, Smart Phone, etc.\(^1\)\(^-\)\(^3\)\.

In the previous paper, we have presented that the Histogram Matching based on Gaussian Distribution (HMGD) processing is one of the automated image arrangement method using Elastic Transformation\(^4\)\(^-\)\(^5\) based on the brightness axis. And through the comparative investigation, we have illustrated that HMGD processing could improve feeling (or Kansei) impression better than original image\(^6\). And we have aimed to improve HMGD processing, we have proposed that how to estimate the variance of reference histogram, which is used in HMGD processing based on curvature computation, and also illustrated these results.

However, we have considered that the histogram of original image is not always ideal shape (i.e., Gaussian distribution, etc). And the variance estimation method
based on curvature computation might not have high reliability. In this paper, we propose how to perform regression analysis using approximation formula of curvature.

2. Principle

2.1. Histogram Matching based on Gaussian Distribution (HMGD)

In the section, we describe the principle of HMGD processing.

Fig. 1 shows the conceptual image of HMGD. Let \( f(x) \) and \( h(y) \) be two probabilistic density functions (PDF) on real variables \( x \) and \( y \), respectively. The PDF is corresponding to histogram of image brightness level which is discretely defined.

In addition, let \( y = \phi(x) \) be a continuous and monotonic increase function corresponding to cumulative histogram of image brightness level between variables \( x \) and \( y \). And let \( y = \phi(x) \) be defined by Eq. (1).

\[
y = \phi(x) = \int_0^x f(x)dx.
\]  

(1)

At first, we have to expand brightness level of original image histogram and convert into uniform distribution histogram, because we aim to match Gaussian distribution. From Eq. (1) and Fig. 1, we can derive Eq. (2) and (3).

\[
f(x) = h(y)\phi'(x) = h(y)\phi(x) \]  

(2)

\[
h(y) = \frac{1}{L} \]  

(3)

We understand the histogram of original image \( f(x) \) becomes uniform distribution \( h(y) \) by Eq. (3). This means that brightness level of original image \( f(x) \) is expanded to \( h(y) \).

Then, let \( \text{Gauss}(z) \) and \( \gamma(z) \) be the function that is defined by Eq. (4) and (5), respectively.

\[
\text{Gauss}(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}
\]  

(4)

\[
y = \gamma(z) = \int_0^z \text{Gauss}(z)dz = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)dz.
\]  

(5)

Here, Fig.1 shows the relationship between \( y = \phi(x) \) and \( y = \gamma(z) \). So we can be obtained following Eq. (6).

\[
L \int_0^z f(x)dx = \int_0^z \text{Gauss}(z)dz = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)dz.
\]  

(6)

And we can derive Eq. (7) from differential of Eq. (6).

\[
\frac{d}{dx} \int_0^z f(x)dx = \frac{d}{dz} \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)dz.
\]  

(7)

If we perform Eq. (7),

\[
L \phi'(x) = L \gamma'(z), \quad f(x) = \text{Gauss}(z).
\]  

(8)

That is, we understand that \( f(x) \) becomes Gaussian distribution \( \text{Gauss}(z) \) when we take the transform function as (1) and (5). Thus, HMGD processing is the function which defined by cumulative histogram transformation the original histogram into Gaussian histogram\(^6\).

![Fig. 1. Conceptual image of HMGD processing\(^6\).](image-url)
2.2. Peak Detection of Histogram

The HMGD processing need to calculate transforms function for brightness peak of histogram. And the solution to detect it is curvature computation of the histogram.

Let \( y \) be a function with respect to \( x \), the definition curvature \( R(x) \) is given by Eq. (9).\(^9\)

\[
R(x) = \left( \frac{d^2y}{dx^2} \right)^2 + \left( \frac{dy}{dx} \right)^2
\]

(9)

Let \( g(x) \) and \( K \) be Gaussian distribution function and a coefficient which is defined by following equation, respectively.

\[
g(x) = \frac{K}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{(x-a)^2}{2\sigma^2} \right)
\]

(10)

Next, let \( y=g(x) \) be a function representing the cumulative histogram which is represented Eq. (11). That is, \( dy/dx \) and \( d^2y/dx^2 \) be described as Eq. (12) and (13), respectively. From Eq. (12) and (13), we obtain the approximation of curvature \( R(x) \) as Eq. (14).

\[
f(x) = \int_{0}^{x} g(u)du = \frac{K}{\sqrt{2\pi\sigma^2}} \int_{0}^{x} \exp \left( -\frac{(u-a)^2}{2\sigma^2} \right) du.
\]

(11)

\[
\frac{dy}{dx} = g(x) = \frac{K}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{(u-a)^2}{2\sigma^2} \right)
\]

(12)

\[
\frac{d^2y}{dx^2} = \frac{dg(x)}{dx} = \frac{(a-x)}{\sigma^2} g(x)
\]

(13)

\[
R(x) = \frac{(a-x)}{\sigma} g(x) \left( 1 + \left( \frac{dy}{dx} \right)^2 \right)^{1/2}
\]

(14)

From Eq. (14), we understand that the curvature \( R(x) \) varies the sign according to the value of \( x \).\(^9\) That is, if \( x<a \rightarrow R>0 \) (downward convex shape), and if \( x<a \rightarrow R<0 \) (upward convex shape).

2.3. Variance Estimation

In this section, we propose how to estimate the variance of original-image histogram.

Fig. 2 shows the conceptual image of the original image histogram which is variance \( \sigma^2 \) and average \( a \). From Eq. (14), we can describe \( R(x) \) following Eq. (15).

\[
R(x) \approx \frac{(a-x)}{\sigma^2} g(x) = \frac{1}{\sigma^2} (a-x) g(x)
\]

(15)

And, let \( C=1/\sigma^2 \) and \( H(x) = (a-x)g(x) \), respectively, we can derive Eq. (16).

\[
R(x) \approx CH(x)
\]

(16)

Now, we can calculate a constant \( C \) by using least-square regression analysis\(^10\) following Eq. (17).

\[
R_i = CH_i + \epsilon_i,
\]

\[
\epsilon_i \sim N(0, \sigma^2) \quad (i = 1, \ldots, n)
\]

(17)

That is, we evaluate \( \sigma^2 \) as follows;

\[
C = \frac{1}{\sum_{i=1}^{n} (H_i R_i)}, \quad \sigma^2 = \frac{1}{C}
\]

(18)

Fig. 2. Conceptual image of the original image histogram.
3. Experimentation

Fig. 3 shows the example of results and the corresponding histogram for original image and HMGD image which applied variance estimation using regression analysis.

In this case, we understand that HMGD image is reducing contrast than original image. And in the HMGD image, the color tones become unnatural.

4. Conclusion

In this paper we proposed we have described that how to estimate the variance of reference histogram, which is used in HMGD processing by using regression analysis. As for the method, first we have detected the brightness peak on the original-image histogram. Then we have performed regression analysis based on the cumulative histogram and its curvature value.

Through the result of experimentation, the variance estimation based on the regression analysis image is reducing contrast than original one, and its color tones become unnatural. That is, we consider that we have to further improvement of this method.

References


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